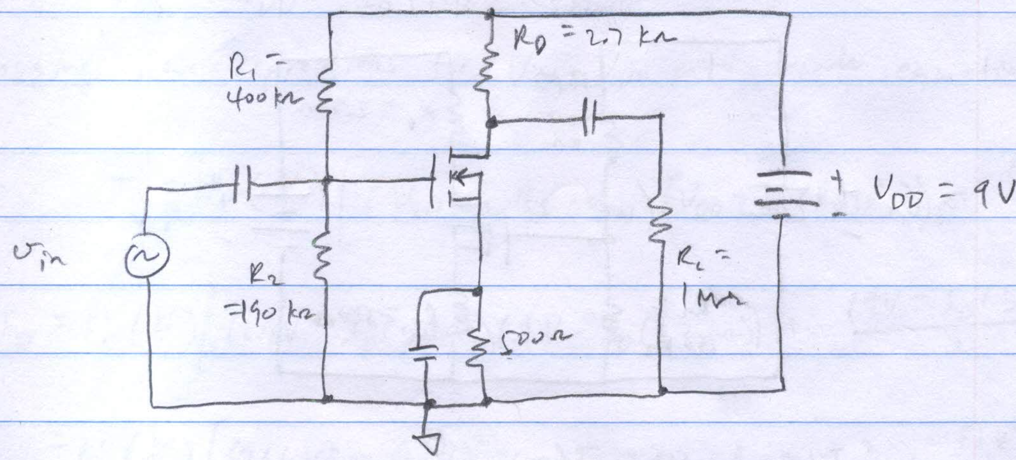
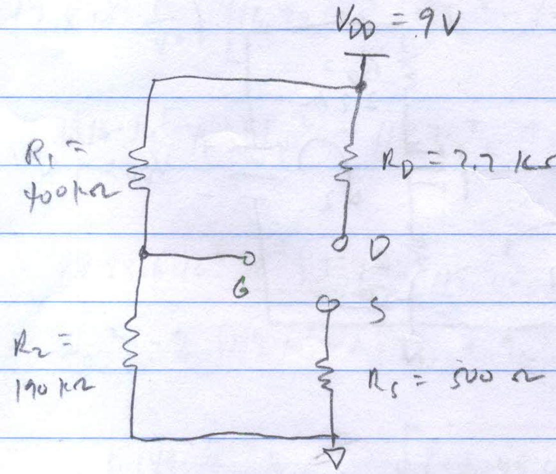
ECE 3040 HW 7

1. Assume $V_{TN} = 0.5\,V$, $K'_n = \mu_n C_{ox} = 100\ \frac{\mu A}{V^2}$, $\frac{W}{L} = 28$



a) assume transistor is in cutoff, find Q-pt.



$I_D = 0$

$V_D = 9V$

$V_G = (9V)\left(\frac{190\,k\Omega}{400\,k\Omega + 190\,k\Omega}\right) = 2.90\,V$

