ECE 3040 Microelectronic Circuits

Exam 1

February 17, 2014

Dr. W. Alan Doolittle

Print your name clearly and largely: SOLUTIONS

Instructions:
Read all the problems carefully and thoroughly before you begin working. You are allowed to use 1 sheet of notes (1 page front and back) as well as a calculator. There are 100 total points. Observe the point value of each problem and allocate your time accordingly. SHOW ALL WORK AND CIRCLE YOUR FINAL ANSWER WITH THE PROPER UNITS INDICATED. Write legibly. If I cannot read it, it will be considered a wrong answer. Do all work on the paper provided. Turn in all scratch paper, even if it did not lead to an answer. Report any and all ethics violations to the instructor. A periodic table is supplied on the last page. Good luck!

Sign your name on **<u>ONE</u>** of the two following cases:

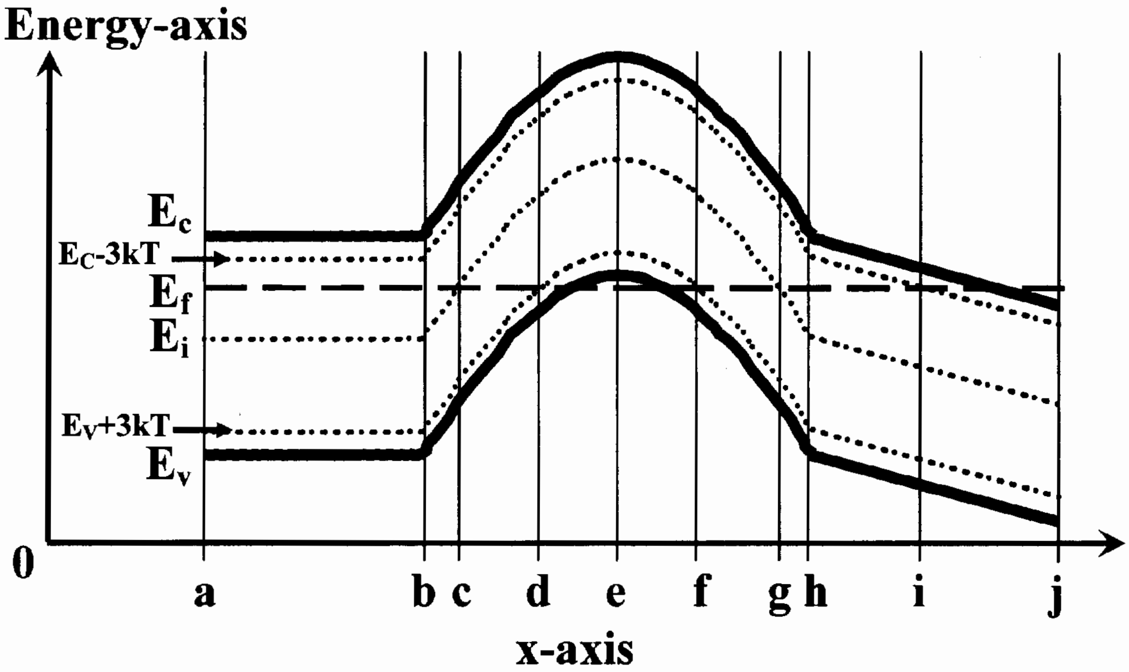
I DID NOT observe any ethical violations during this exam:

I observed an ethical violation during this exam:

True/False
(Circle the letter of the most correct *answer or answers*)

1.) (2-points) [✓ True] or False: The energy bandgap results from hybridization (intermixing) of s and p orbitals and defines allowed and disallowed energies for electrons in a semiconductor or insulator.
2.) (2-points) [✓ True] or False: Atoms with weak (small) chemical bond strengths usually result in smaller energy bandgaps and large mobility (due to infrequent collisions resulting from low atomic density).
3.) (2-points) [✓ True] or False: The fermi distribution function defines whether a state will be likely to be empty or filled.
4.) (2-points) True or [✓ False]: $In_{0.3}Ga_{0.7}N_2$ is a valid semiconductor formula in standard semiconductor notation.
5.) (2-points) [✓ True] or False: The density of states describes the number of allowed states at a given energy and far away (in energy) from the band edges has a decreasing exponential form.
6.) (2-points) True or [✓ False]: To determine the hole concentration, one must multiply the density of states by the Fermi distribution function and integrate that product over all energies.

