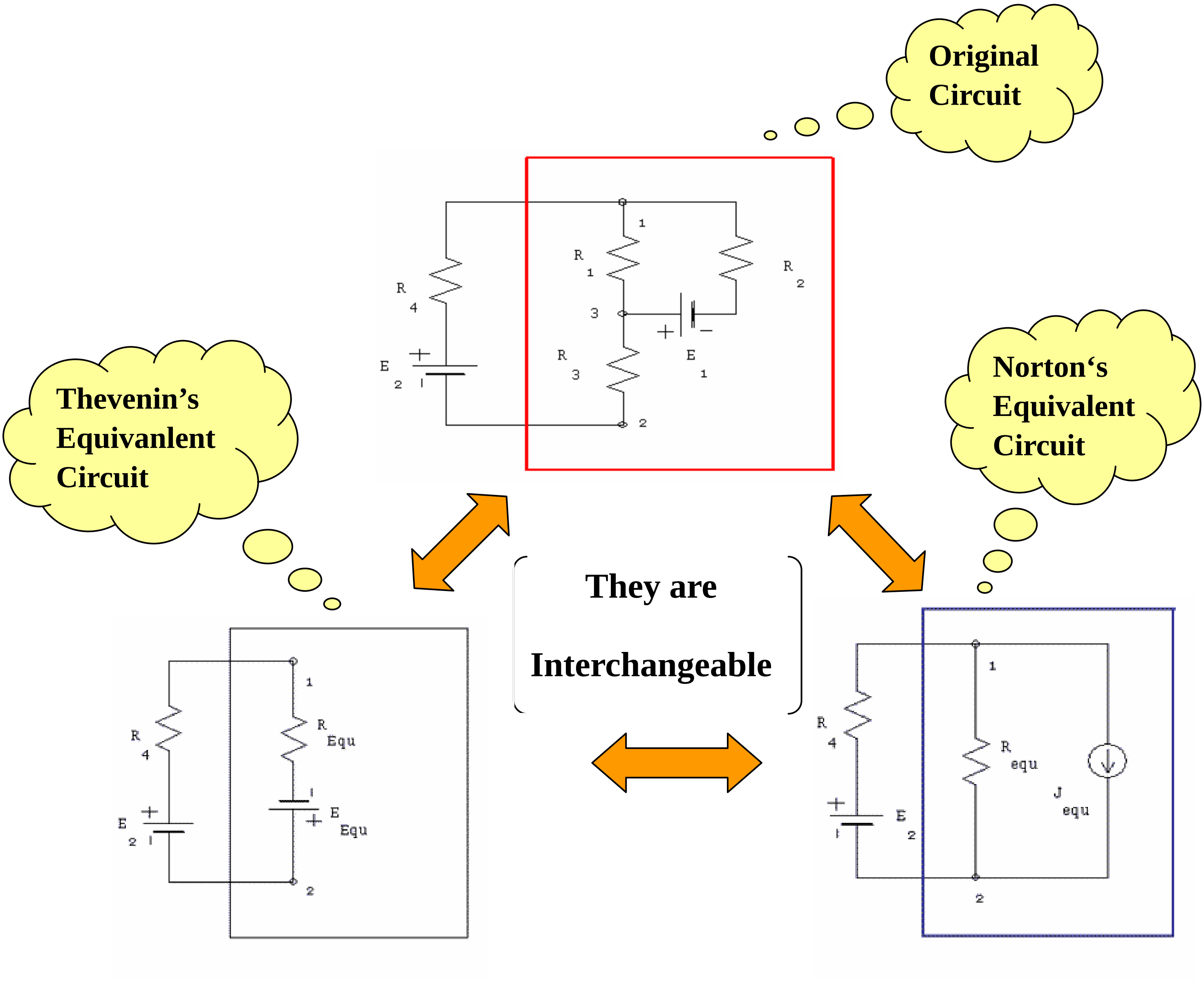
Thevenin's and Norton's Equivalent Circuit Tutorial. (by Kim, Eung)

Thevenin's Theorem states that we can replace entire network by an equivalent circuit that contains only an independent voltage source in series with an impedance (resistor) such that the current-voltage relationship at the load is unchanged.

