

12:40 - 1:02

ECE 3040 Microelectronic Circuits

Exam 1

September 23, 2009

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Print your name clearly and largely:

Solutions

Instructions:

Read all the problems carefully and thoroughly before you begin working. You are allowed to use 1 sheet of notes (1 page front and back) as well as a calculator. There are 100 total points. Observe the point value of each problem and allocate your time accordingly. **SHOW ALL WORK AND CIRCLE YOUR FINAL ANSWER WITH THE PROPER UNITS INDICATED.** Write legibly. If I cannot read it, it will be considered a wrong answer. Do all work on the paper provided. Turn in all scratch paper, even if it did not lead to an answer. Report any and all ethics violations to the instructor. Good luck!

Sign your name on **ONE** of the two following cases:

I DID NOT observe any ethical violations during this exam:

I observed an ethical violation during this exam:

First 33% Multiple Choice and True/False
(Circle the letter of the most correct answer or answers)

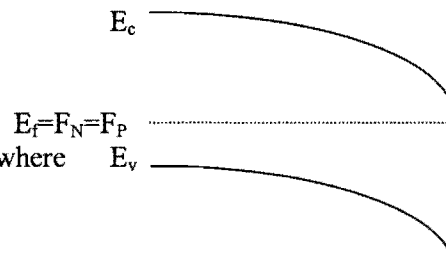
- 1.) (3-points) True or False: Due to their completely disordered arrangement of atoms, amorphous materials are only useful for things like insulators and are never used for semiconductor devices .
- 2.) (3-points) True or False: Atoms with large chemical bond strengths usually result in small energy bandgaps.
- 3.) (3-points) True or False: $f(E=E_f)$ is the fermi distribution function evaluated at the Fermi energy. It indicates that at all temperatures greater than 0 Kelvin, an equal likelihood of having filled and empty states provided there are actually states at that energy.
- 4.) (3-points) True or False: $In_{0.33}Ga_{0.44}N_{0.23}$ is a valid semiconductor formula in standard semiconductor notation.
- 5.) (3-points) True or False: The law of mass action describes the interplay of electrons being created at the expense of holes (and vice versa) but is only valid in equilibrium.
- 6.) (3-points) True or False: The density of states predicts more states at energies far away from the energy bandgap edge ($E=E_g$).
- 7.) (3-points) True or False: Auger recombination involves three particles like two electrons and one hole and results in an annihilated hole, annihilated electron and an electron with higher kinetic energy.

Select the **best** answer or answers for 8-10:

- 8.) (4-points) Partial Ionization ...
 - a.) ... always predicts approximately the same results as total ionization.
 - b.) ... predicts approximately the same results as total ionization for low doping cases
 - c.) ... still requires charge neutrality to be maintained.
 - d.) ... predicts more free carriers if the dopant energies are close to the band edges.
 - e.) ... should never be covered at Ga Tech because the math is too hard!

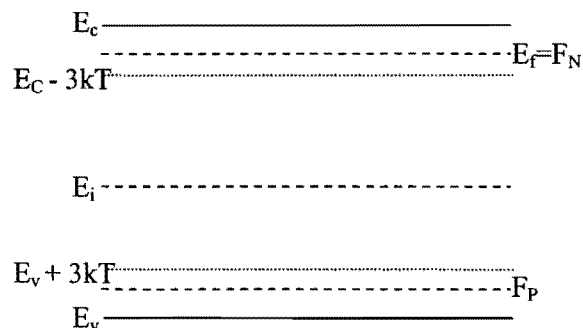
- 9.) (4-points) Select all of the following that are true.