$V_S = 0\,V$

$V_{DS} = 9V$

$V_{GS} = 2.90\,V$

Q-pt $= (I_D, V_{DS})$

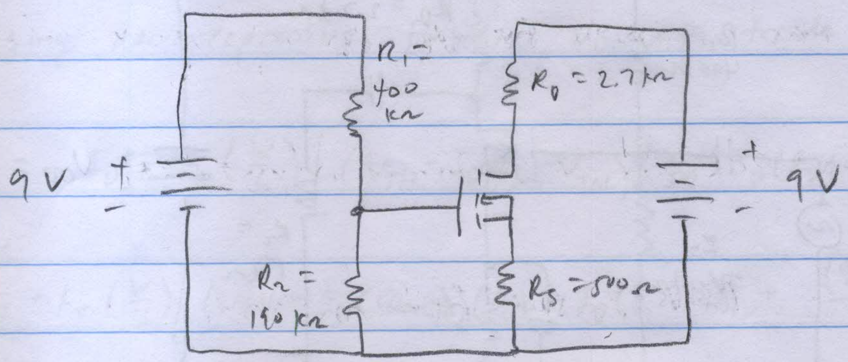
Q-pt $= (0\,A, 9\,V)$

$V_{DS} > V_{GS} - V_T$

$9\,V > 2.4\,V$

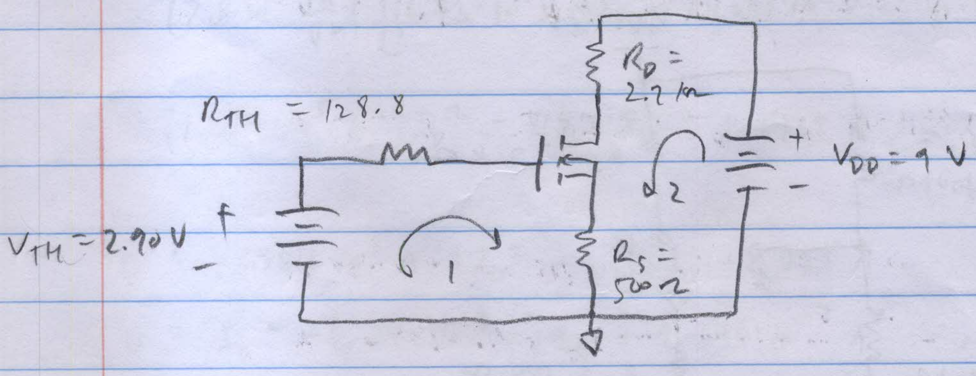
assumption untrue

b) assume transistor is in triode



$$V_{TH} = (9V)\left(\frac{190\ k\Omega}{400\ k\Omega + 190\ k\Omega}\right) = 2.90\ V$$

$$R_{TH} = R_1 \| R_2 = 400\ k\Omega \| 190\ k\Omega = 128.8\ k\Omega$$



$V_G = 2.90\ V$

KVL Loop 1 $\quad V_{TH} = V_{GS} + I_D R_S$

KVL Loop 2 $\quad V_{DD} = I_D R_D + V_{DS} + I_D R_S$

triode $\quad I_D = k_n' \left(\frac{W}{L}\right)\left[(V_{GS} - V_{TN})V_{DS} + \frac{V_{DS}^2}{2}\right]$

using Loop 1 to solve for V_{GS}

$$V_{GS} = V_{TH} - I_D R_S$$

using Loop 2 to solve for V_{DS}

$$V_{DS} = V_{DD} - I_D(R_D + R_S)$$

plugging new expressions for V_{GS}, V_{DS} into triode equation:

$$I_D = k'_n\left(\frac{W}{L}\right)\left[\left(V_{TH} - I_D R_S - V_{TN}\right)\left(V_{DD} - I_D(R_D + R_S)\right) + \frac{\left(V_{DD} + I_D(R_D+R_S)\right)^2}{2}\right]$$

$$I_D = k'_n\left(\frac{W}{L}\right)\left[\left(2.4V - I_D(500\Omega)\right)\left(9V - I_D(3.2k\Omega)\right) + \frac{\left(9V - I_D(3.2k\Omega)\right)^2}{2}\right]$$

$$= k'_n\left(\frac{W}{L}\right)\left[\left(21.6V^2 - (1.218\cdot10^4\, V\cdot\Omega)I_D + (1.6\cdot10^6\,\Omega^2)I_D^2 + \frac{81V^2 - (5.76\cdot10^4\, V\cdot\Omega)I_D + (1.024\cdot10^7\,\Omega^2)I_D^2}{2}\right]$$

$$= k'_n\left(\frac{W}{L}\right)\left[(6.72\cdot10^7\,\Omega^2)I_D^2 - (4.098\cdot10^4\, V\cdot\Omega)I_D + 62.1\,V^2\right]$$

$$= \left(2.8\cdot10^{-3}\,\frac{A}{V^2}\right)\left[(6.72\cdot10^7\,\Omega^2)I_D^2 - (4.098\cdot10^4\, V\cdot\Omega)I_D + 62.1\,V^2\right]$$

$$= (1.8816\cdot10^5\,A^{-1})I_D^2 - 114.7\,I_D + 0.1739\,A$$

$$0 = (1.8816\cdot10^5\,A^{-1})I_D^2 - 115.7\,I_D + 0.1739\,A$$

$$0 = I_D^2 - (6.149\cdot10^{-4}\,A)I_D + 9.242\cdot10^{-7}\,A^2$$

$$I_D = \frac{6.149\cdot10^{-4}\,A \pm \sqrt{(-6.149\cdot10^{-4}\,A)^2 - 4(9.242\cdot10^{-7}\,A^2)}}{2}$$

$$\boxed{I_D = \text{non real answers}}$$

$\boxed{\text{assumption incorrect}}$

c) same equivalent circuit and loop equations as triode solution.

