Lecture 11

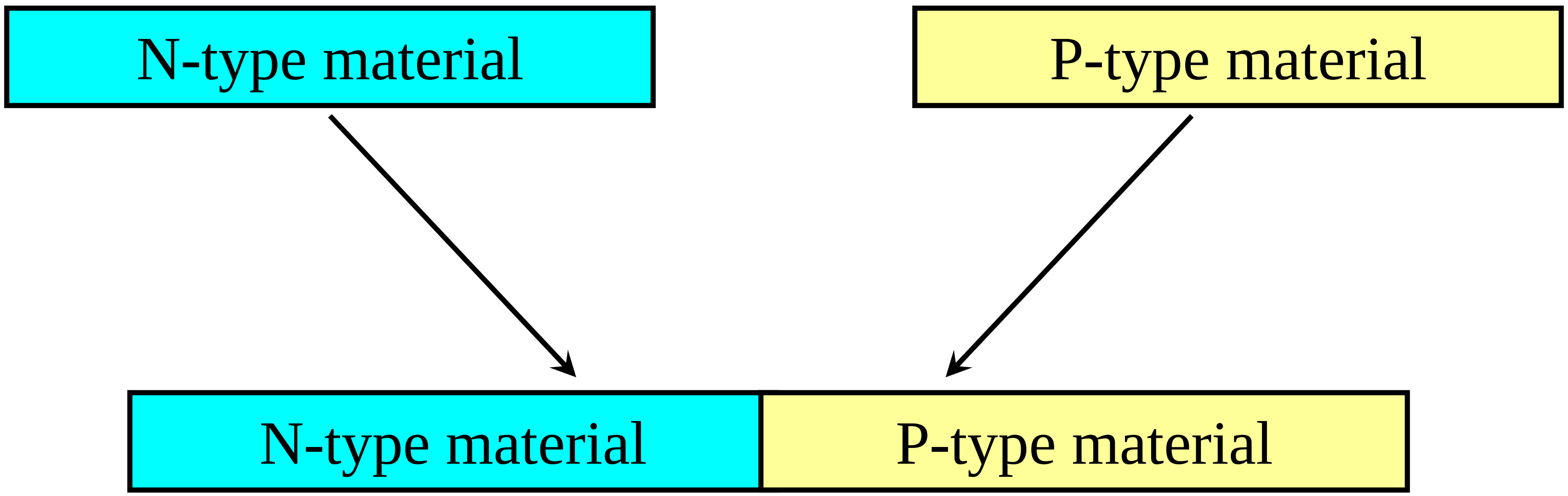
P-N Junction Diodes: Part 1

How do they work? (postponing the math)

Reading:

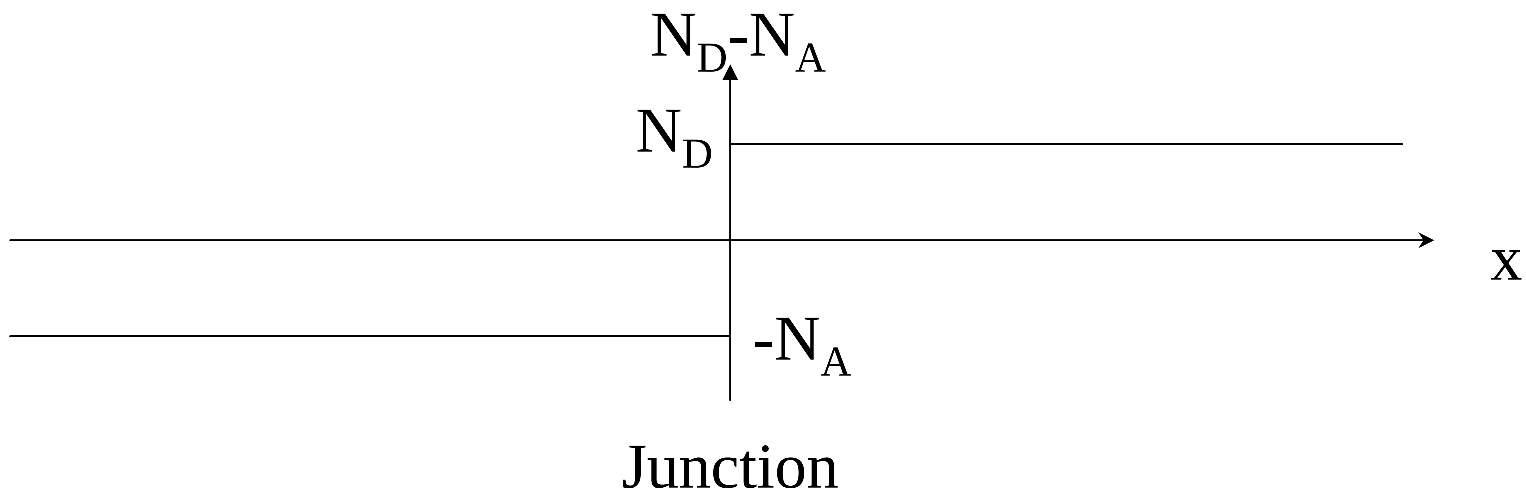
(Cont'd) Notes and Anderson[2] sections Part 2 preface and sections 5.0-5.3