- a.) The material is in equilibrium
- b.) The material is in steady state non-equilibrium
- c.) The material has non-uniform doping
- d.) The material has uniform doping
- e.) The material has a non-zero electric field everywhere
- f.) The material has zero electric field everywhere
- g.) The material clearly has a net current flow
- h.) The material clearly has no net current flow



- 10.) (4-points) The following energy band diagram indicates the material is:

- a.) In equilibrium
- b.) Degenerate and n-type
- c.) Degenerate and p-type
- d.) Non-degenerate n-type
- e.) Non-degenerate p-type
- f.) In low level injection
- g.) In high level injection
- h.) In steady state



Second 17% Short Answer (“Plug and Chug”):

For the following problems (11-12) use the following material parameters and assuming total ionization:

$n_i = 1 \times 10^{14} \text{ cm}^{-3}$ $N_D = 2 \times 10^{16} \text{ cm}^{-3}$ donors $N_A = 1 \times 10^{16} \text{ cm}^{-3}$ acceptors $m_p^* = 0.55m_0$ $m_n^* = 0.36m_0$
 $E_G = 0.66 \text{ eV}$ Electron mobility, $\mu_n = 2200 \text{ cm}^2/\text{Vsec}$ Hole mobility, $\mu_p = 500 \text{ cm}^2/\text{Vsec}$
 Temperature = 27 degrees C

11.) (7-points) Where is the fermi energy (relative to the valence band which is referenced to zero energy)?

Note: Since I did not use a n_i that is correct (i.e. does not match the values of m_p^* and m_n^* there are actually 3 acceptable answers instead of 1 depending on the choice of equations. Regardless of equation used, you MUST recognize that there are both acceptors and donors present so you must use the equation:

$$n = \frac{N^+D - N^-A}{2} + \sqrt{\left(\frac{N^+D - N^-A}{2}\right)^2 + n_i^2} = 1 \times 10^{16} \text{ cm}^{-3}$$

Then the law of mass action:

$$p = \frac{n_i^2}{n} = 1 \times 10^{12} \text{ cm}^{-3}$$

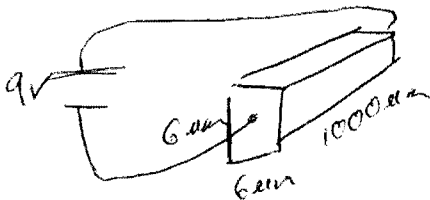
Noting that $N_v = 2.51 \times 10^{19} (0.55m_0/m_0)^{3/2} = 1.02 \times 10^{19} \text{ cm}^{-3}$ or $N_c = 2.51 \times 10^{19} (0.36m_0/m_0)^{3/2} = 5.4 \times 10^{18} \text{ cm}^{-3}$ at 300 K.

| Problem as posed | $n_i =$ | $1.00 \times 10^{14} \text{ cm}^{-3}$ |
|--|--|---|
| $E_i = \frac{E_c + E_v}{2} + \frac{3kT}{4} \ln\left(\frac{m_p^*}{m_n^*}\right)$ $E_i = 0.338233 \text{ eV}$ | $n = n_i e^{(E_f - E_i)/kT}$ $E_f = 0.4575 \text{ eV}$ | $p = n_i e^{(E_i - E_f)/kT}$ $E_f = 0.4575 \text{ eV}$ |
| | $n = N_c e^{(E_f - E_c)/kT}$ $E_f - E_c = -0.16305 \text{ eV}$ $E_f = 0.496948 \text{ eV}$ | $p = N_v e^{(E_v - E_f)/kT}$ $E_f - E_v = 0.4181 \text{ eV}$ |

If I had used the correct $n_i = 2.8 \times 10^{13} \text{ cm}^{-3}$, then there would only be one answer.