Norton's Thereom is identical to Thevenin's Theorem except that the equivalent circuit is an independent current source in parallel with an impedance (resistor). Therefore, the Norton equivalent circuit is a source transformation of the Thevenin equivalent circuit.



How to find Thevenin's Equivalent Circuit?

If the circuit contains	You should do
Resistors and independent sources	1) Connect an open circuit between a and b. 2) Find the voltage across the open circuit which is Voc. Voc = Vth. 3) Deactivate the independent sources. Voltage source ➔ open circuit Current source ➔ short circuit 4) Find Rth by circuit resistance reduction
Resistors and dependent sources or independent shorces	1) Connect an open circuit between a and b. 2) Find the voltage across the open circuit which is Voc. Voc = Vth. If there are both dependent and independent sources. 3) Connect a short circuit between a and b. 4) Determine the current between a and b. 5) Rth = Voc / Iab If there are only dependent sources. 3) Connect 1 Ampere current source flowing from terminal b to a. It = 1 [A] 4) Then Rth = Voc / It = Voc / 1

Note: When there are only dependent sources, the equivalent network is merely $R_{Th,}$ that is, no current or voltage sources.

How to find Norton's Equivalent Circuit?

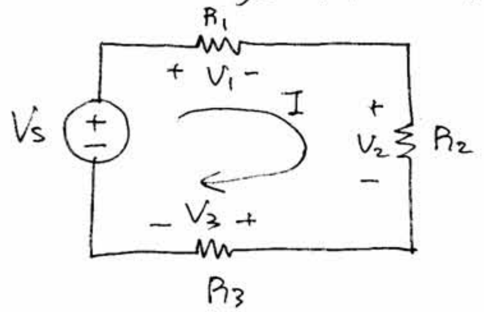
If the circuit contains	You should do
Resistors and independent sources	- Deactivate the independent sources. Voltage source ➔ open circuit Current source ➔ short circuit - Find Rt by circuit resistance reduction - Connect an short circuit between a and b. - Find the current across the short circuit which is Isc.
Resistors and dependent sources or Independent sources	1) Connect a short circuit between a and b. 2) Find the current across the short circuit which is Isc. Isc = In. If there are both dependent and independent sources. 3) Connect a open circuit between a and b. 4) Determine the voltage between a and b. Voc = Vab 5) Rn = Voc / Isc If there are only dependent sources. 3) Connect 1 Ampere current source flowing from terminal b to a. It = 1 [A] 4) Then Rn = Voc / It = Voc / 1

Note: When there are only dependent sources, the equivalent network is merely R_{Th}, that is, no current or voltage sources.

References

1. Introduction to Electric Circuits 5th Edition. Richard C. D and James A. S. 2001. John Wiley & Sons, Inc.

* Voltage Divider.



When you have multiple resistors in a single loop, the source voltage will be divided according to KVL.

$$V_s = V_1 + V_2 + V_3$$

And since there exists only one loop, the current flowing through each resistor is the same as I.

$$\begin{pmatrix} V_1 = I \cdot R_1 \\ V_2 = I \cdot R_2 \\ V_3 = I \cdot R_3 \end{pmatrix} \quad \text{therefore} \quad \begin{aligned} V_s &= V_1 + V_2 + V_3 \\ &= I \cdot R_1 + I \cdot R_2 + I \cdot R_3 \\ &= I(R_1 + R_2 + R_3) \end{aligned}$$