Our First Device: p-n Junction Diode



A p-n junction diode is made by forming a p-type region of material directly next to a n-type region.

Our First Device: p-n Junction Diode



In regions far away from the "junction" the band diagram looks like:

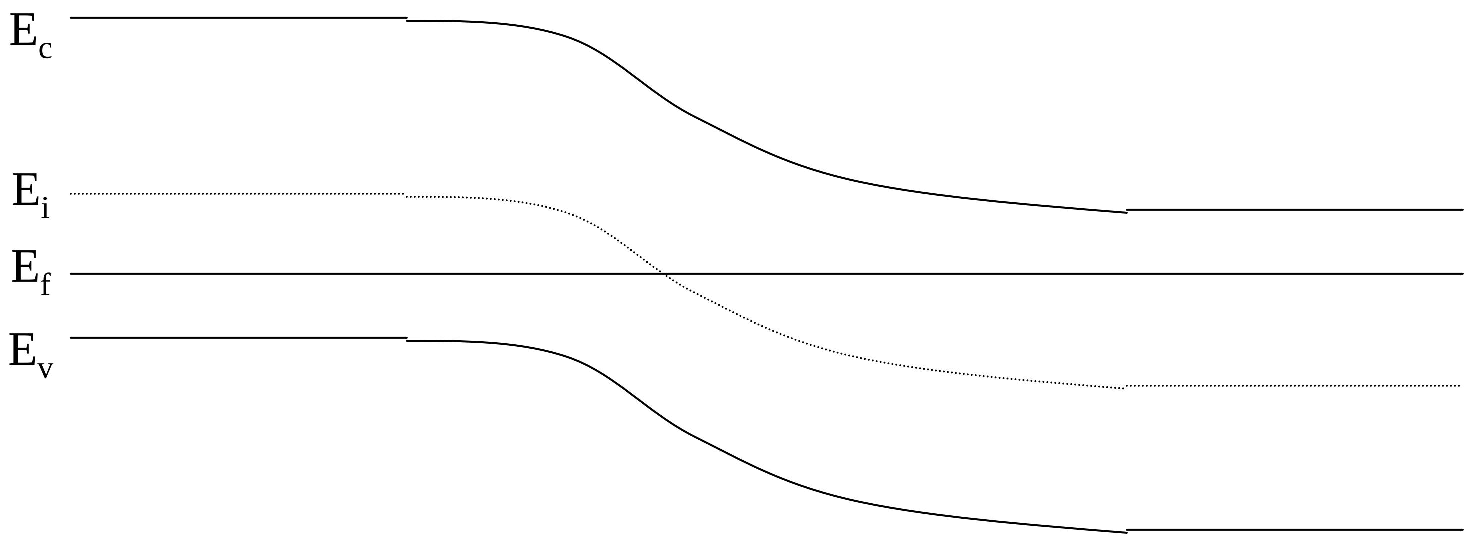
Our First Device: p-n Junction Diode

But when the device has no external applied forces, no current can flow. Thus, the fermi-level must be flat!