| | | | |
|--|--|--|--|
| Problem with Correct n_i | | $n_i = \sqrt{N_c N_v} e^{-E_G / 2kT}$ $n_i = 2.18 \times 10^{13} \text{ cm}^{-3}$ | |
| $E_i = \frac{E_c + E_v}{2} + \frac{3kT}{4} \ln\left(\frac{m_p^*}{m_n^*}\right)$ $E_i = 0.338233 \text{ eV}$ | $n = n_i e^{(E_f - E_i) / kT}$ $E_f = 0.4969 \text{ eV}$ | $p = n_i e^{(E_i - E_f) / kT}$ $E_f = 0.4969 \text{ eV}$ | |
| | $n = N_c e^{(E_f - E_c) / kT}$ $E_f - E_c = -0.16305 \text{ eV}$ $E_f = 0.496945 \text{ eV}$ | $p = N_v e^{(E_v - E_f) / kT}$ $E_f - E_v = 0.4969 \text{ eV}$ | |

- 12.) (10-points) A $6 \mu\text{m} \times 6 \mu\text{m} \times 1 \text{mm}$ rectangular semiconductor resistor is made from the semiconductor from problem 11. It is biased on two opposing sides (longest dimension) with 9 volts. Determine both the electron and hole current density and currents flowing in the device.



$$A = (6 \times 10^{-4})^2 = 3.6 \times 10^{-7} \text{ cm}^2$$

$$L = 0.1 \text{ cm}$$

Electrons:

$$J_n = \sigma_n [E] = q \mu_n [9V / 0.1 \text{ cm}]$$

$$= (1.6 \times 10^{-19}) (2200 \text{ cm}^2 / \text{V-s}) (1 \times 10^{16} \text{ cm}^{-3}) (90 \text{ V/cm})$$

$$J_n = 316.8 \text{ A/cm}^2 \quad \text{or } I = JA$$

$$I_n = 114 \mu\text{A}$$

Holes:

$$J_p = \sigma_p [E] = 1.6 \times 10^{-19} (500) (1 \times 10^{16}) (90)$$

$$J_p = 0.0072 \text{ A/cm}^2 \quad (7.2 \mu\text{A/cm}^2)$$

$$I_p = 2.59 \text{ nA}$$

13.) (17-points total) The material in problems 11 ~~(10)~~ is exposed to a laser light that generates a steady state $2 \times 10^{16} \text{ cm}^{-3}$ extra minority carriers.

a) (5 points) Is this low level or high level injection?

$$\Delta n = \Delta p = 2 \times 10^{16} \text{ cm}^{-3} > n_0 \Rightarrow \text{High level injection}$$

b) (12 points) Draw the 1 dimensional energy band diagram showing the placement of both the quasi-fermi levels (numeric answer).

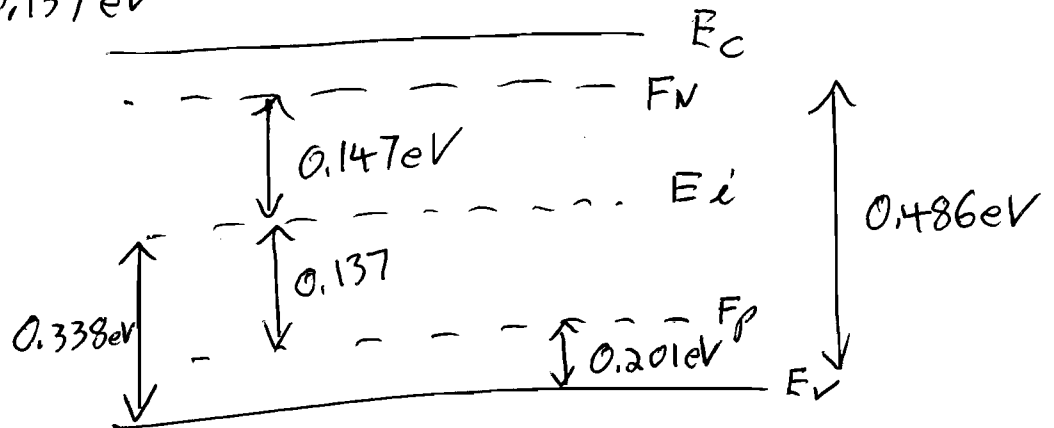
$$n = 1 \times 10^{16} \text{ cm}^{-3} + 2 \times 10^{16} \text{ cm}^{-3} = 1 \times 10^{17} \text{ cm}^{-3}$$