Short Answer
7.) (22 points total)
Write in the blanks (on the next page) the answer to the following questions using the information below:
Given a semiconductor at room temperature (27 degrees C) has the following parameters and energy band diagram:
Hole Diffusion coefficient, D_p=11.86 cm^2/Sec
Electron Diffusion coefficient, D_n=33.625 cm^2/Sec
Substrate intrinsic concentration, n_i=1e10 cm^{-3}



Example Answer: " region c<x<e, region h<x<I and point f"

a.) (4-points) Which region(s) contain nonzero electric fields?

$b < x < e$ and $e < x < j$

b.) (4-points) Which region(s) are n-type? $a \le x \le c$ and $g \le x \le j$

c.) (4-points) Which region(s) are degenerate? $d \le x \le f$ and $i \le x \le j$

d.) (4-points) Which region(s) has zero total (sum) current? $a < x < j$ Fermi energy level is flat throughout

e.) (6-points) If the slope of E_C for the region h<x<j is 1000 eV/cm what is the electron drift velocity at point i ? (Hint: Be careful of your units.)

$$\mathcal{E} = \frac{1}{q}\frac{dE_c}{dx} \quad (E_c \text{ is in units of Joules})$$

$$= \left(\frac{1}{1.6 \cdot 10^{-19}\,C}\right)\left(1000\,\frac{eV}{cm}\right)\left(1.6 \cdot 10^{-19}\,\frac{J}{eV}\right)$$

$$= 1000\,\frac{J}{C \cdot cm} = 1000\ V/cm$$

$$v_{dn} = \mu_n \mathcal{E}$$

$$= \frac{D_N}{kT/q}\,\mathcal{E}$$

$$= \left(\frac{33.625\ cm^2/s}{0.0259\ V}\right)\left(1000\ V/cm\right)$$

$$\boxed{v_{dn} = 1.30 \cdot 10^{6}\ cm/s}$$

Short Answer ("Plug and Chug"):
For the following problems (8-10) use the following material parameters and assuming total ionization:

n_i=2e6 cm^{-3} N_D=2e15 cm^{-3} donors N_A=1e15 cm^{-3} acceptors m_p^*=0.55m$_o$ m_n^*=0.36m$_o$
E_G=1.45 eV Electron mobility, μ_n = 2200 cm^2/Vsec Hole mobility, μ_p = 500 cm^2/V-sec
Temperature=27 degrees C

8) (7-points) Where is the Fermi-energy (relative to the valence band which is referenced to zero energy)?

$$n = \frac{N_D - N_A}{2} + \left[\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2\right]^{1/2}$$

$$= \frac{2\cdot10^{15} - 1\cdot10^{15}}{2} + \left[\left(\frac{2\cdot10^{15} - 1\cdot10^{15}}{2}\right)^2 + (2\cdot10^6)^2\right]^{1/2} = 1\cdot10^{15}\ cm^{-3}$$

$$p = \frac{n_i^2}{n} = \frac{(2\cdot10^6\ cm^{-3})^2}{1\cdot10^{15}\ cm^{-3}} = 4\cdot10^{-3}\ cm^{-3}$$

$$p = N_V e^{(E_V - E_F)/kT} \rightarrow E_F = E_V - kT\ln\left(\frac{p}{N_V}\right)$$

$$N_V = (2.51\cdot10^{19}\ cm^{-3})(0.55)^{3/2} = 1\cdot10^{19}\ cm^{-3}$$

$$E_F = (0\ eV) - (0.0259\ eV)\ln\left(\frac{4\cdot10^{-3}\ cm^{-3}}{1\cdot10^{19}\ cm^{-3}}\right) = 1.28\ eV$$

$$\boxed{E_F = 1.28\ eV}$$

There are several alternative solutions

9) (10-points) A 10 um x 10 um x 500 um rectangular semiconductor resistor is made from the semiconductor from problem 8. It is biased on two opposing sides (longest dimension) with 9 volts. Determine both the electron and hole current density and currents flowing in the device.