$$\therefore I = \frac{V_s}{R_1 + R_2 + R_3}$$

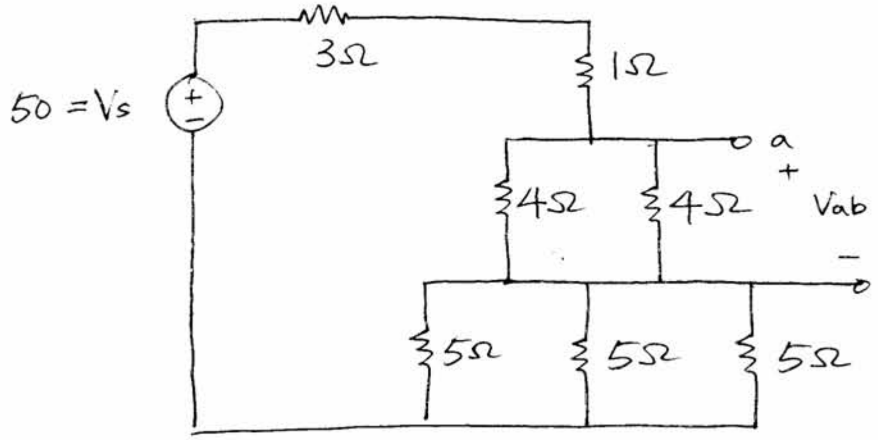
Thus, the voltage across the nth resistor R_n can be found as

$$V_n = I \cdot R_n = \left(\frac{V_s}{R_1 + R_2 + R_3} \right) \cdot R_n$$

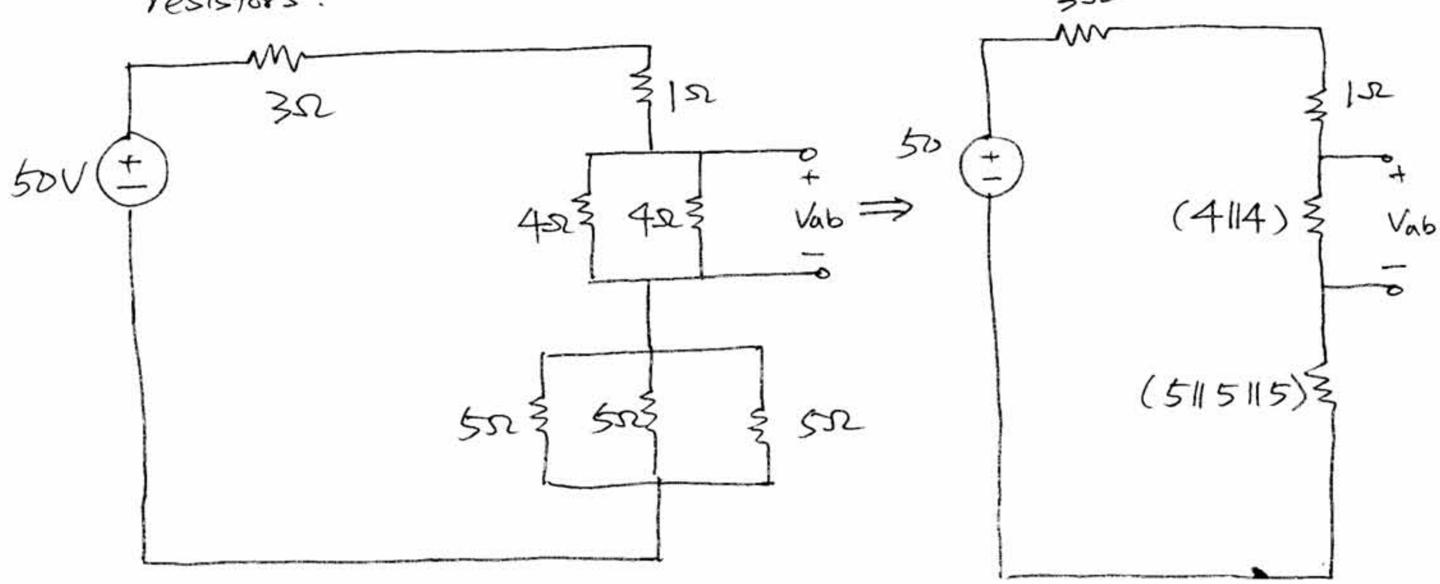
In general, we may repeat the voltage devider principle by

$$V_n = \left(\frac{V_s}{\sum_{i=1}^{n} R_i} \right) \cdot R_n$$

example 1)