$$V_{DS} = V_{DD} - I_D(R_D + R_S)$$

$$V_{GS} = V_{TH} - I_D R_S$$

saturation current: $I_D = \frac{k'_n}{2}\left(\frac{W}{L}\right)(V_{GS} - V_{TN})^2$

plugging expression for V_{GS} into I_D

$$I_D = \frac{k'_n}{2}\left(\frac{W}{L}\right)(V_{TH} - I_D R_S - V_{TN})^2$$

$$= \left(1.4\cdot10^{-3}\ \frac{A}{V^2}\right)(2.4\ V - I_D(500\ \Omega))^2$$

$$= \left(1.4\cdot10^{-3}\ \frac{A}{V^2}\right)\left[5.76\ V^2 - (2.4\cdot10^3\ V\cdot\Omega)I_D + (2.5\cdot10^5\ \Omega^2)I_D^2\right]$$

$$= (8.064\cdot10^{-3}\ A) - 3.36\,I_D + (350\ A^{-1})I_D^2$$

$$0 = (350\ A^{-1})I_D^2 - 4.36\,I_D + 8.064\cdot10^{-3}\ A$$

$$0 = I_D^2 - (1.246\cdot10^{-2}\ A)I_D + 2.304\cdot10^{-5}\ A^2$$

$$I_D = \frac{1.246\cdot10^{-2} \pm \sqrt{(-1.246\cdot10^{-2}\ A)^2 - 4(2.304\cdot10^{-5}\ A^2)}}{2}$$

$$= 6.23 \pm 3.97\ mA = 10.2\ mA \text{ or } 2.26\ mA$$

arbitrarily test $I_D = 10.2\ mA$ first

$$V_{DS} = 9\ V - (10.2\ mA)(3.2\ k\Omega) = -23.64\ V$$

$$V_{GS} = 2.90\ V - (10.2\ mA)(500\ \Omega) = -2.2\ V$$

$$\boxed{\begin{array}{l} V_{DS} < V_{GS} - V_T \quad \text{assumption untrue} \\ -23.64\ V < -2.7\ V \end{array}}$$

$I_D = 2.26 \text{ mA}$

$V_{DS} = 9\text{ V} - (2.26 \text{ mA})(3.2 \text{ k}\Omega) = 1.77 \text{ V}$

$V_{GS} = 2.90 \text{ V} - (2.26 \text{ mA})(500\ \Omega) = 1.77 \text{ V}$

$V_{DS} > V_{GS} - V_T$

$\boxed{1.77 \text{ V} > 1.27 \text{ V} \quad \text{assumption true}}$

$\boxed{\text{Q-pt: } (2.26 \text{ mA}, 1.77 \text{ V})}$

d) $\boxed{\text{saturation is valid assumption}}$

e) $I_D = \frac{k'n}{2}\left(\frac{W}{L}\right)(V_{GS} - V_{TN})^2(1 + \lambda V_{DS}) \qquad \lambda = 0.1 \text{ V}^{-1}$

$$I_D = \frac{k'n}{2}\left(\frac{W}{L}\right)(V_{TH} - I_D R_S - V_{TN})^2\left[1 + \lambda(V_{DD} - I_D(R_D + R_S))\right]$$
$$= \left[(8.064\cdot10^{-3} \text{ A}) - 3.36 I_D + (350 \text{ A}^{-1}) I_D^2\right]\left[1 + 0.9 - (320 \text{ A}^{-1}) I_D\right]$$
$$= (1.532\cdot10^{-2} \text{ A}) - 6.384 I_D + (665 \text{ A}^{-1}) I_D^2 - 2.580 I_D + (1.075\cdot10^3 \text{ A}^{-1}) I_D^2 - (1.12\cdot10^5 \text{ A}^{-2}) I_D^3$$