Recall: $n = 1\cdot10^{15}\ cm^{-3}$, $p = 4\cdot10^{-3}\ cm^{-3}$

$$\mathcal{E} = \frac{9\ V}{0.05\ cm} = 180\ \frac{V}{cm}$$

$$J_N = q\mu_n n\mathcal{E}$$

$$= (1.6\cdot10^{-19}\ C)(2200\ \tfrac{cm^2}{V\cdot s})(1\cdot10^{15}\ cm^{-3})(180\ \tfrac{V}{cm})$$

$$\boxed{J_N = 63.36\ A/cm^2}$$

$$I_N = AJ_N = (10^{-6}\ cm^2)(63.36\ A/cm^2)$$

$$\boxed{I_N = 63.36\ \mu A}$$

$$J_p = q\mu_p p\mathcal{E} = (1.6\cdot10^{-19}\ C)(500\ \tfrac{cm^2}{V\cdot s})(4\cdot10^{-3}\ cm^{-3})(180\ V/cm)$$

$$\boxed{J_p = 5.76\cdot10^{-17}\ A/cm^2}$$

$$I_p = AJ_p = (10^{-6}\ cm^2)(5.76\cdot10^{-17}\ A/cm^2)$$

$$\boxed{I_p = 5.76\cdot10^{-23}\ A}$$

Section 3 (more short answer)

10) (10-points total) The material in problems 8 and 9 is exposed to a laser light that generates 2e16 cm^{-3} extra minority carriers.

a) (2 points) Is this low level or high level injection?

$$\Delta n = \Delta p = 2\cdot 10^{16}\ cm^{-3}$$

$$n_0 = 1\cdot 10^{15}\ cm^{-3}$$

$$\Delta n > n_0 \longrightarrow \boxed{\text{high level injection}}$$

b) (8 points) Draw the 1 dimensional energy band diagram showing the placement of both the quasi-fermi levels (numeric answer) relative to E_i, E_c, and E_v.

$$n = n_0 + \Delta n = 10^{15}\ cm^{-3} + 2\cdot 10^{16}\ cm^{-3} = 2.1\cdot 10^{16}\ cm^{-3}$$

$$p = p_0 + \Delta p = 4\cdot 10^{-3}\ cm^{-3} + 2\cdot 10^{16}\ cm^{-3} \approx 2\cdot 10^{16}\ cm^{-3}$$

$$n = n_i e^{(F_N - E_i)/kT} \longrightarrow F_N = E_i + kT\ln\left(\frac{n}{n_i}\right)$$

$$p = n_i e^{(E_i - F_P)/kT} \longrightarrow F_P = E_i - kT\ln\left(\frac{p}{n_i}\right)$$

$$E_i = \frac{E_G}{2} + \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right) = \frac{1.45\ eV}{2} + \frac{3}{4}(0.0259\ eV)\ln\left(\frac{0.55}{0.36}\right)$$

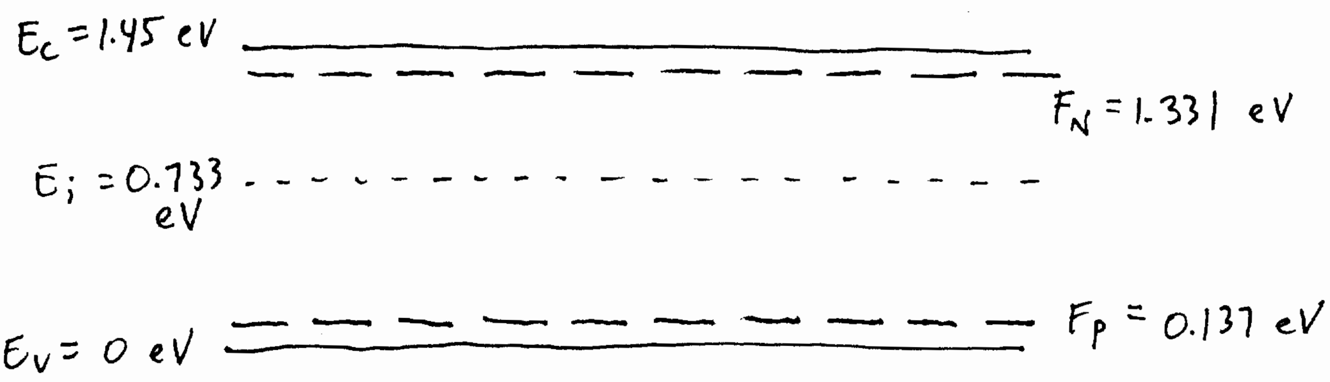
$$= 0.733\ eV$$

$$F_N = 0.733\ eV + (0.0259\ eV)\ln\left(\frac{2.1\cdot 10^{16}\ cm^{-3}}{2\cdot 10^{6}\ cm^{-3}}\right)$$