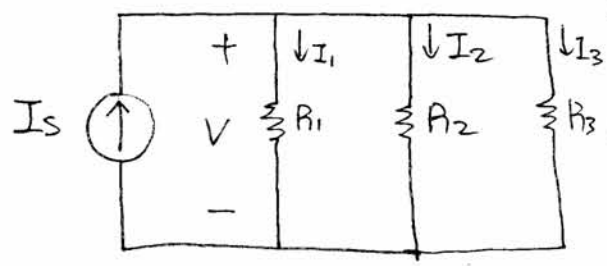


We can seperate the resistors into some groups of parallel-connected resistors.



$$\therefore V_{ab} = \frac{50}{3+1+(4\|4)+(5\|5\|5)} \cdot (4\|4)$$

* Current Divider.



When we have multiple resistors in parallel connection ; the source current will be divided into each parallel branch according to KCL

$$I_s = I_1 + I_2 + I_3$$

Since parallel-connected resistors can be simplified as one single resistor as $(R_1 \parallel R_2 \parallel R_3)$, the voltage across each resistor is the same as V.

$$\left(\begin{array}{l} I_1 = \frac{V}{R_1} \\ I_2 = \frac{V}{R_2} \\ I_3 = \frac{V}{R_3} \end{array} \right) \text{ therefore } \quad I_s = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\therefore V = \frac{I_s}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

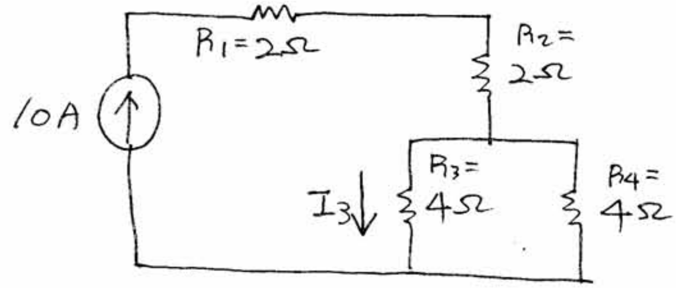
Thus, the current flowing through nth resistor R_n can be found as

$$I_n = \frac{V}{R_n} = \frac{I_s}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)} \cdot \frac{1}{R_n}$$

In general, we can repeat the current divider principle by

$$I_n = \frac{I_s}{\sum_{i=1}^{n} G_n} \cdot G_n \qquad \left(\begin{array}{c} G \text{ is conductance} \\ G_n = \frac{1}{R_n} \end{array} \right)$$

example 1)

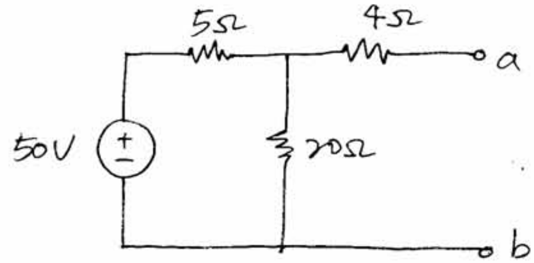


R_1 and R_2 is series-connected to the current source, therefore the current flowing across R_1 and R_2 is just the same as 10A. However R_3 and R_4 are parallel-connected, so the current will be divided into two branches.

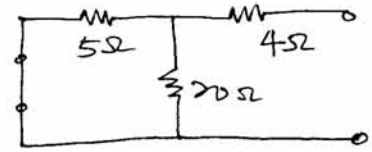
$$I_3 = \frac{10}{\left(\frac{1}{4} + \frac{1}{4}\right)} \cdot \left(\frac{1}{4}\right)$$

* Thevenin's and Norton's Equivalent Circuits

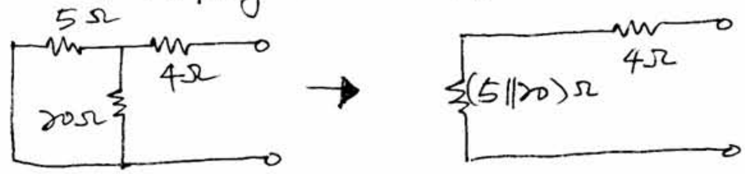
example 1).



sol) ① deactivate voltage source
(Voltage Source ⇒ short circuit).