$$0 = -(1.12\cdot10^5 \text{ A}^{-2}) I_D^3 + (1.740\cdot10^3 \text{ A}^{-1}) I_D^2 - 9.964 I_D + (1.532\cdot10^{-2} \text{ A})$$
$$0 = I_D^3 - (1.554\cdot10^{-2} \text{ A}) I_D^2 + (8.896\cdot10^{-5} \text{ A}^2) I_D - (1.368\cdot10^{-7} \text{ A}^3)$$
$$= I_D^3 + p I_D^2 + q I_D + r$$

let $a = \frac{1}{3}(3q - p^2) = 8.463\cdot10^{-6} \text{ A}^2$

$b = \frac{1}{27}(2p^3 - 9pq + 27r) = 4.603\cdot10^{-8} \text{ A}^3$

let $A = \sqrt[3]{-\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}} = 7.844\cdot10^{-4} \text{ A}$

let

$$B = -\sqrt[3]{+\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}} = -3.596 \cdot 10^{-3}\ A$$

$$x = A + B = -2.812 \cdot 10^{-3}\ A \rightarrow \text{real}$$

or

$$x = -\frac{A+B}{2} + \frac{A-B}{2}\sqrt{-3} =$$

or

$$x = -\frac{A+B}{2} - \frac{A-B}{2}\sqrt{-3} =$$

complex

use real root

$$I_D = y = x - \frac{p}{3} = -2.812 \cdot 10^{-3} A - \frac{(-1.554 \cdot 10^{-2}\ A)}{3}$$

$$I_D = 2.368\ mA$$

$$V_{DS} = 9\ V - (2.368\ mA)(3.2\ k\Omega) = 1.422\ V$$

$$V_{GS} = 2.90\ V - (2.368\ mA)(500\ \Omega) = 1.716\ V$$

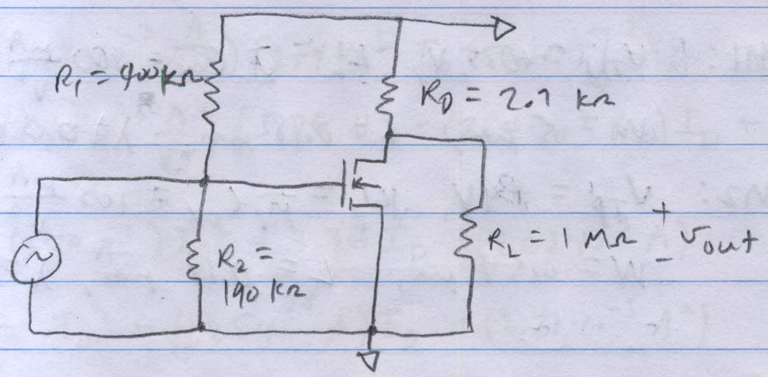
Q-pt: (2.368 mA, 1.422 V)

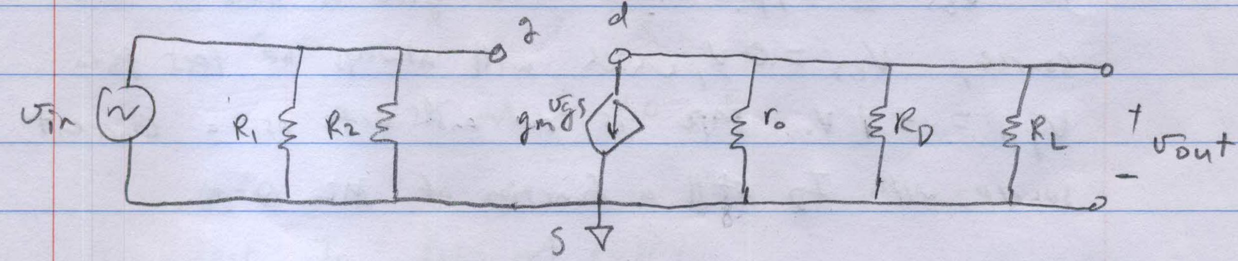
$V_{DS} > V_{GS} - V_{TN}$

$1.422\ V > 1.216\ V$

saturation assumption verified

f)