$$\boxed{F_N = 1.331\ eV}$$

$$F_P = 0.733\ eV - (0.0259\ eV)\ln\left(\frac{2\cdot 10^{16}\ cm^{-3}}{2\cdot 10^{6}\ cm^{-3}}\right)$$

$$\boxed{F_P = 0.137\ eV}$$



Pulling all the concepts together for a useful purpose:

11.) (39-points)

steady state

A semi-infinite length of semiconductor in a device flies on a satellite launched in 1966 and has been exposed to constant cosmic radiation and sunlight. The light is absorbed uniformly in the semiconductor. Over time, the radiation damages the front surface. This problem deals with the performance after being exposed to the radiation for a very long time (i.e. not the initial pristine condition). The semiconductor is doped p-type with an acceptor concentration of 1e15 cm^{-3} and has a minority carrier lifetime, of 400 picoseconds (400e-12 seconds). Photons absorbed in the semiconductor generate 1e19 extra electron-hole pairs per cm^3 per second. It is found that at the surface exposed to the radiation, x=0, the excess electron concentration is smaller than in the rest of the device, $\Delta n(x=0)=2e9\text{cm}^{-3}$. If the semiconductor is held at room temperature (27 degrees C), determine the minority carrier diffusion current density at all positions in the semiconductor ($x \geq 0$ um). Assume a minority carrier mobility of 100 cm^2/Vsec, 27 degree C operation and the intrinsic concentration is 1e6cm^{-3}.

Given: $0 = D_n \frac{d^2\Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n}$ General Solution is: $\Delta n_p(x) = Ae^{-x/L_n} + Be^{+x/L_n}$

this eqn applies for $x \neq \infty$

$$\boxed{\text{Given: } 0 = D_n \frac{d^2\Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n} + G_L \quad \text{General Solution is: } \Delta n_p(x) = Ae^{-x/L_n} + Be^{+x/L_n} + G_L\tau_n}$$

Given: $0 = D_n \frac{d^2\Delta n_p}{dx^2}$ General Solution is: $\Delta n_p(x) = A + Bx$

Given: $0 = D_n \frac{d^2\Delta n_p}{dx^2} + G_L$ General Solution is: $\Delta n_p(x) = Ax^2 + Bx + C$

Given: $0 = D_n \frac{d^2\Delta n_p}{dx^2} + G_{LO} f(x)$ General Solution is: $\Delta n_p(x) = \left[-\frac{G_{LO}}{D_N}\iint f(x)dx\right] + Bx + C$

Given: $\frac{d\Delta n_p}{dt} = -\frac{\Delta n_p}{\tau_n}$ General Solution is: $\Delta n_p(t) = \Delta n_p(t=0)e^{-t/\tau_n}$

this eqn applies for $x = \infty$

$$\boxed{\text{Given: } 0 = -\frac{\Delta n_p}{\tau_n} + G_L \quad \text{General Solution is: } \Delta n_p = G_L\tau_n}$$

steady state: $\frac{\partial \Delta n_p}{\partial t} = 0$ (no change with time)

there is a concentration gradient near the surface of the device, inferred from $\Delta n_p(0) \neq G_L\tau_n$.

However, at depths further from the surface, the concentration gradient decreases