$$\Rightarrow F_n - E_i = \frac{1}{2} kT \ln \left(\frac{1 \times 10^{16} + 2 \times 10^{16}}{1 \times 10^{17}} \right) = 0.1477 \text{ eV}$$

$$E_i - F_p = \frac{1}{2} kT \ln \left(\frac{1 \times 10^{12} + 2 \times 10^{16}}{1 \times 10^{14}} \right) = 0.137 \text{ eV}$$

$$n = n_i e^{(F_n - E_i)/kT}$$

$$p = n_i e^{(E_i - F_p)/kT}$$



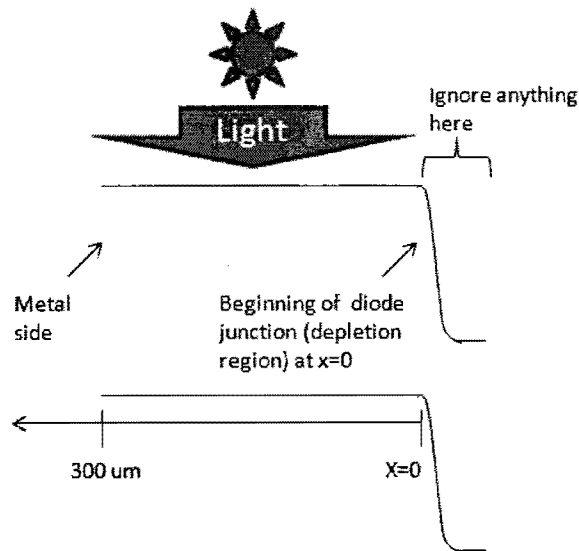
$$E_i = \frac{E_g}{2} + \frac{3}{4} kT \ln \left(\frac{m_p^*}{m_n^*} \right) = \frac{0.66 \text{ eV}}{2} + \frac{3}{4} (0.0259 \text{ eV}) \ln \left(\frac{0.53}{0.36} \right)$$

$$= 0.338 \text{ eV}$$

Pulling all the concepts together for a useful purpose:

14.) (33-points)

NOTE: Do not panic! I am not asking you to analyze an entire diode, only one side (one "quasi neutral region") which is simply a slab of semiconductor material just like all other problems described in class/old exams/homework.



A 300 um length of silicon (the length of the region you are asked to analyze, ignoring all other layers) is to be used as a solar cell at Clemson University. The silicon is doped p-type with an acceptor concentration of $1 \times 10^{15} \text{ cm}^{-3}$ and has a minority carrier lifetime, of 200×10^{-6} seconds. The sun has been shining uniformly on the entire semiconductor for days and the light is uniformly absorbed, generating 5×10^{17} extra electron-hole pairs per cm^3 per second. One side of the device, $x=0$ (the junction side), has an excess electron concentration of $8.7 \times 10^{13} \text{ cm}^{-3}$. Unfortunately, the Clemson student puts metal directly on the other end of the semiconductor (not a good design). Since only majority carriers are passed by an ohmic metal contact (resistor), all excess minority carriers recombine at this point ($x=300 \text{ um}$) making the excess minority carrier concentration 0 cm^{-3} at the metal end. If the semiconductor is held at room temperature (27 degrees C), determine the minority carrier diffusion current density at all positions in the semiconductor ($300 \text{ um} \geq x \geq 0 \text{ um}$). Assume a minority carrier mobility of $100 \text{ cm}^2/\text{Vsec}$ and the intrinsic concentration is $1 \times 10^{10} \text{ cm}^{-3}$.