$$g_m = \frac{2 I_D}{V_{GS} - V_{TN}} = \frac{2(2.368\ mA)}{1.216\ V} = 3.895\ mS$$

$$r_o = \frac{\frac{1}{\lambda} + V_{DS}}{I_D} = \frac{10\ V + 1.422\ V}{2.368\ mA} = 4.823\ k\Omega$$

$$v_{gs} = v_{in}$$

$$v_{out} = -g_m v_{gs} \underline{(r_o \| R_D \| R_L)} = -g_m v_{in} (r_o \| R_D \| R_L)$$

$$A_v = -g_m (r_o \| R_D \| R_L)$$
$$= -(3.895\ mS)(1.728\ k\Omega)$$
$$\boxed{A_v = -6.73\ V/V}$$

g) to remain in small signal mode, $v_{DS} \geq v_{GS} - V_{TN}$

$$V_{DS} - 6.73 v_{gs,max} > V_{GS} + v_{gs,max} - V_{TN}$$
$$V_{DS} - V_{GS} + V_{TN} > 7.73 v_{gs,max}$$
$$27\ mV > v_{gs,max}$$

$$\boxed{\begin{array}{l}\text{biggest } V_{out} \text{ DC swing} \\ = 6.73 v_{gs,max} \\ = 179\ mV\end{array}}$$

2. M1: $V_{TN} = +0.5\text{ V}$, $K'_n = \bar{\mu}_n C_{ox} = 100 \frac{\mu A}{V^2}$

$W = 5\ \mu m$, $L = 0.18\ \mu m$, $\lambda = 0.0\ V^{-1}$, $\gamma = 0.0\ V^{1/2}$

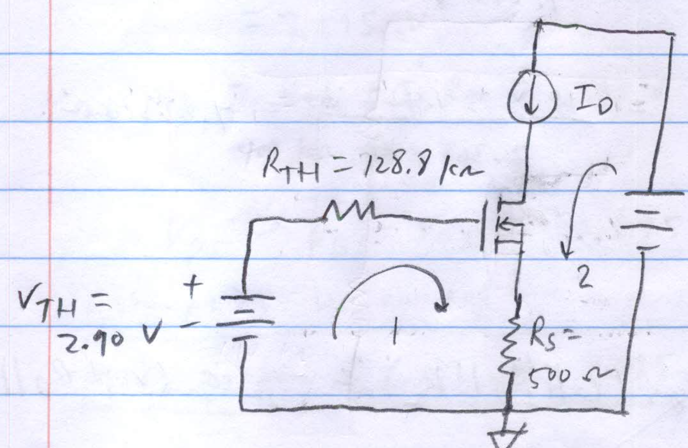
M2: $V_{TP} = +4V$, $K'_p = \bar{\mu}_p C_{ox} = 100 \frac{\mu A}{V^2}$

$W = 0.54\ \mu m$, $L = 0.18\ \mu m$, $\lambda = 0.0\ V^{-1}$, $\gamma = 0.0\ V^{1/2}$

For a pMOSFET like M2, the transistor is on if $V_{GS} \leq V_{TP}$. Since the gate is tied to the source, $V_{GS} = 0\text{ V}$, which will always be less than $V_{TP} = +4\text{ V}$. M2 is on and acts as a current source with I_D still a function of M1 Q-pt.

a) DC equivalent circuit

V_{TH} and R_{TH} are same as problem 1



Loop 1: $V_{TH} = V_{GS} + I_D R_S$

Loop 2: $V_{DD} = V_{DS} + I_D R_S$

assuming saturation: $I_D = \frac{K'_n}{2}\left(\frac{W}{L}\right)(V_{GS} - V_{TN})^2$

solving Loop 1 for V_{GS}: $V_{GS} = V_{TH} - I_D R_S$

plug into I_D:

$$I_D = \frac{K'_n}{2}\left(\frac{W}{L}\right)(V_{TH} - I_D R_S - V_{TN})^2$$

$$I_D = \left(1.4\cdot10^{-3}\frac{A}{V^2}\right)\left[2.4V - I_D(500\Omega)\right]^2$$