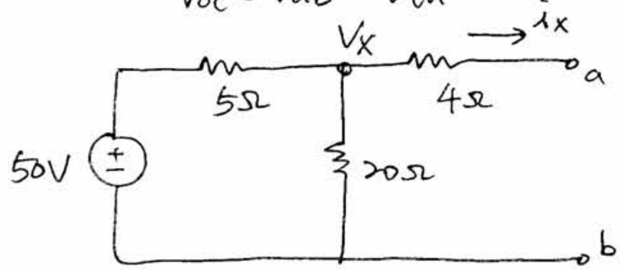
② Simplify the circuit and find R_{th}.



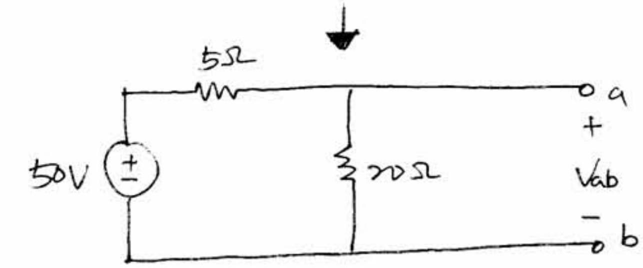
$\therefore R_{th} = (5 \| 20) + 4 = 8\Omega$

③ Find open-circuit voltage across ab

$V_{oc} = V_{ab} = V_{th}$



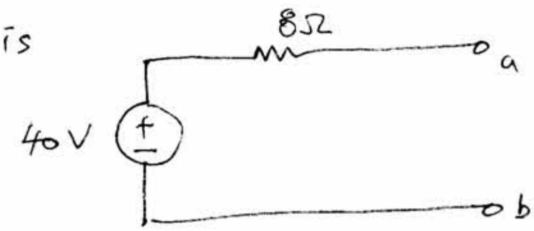
$i_x = 0$ since node a is open.
Therefore $V_x = V_a$.
(no voltage drop across 4Ω resistor).
We can remove 4Ω resistor from the circuit.



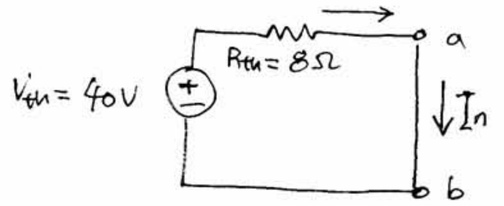
$$V_{ab} = V_{th} = 50 \times \frac{20}{20+5} = 40\,V$$

Thevenin's Circuit is

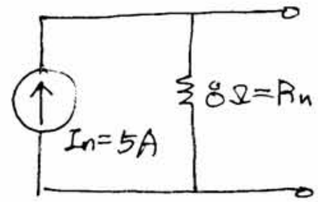
$\therefore$



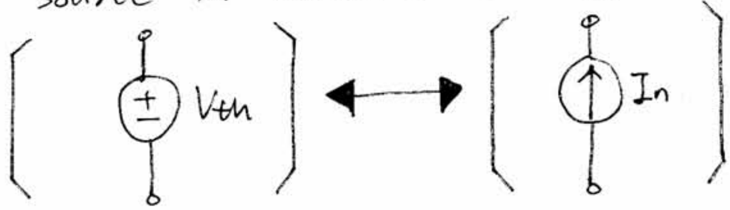
This Thevenin's Circuit can be converted to Norton's Circuit as



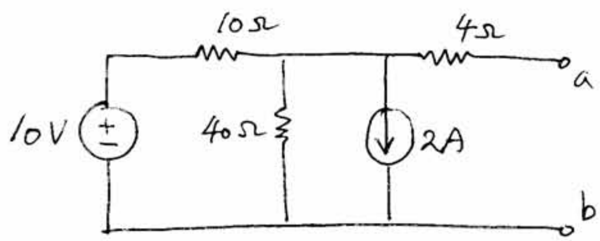
$$\left(\begin{array}{l} I_n = \dfrac{40V}{8\Omega} = 5A \\ R_{th} = R_n \end{array} \right. \Rightarrow$$



note: when you convert Thevenin's Circuit to Norton's Circuit, the direction of current flow of the current source in the Norton's Circuit should be matched with the voltage source in Thevenin's Circuit.