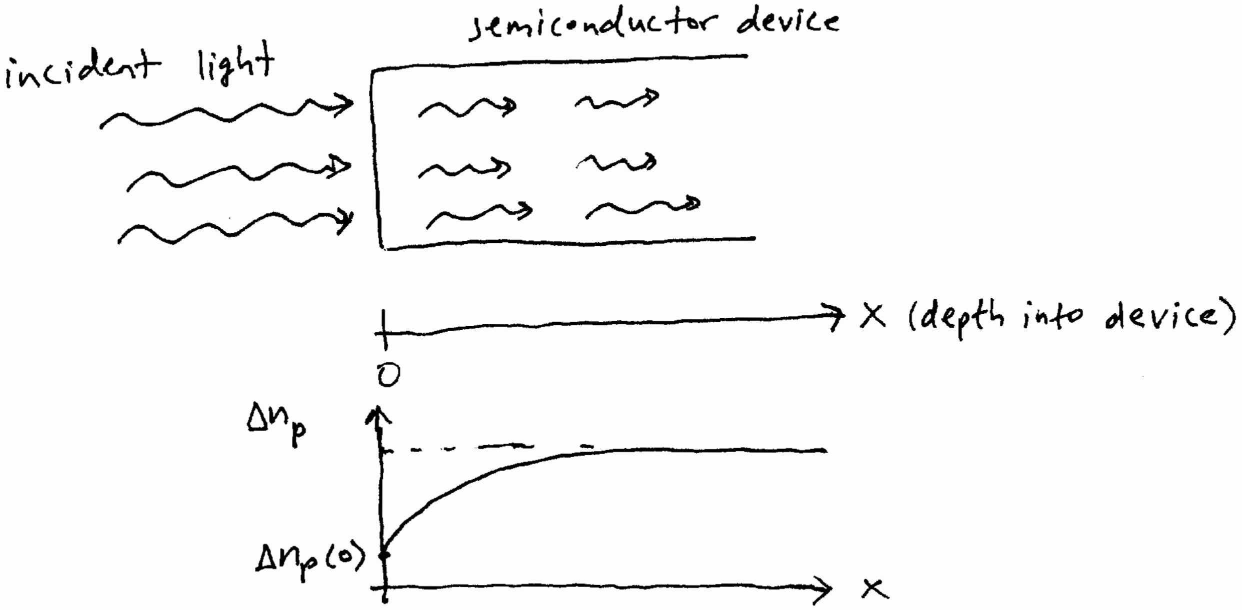
$$\Delta n_p(\infty) = G_L\tau_n$$

$$\left.\frac{\partial^2 \Delta n_p}{\partial x^2}\right|_{x=\infty} = 0$$

Problem 11 cont'd

Extra work can be done here, but clearly indicate which problem you are solving.

Helpful diagram



Boundary conditions:

$\Delta n_p(0) = 2\cdot10^{9}\ \text{cm}^{-3}$

$\Delta n_p(\infty) = G_L\tau_n = (10^{19}\ \frac{\text{cm}^{-3}}{s})(4\cdot10^{-10}\ s)$

$\Delta n_p(\infty) = 4\cdot10^{9}\ \text{cm}^{-3}$

General solution:

$$0 = D_N \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L$$

$$\Delta n_p(x) = A e^{-x/L_N} + B e^{+x/L_N} + G_L\tau_n$$

apply B.C.s

$$\Delta n_p(0) = A + B + 4\cdot10^{9}\ \text{cm}^{-3} = 2\cdot10^{9}\ \text{cm}^{-3}$$

$$A + B = -2\cdot10^{9}\ \text{cm}^{-3}$$

$$\Delta n_p(\infty) = 0 + B(\infty) + 4\cdot10^{9}\ \text{cm}^{-3} = 4\cdot10^{9}\ \text{cm}^{-3}$$

$$B = 0$$

$$A = -2\cdot10^{9}\ \text{cm}^{-3}$$

Problem 11 cont'd

Extra work can be done here, but clearly indicate which problem you are solving.

$$D_N = kT/q \cdot \mu_n = (0.0259\ V)(100\ cm^2/Vs) = 2.59\ cm^2/s$$

$$L_N = \sqrt{D_N \tau_n} = \sqrt{(2.59\ cm^2/s)(4 \cdot 10^{-10}\ s)} = 3.22 \cdot 10^{-5}\ cm$$

$$= 0.322\ \mu m$$

$$\Delta n_p(x) = 4 \cdot 10^{9}\ cm^{-3} - (2 \cdot 10^{9}\ cm^{-3}) e^{-x/0.322\ \mu m}$$

$$J_N = q D_N \nabla n = q D_N \frac{d \Delta n_p}{dx}$$

$$= - \frac{(1.6 \cdot 10^{-19}\ C)(2.59\ cm^2/s)(-2 \cdot 10^{9}\ cm^{-3})}{3.22 \cdot 10^{-5}\ cm} e^{-x/0.322\ \mu m}$$

$$\boxed{J_N = 25.74\, e^{-x/0.322\ \mu m} \quad \mu A/cm^2}$$