$$= \left(1.4\cdot10^{-3}\frac{A}{V^2}\right)\left[5.76\ V^2 - (2.4\cdot10^3\ V\cdot\Omega)I_D + (2.5\cdot10^5\ \Omega^2)I_D^2\right]$$

$$I_D = (350\ A^{-1})I_D^2 - 3.36\ I_D + (8.07\cdot10^{-3}\ A)$$

$$0 = I_D^2 - (1.25\cdot10^{-2}\ A)I_D + (2.31\cdot10^{-5}\ A^2)$$

$$I_D = \frac{+1.25\cdot10^{-2}A \pm \sqrt{(-1.25\cdot10^{-2}A)^2 - 4(2.31\cdot10^{-5}A^2)}}{2}$$

$$= +6.25\ mA \pm 4.0\ mA$$

test $I_D = 2.25\ mA$ first:

$$V_{DS} = V_{DD} - I_D R_S = 9\ V - (2.25\ mA)(500\Omega)$$
$$= 7.875\ V$$

$$V_{GS} = V_{TH} - I_D R_S = 2.90\ V - (2.25\ mA)(500\Omega)$$
$$= 1.775\ V$$

$$V_{DS} > V_{GS} - V_{TN} = 1.275\ V$$

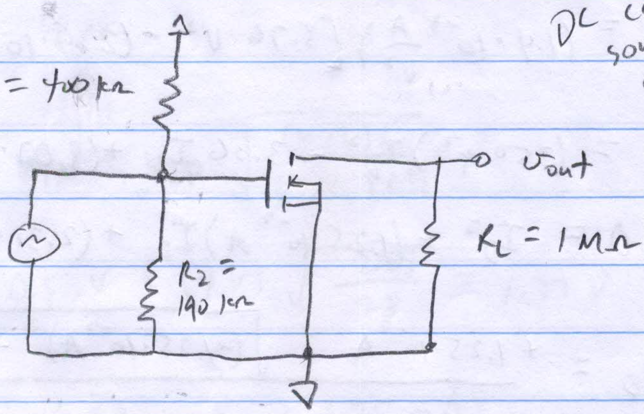
assumption is correct

$$\boxed{M1\ \text{Q-pt}: (2.25\ mA,\ 7.875\ V)}$$

$$g_{m1} = \frac{2I_D}{V_{GS} - V_{TN}} = \frac{2(2.25\ mA)}{1.275\ V} = 3.53\ mS$$

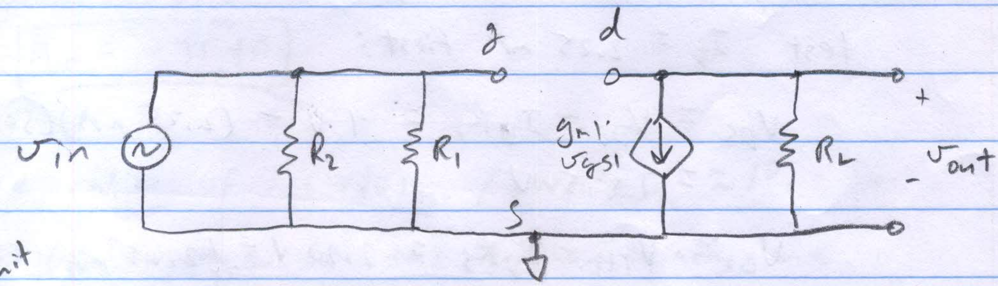
$$r_{o1} = \frac{1/\lambda + V_{DS}}{I_D} = \frac{\infty + 7.875\ V}{2.25\ mA} = \infty\ \Omega$$

ac equivalent circuit



DC current source is an open circuit here

Small signal model



Recall: $r_o = \infty\,\Omega$ = open circuit

$v_{in} = v_{gs1}$

$v_{out} = -g_{m1} v_{gs1} R_L = -g_{m1} v_{in} R_L$

$A_v = \frac{v_{out}}{v_{in}} = -g_{m1} R_L = -(3.53\,mS)(10^6\,\Omega)$

$\boxed{A_v = -3530\ V/V}$

b) We tie the bulk/body and source terminal together to eliminate a drifting of the expected V_{TP}