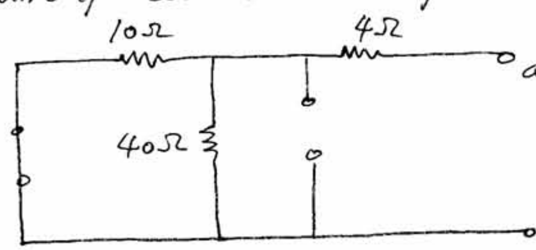


example 2).

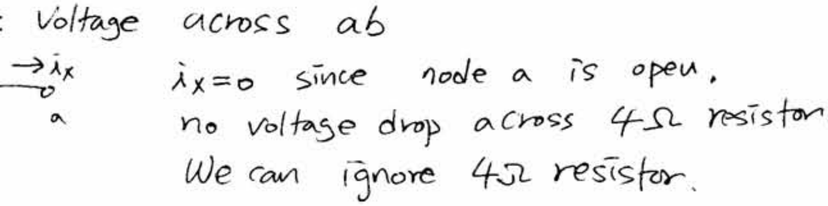


① deactivate independent sources

(Voltage Source → short
(Current source → open.



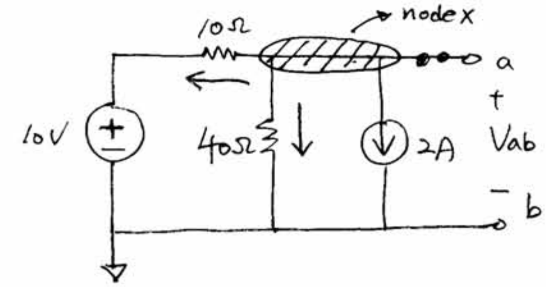
② Simplify the circuit and find R_{th}.



$\therefore R_{th} = (10\|40) + 4 = 12\,\Omega$

③ Find open-circuit Voltage across ab

$i_x = 0$ since node a is open,
no voltage drop across 4Ω resistor.
We can ignore 4Ω resistor.

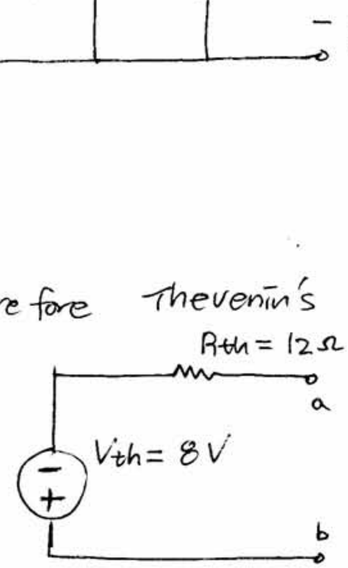


If we set the minus terminal of the voltage source as ground, then the voltage at node-x is V_{ab}. Apply KCL to node-x.

$$\frac{V_{ab}-10}{10}+\frac{V_{ab}}{40}+2=0$$

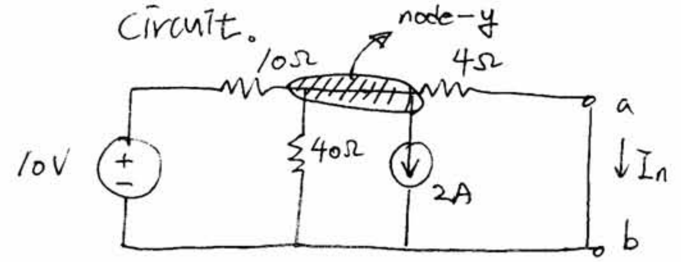
$$\therefore V_{ab}=-8\,V=V_{th}.$$

Therefore Thevenin's Circuit is.