Steady State
 $\frac{d\Delta n_p}{dx} = 0$

$\Delta n_p(x=0) = 1 \times 10^{14} \text{ cm}^{-3}$

One side of the device, $x=0$ (the junction side), has an excess electron concentration of $8.7 \times 10^{13} \text{ cm}^{-3}$. Unfortunately, the Clemson student puts metal directly on the other end of the semiconductor (not a good design). Since only majority carriers are passed by an ohmic metal contact (resistor), all excess minority carriers recombine at this point ($x=300 \text{ um}$) making the excess minority carrier concentration 0 cm^{-3} at the metal end. If the semiconductor is held at room temperature (27 degrees C), determine the minority carrier diffusion current density at all positions in the semiconductor ($300 \text{ um} \geq x \geq 0 \text{ um}$). Assume a minority carrier mobility of $100 \text{ cm}^2/\text{Vsec}$ and the intrinsic concentration is $1 \times 10^{10} \text{ cm}^{-3}$.

correct in student version so all options are on same page

Given: $0 = D_n \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n}$ General Solution is: $\Delta n_p(x) = Ae^{-x/L_n} + Be^{+x/L_n}$

Given: $0 = D_n \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n} + G_L$ General Solution is: $\Delta n_p(x) = Ae^{-x/L_n} + Be^{+x/L_n} + G_L \tau_n$

Given: $0 = D_n \frac{d^2 \Delta n_p}{dx^2}$ General Solution is: $\Delta n_p(x) = A + Bx$

Given: $0 = D_n \frac{d^2 \Delta n_p}{dx^2} + G_L$ General Solution is: $\Delta n_p(x) = Ax^2 + Bx + C$

Given: $0 = D_n \frac{d^2 \Delta n_p}{dx^2} + G_{LO} f(x)$ General Solution is: $\Delta n_p(x) = \left[-\frac{G_{LO}}{D_n} \iint f(x) dx \right] + Bx + C$

Given: $\frac{d\Delta n_p}{dt} = -\frac{\Delta n_p}{\tau_n}$ General Solution is: $\Delta n_p(t) = \Delta n_p(t=0) e^{-t/\tau_n}$

Extra work can be done here, but clearly indicate which problem you are solving.

Boundary Conditions:

Junction 1) $\Delta n_p(x=0) = A + B + G_L \tau_n$

$$10^{14} \text{ cm}^{-3} = A + B + 10^{14} \text{ cm}^{-3}$$

$$A = -B$$

Metal 2) $\Delta n_p(x=300 \mu\text{m}) = 0$

$$0 = A e^{-300/227.6} + B e^{+300/227.6} + 1e^{14}$$

$$A (e^{-300/227.6} - e^{+300/227.6}) = -1e^{14}$$

$$-B = A = 2.883e^{13} \text{ cm}^{-3}$$

$$\Delta n_p(x) = 2.883e^{13} [\text{cm}^{-3}] \left(e^{-x/227.6 \mu\text{m}} - e^{x/227.6 \mu\text{m}} \right)$$

$$J_n = q D_n \frac{d(\Delta n_p(x))}{dx}$$

$$= \frac{q D_n}{L_n} 2.883e^{13} \left(-e^{-x/227.6 \mu\text{m}} + e^{x/227.6 \mu\text{m}} \right)$$

$$J_n = [-0.525 \text{ mA/cm}^2] \left[e^{-x/227.6 \mu\text{m}} + e^{x/227.6 \mu\text{m}} \right]$$

$$L_n = \sqrt{D_n \tau_n}$$

$$= \sqrt{\mu_n \left(\frac{kT}{q} \right) \tau_n}$$

$$= \sqrt{(100 \text{ cm}^2/\text{V}\cdot\text{s}) (0.0259 \text{ V}) \dots}$$

$$\dots \sqrt{200e^{-6}}$$

$$L_n = 0.02276 \text{ cm or } 227.6 \mu\text{m}$$

$$D_n = \mu_n \left(\frac{kT}{q} \right)$$

$$= (100 \text{ cm}^2/\text{V}\cdot\text{s}) (0.0259 \text{ V})$$

$$D_n = 2.59 \text{ cm}^2/\text{sec}$$