c) $\left(\frac{W}{L}\right)_{M2} = 2.81$

$\left(\frac{W}{L}\right)_{M1} = 28$

$$V_{GS1} = V_{TN} + V_{TP}\sqrt{\frac{(W/L)_{M2}}{(W/L)_{M1}}}$$

$$= 0.5\ V + (4\ V)\sqrt{\frac{2.81}{28}} = 1.77\ V$$

$$g_{m1} = \frac{2 I_D}{V_{GS} - V_{TN}} = \frac{2(2.25\ mA)}{1.27\ V} = 3.54\ mS$$

$$A_v = -g_{m1} R_L = (3.54\ mS)(1\ M\Omega)$$

$$\boxed{A_v = -3540}$$

d) The ratio of $(W/L)_{M1} / (W/L)_{M2} \approx 10$. To take advantage of this higher gain, one should make $W_{M1} = 10 W_{M2}$, and keep the gate lengths as low as possible.

3. a) Stealing Q-pt from DC analysis of 1(e),

$V_{DS} = 1.422\ V$

$V_{GS} = 1.716\ V$

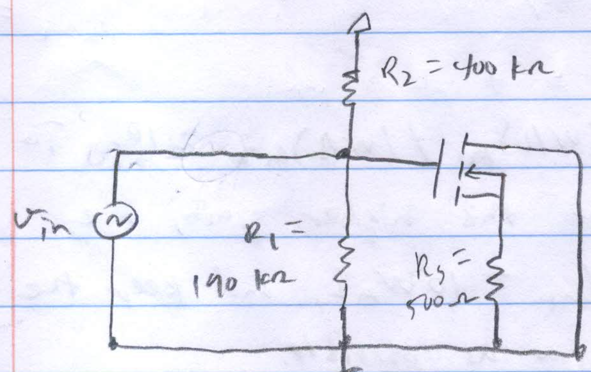
$I_D = 2.368\ mA$

Stealing the transistor parameters from 1(f)

$g_m = 3.895\ mS$

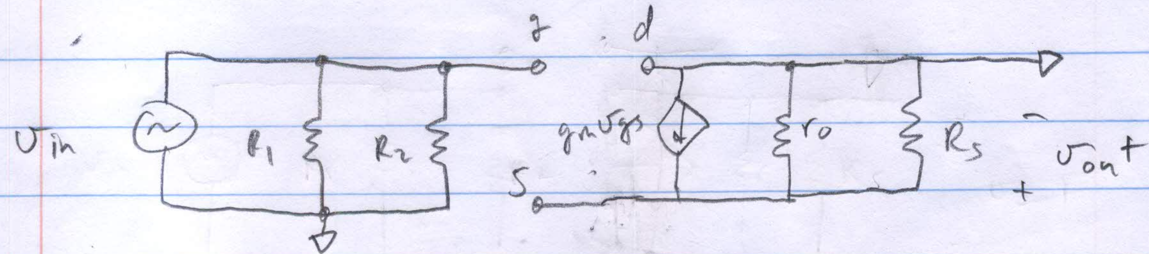
$r_o = 4.823\ k\Omega$

ac equivalent circuit:



Note: common drain config

small signal model:



$v_{out} = g_m v_{gs} (r_o \| R_s)$

$v_{in} = g_m v_{gs} (r_o \| R_s) + v_{gs} = v_{gs}\left[1 + g_m (r_o \| R_s)\right]$

$r_o \| R_S = 4.823\ k\Omega \| 500\ \Omega = 453\ \Omega$

$$A_v = \frac{v_{out}}{v_{in}} = \frac{g_m (r_o \| R_S)}{1 + g_m (r_o \| R_S)} = \frac{(3.895 \cdot 10^{-3} S)(453\ \Omega)}{1 + (3.895 \cdot 10^{-3} S)(453\ \Omega)}$$

$$\boxed{A_v = 0.638\ V/V}$$

b) the phase of the circuit is $0°$ (v_{out} is in phase with v_{in})

c) $g_m \to \infty$ $\boxed{A_v = 1}$