(The polarity of the voltage source is reversed since Thevenin voltage source is minus value.)

As you know, the impedence in Thevenin's Circuit is the same as the impedence in Norton's Circuit. So, if you find short-circuit current across ab at ③ instead of open-circuit voltage across ab, you can find Norton's Equivalent Circuit.

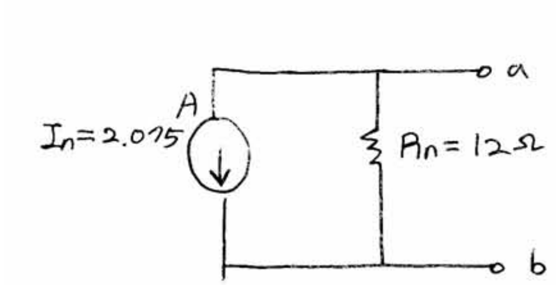


Let's set the voltage at node-y as V_y. Apply KCL to node-y

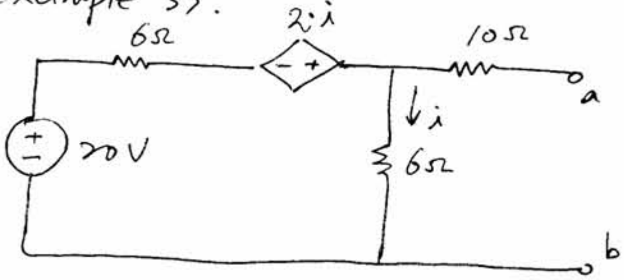
$$\frac{V_y-10}{10}+\frac{V_y}{40}+2+\frac{V_y}{4}=0$$

$$\therefore V_y=-8.3\,V$$

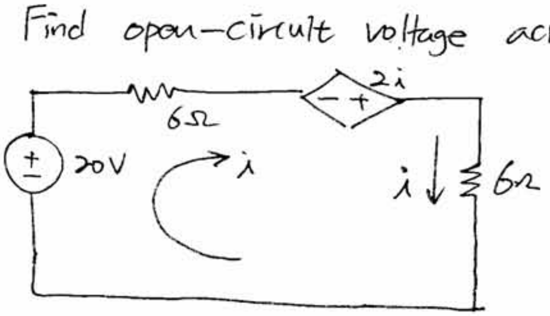
$$\therefore I_n=\frac{V_y}{4\Omega}=\frac{-8.3V}{4\Omega}=-2.075\,A$$



example 3).



① Find open-circuit voltage across ab.



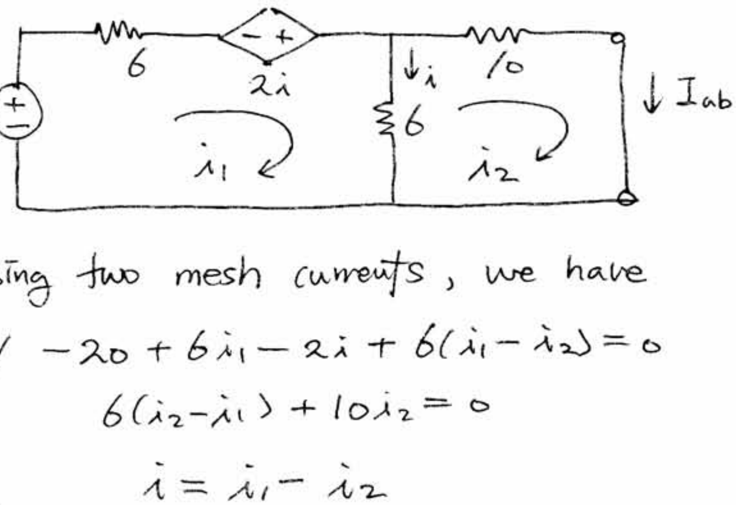
10 Ω resistor can be ignored since node a is open.

Apply KVL. $6i - 2i + 6i - 20 = 0$

$\therefore i = 2\,A$

Therefore $V_{ab} = 6i = 12V$

② Find short-circuit current across ab.



Using two mesh currents, we have

$$\begin{cases} -20 + 6i_1 - 2i + 6(i_1 - i_2) = 0 \\ 6(i_2 - i_1) + 10 i_2 = 0 \\ i = i_1 - i_2 \end{cases}$$

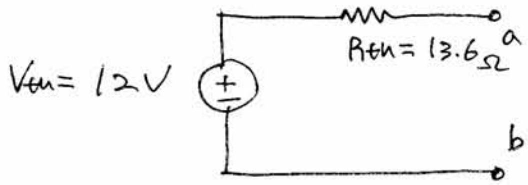
From these three equations, we obtain

$$i_2 = \frac{120}{136}\,A = I_{ab}$$

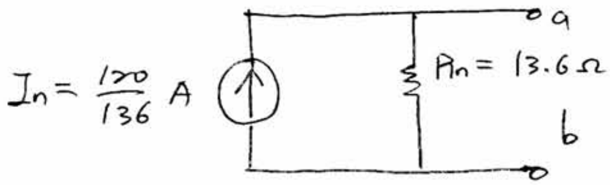
③ From V_{ab} (open-circuit) and I_{ab} (short-circuit), find R_{th}

$$R_{th} = \frac{V_{ab}}{I_{ab}} = \frac{12}{120/136} = 13.6\,\Omega$$

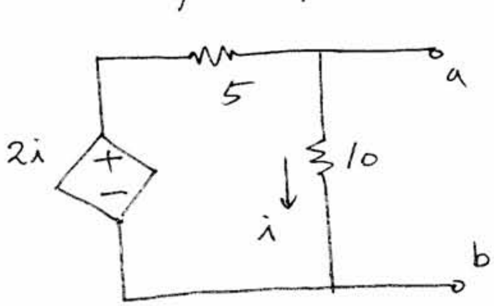
Therefore Thevenin's Equivalent Circuit is



Norton's Equivalent Circuit is



example 4).

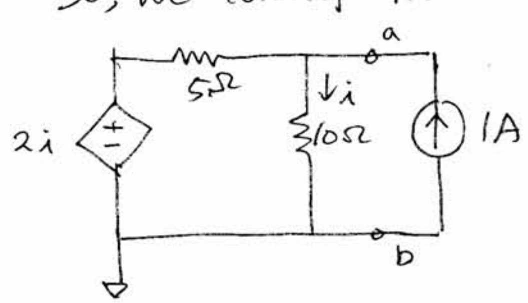


※ Since the circuit has no independent source, $i=0$ when we connect an open circuit to ab.

Therefore $\underset{(\text{open})}{V_{ab}} = 0$ and $\underset{(\text{close})}{I_{ab}} = 0$.

So, we can not use $R_{th} = \frac{V_{ab}}{I_{ab}}$ like example 3).

So, we connect 1A test current source to ab. Then we can say



$$R_{th} = \frac{V_{ab}}{1\,A}$$

Let's set the minus node of the voltage source as ground for reference.

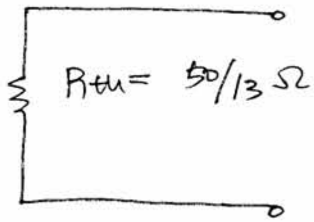
Apply KCL to node a.

$$\left(\begin{array}{l} \frac{V_{ab} - 2i}{5} + \frac{V_{ab}}{10} - 1 = 0 \\ \text{and } i = \frac{V_{ab}}{10} \end{array} \right.$$

Therefore $\frac{V_{ab} - 2\left(\frac{V_{ab}}{10}\right)}{5} + \frac{V_{ab}}{10} - 1 = 0 \quad \therefore V_{ab} = \frac{50}{13}\,V$

$\therefore R_{th} = \frac{V_{ab}}{1} = \frac{50}{13}\,\Omega$

∴ Thevenin's Equivalent Circuit is



Norton's Equivalent Circuit is the same as Thevenin's Circuit.