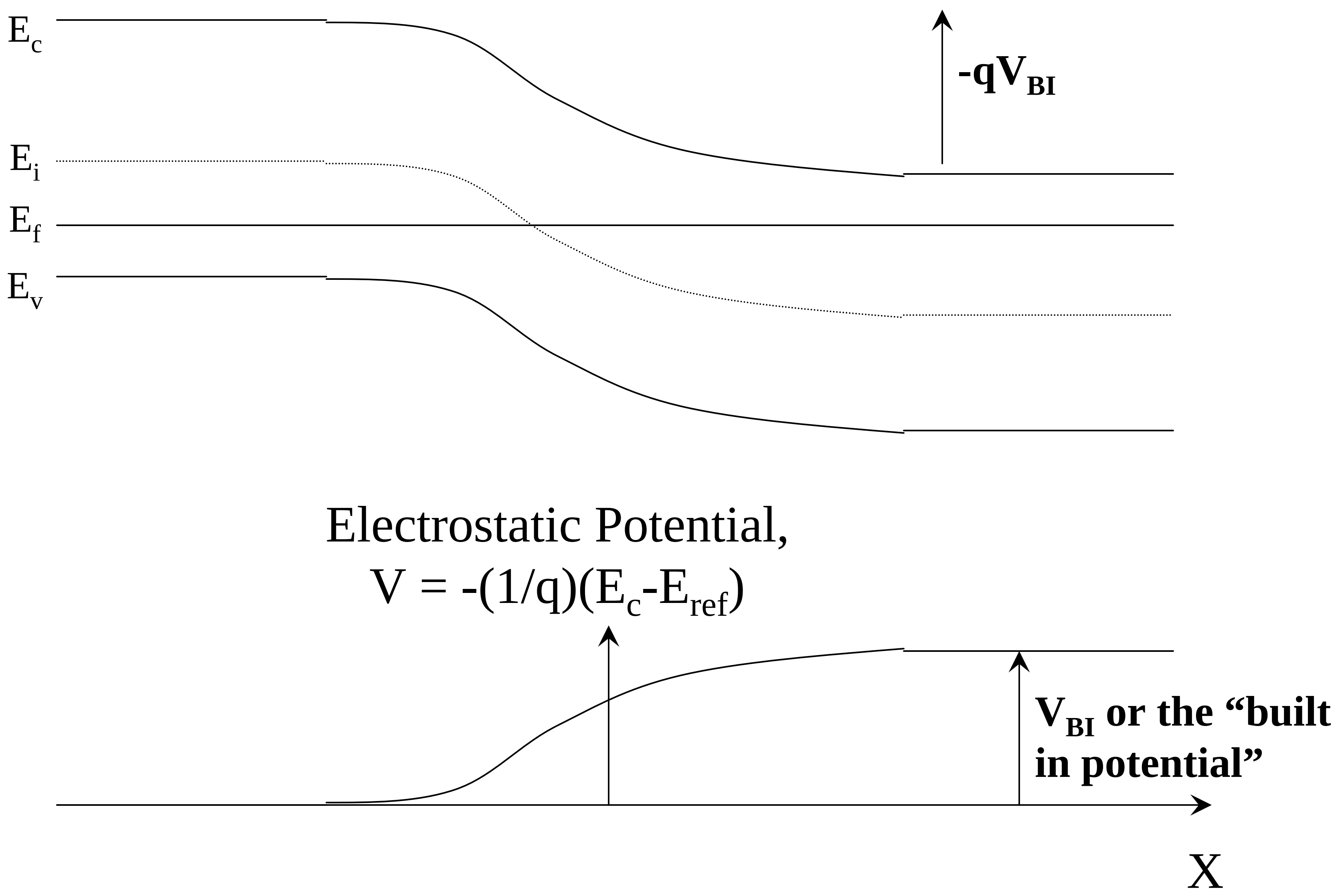
We can then fill in the junction region of the band diagram as:

or…

Our First Device: p-n Junction Diode



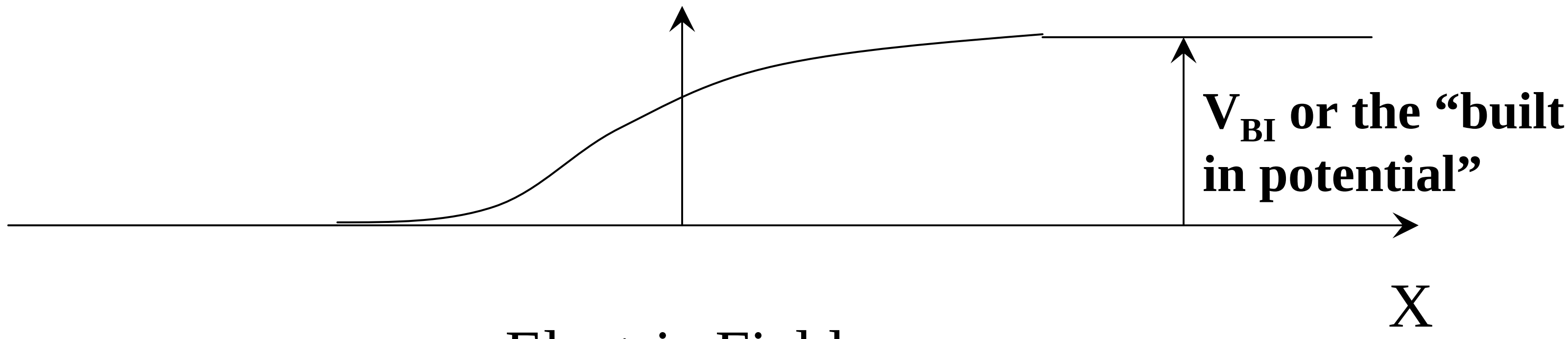
Our First Device: p-n Junction Diode



Our First Device: p-n Junction Diode

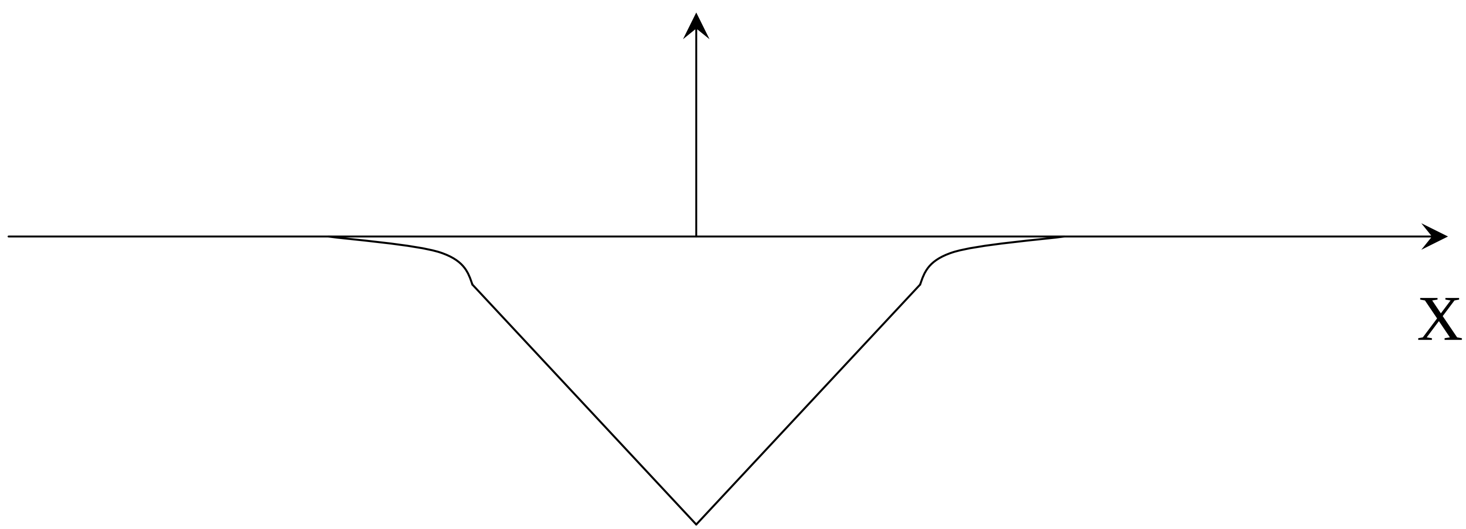
Electrostatic Potential,

$V=-(1/q)(E_c-E_{ref})$



Electric Field

$E=-dV/dx$



Our First Device: p-n Junction Diode

Poisson's Equation:

Electric Field **Charge Density (NOT resistivity)**

$$\nabla \bullet E = \frac{\rho}{K_s \varepsilon_o} \quad or\ in\ 1D,\ \frac{dE}{dx} = \frac{\rho}{K_s \varepsilon_o}$$

Permittivity of free space

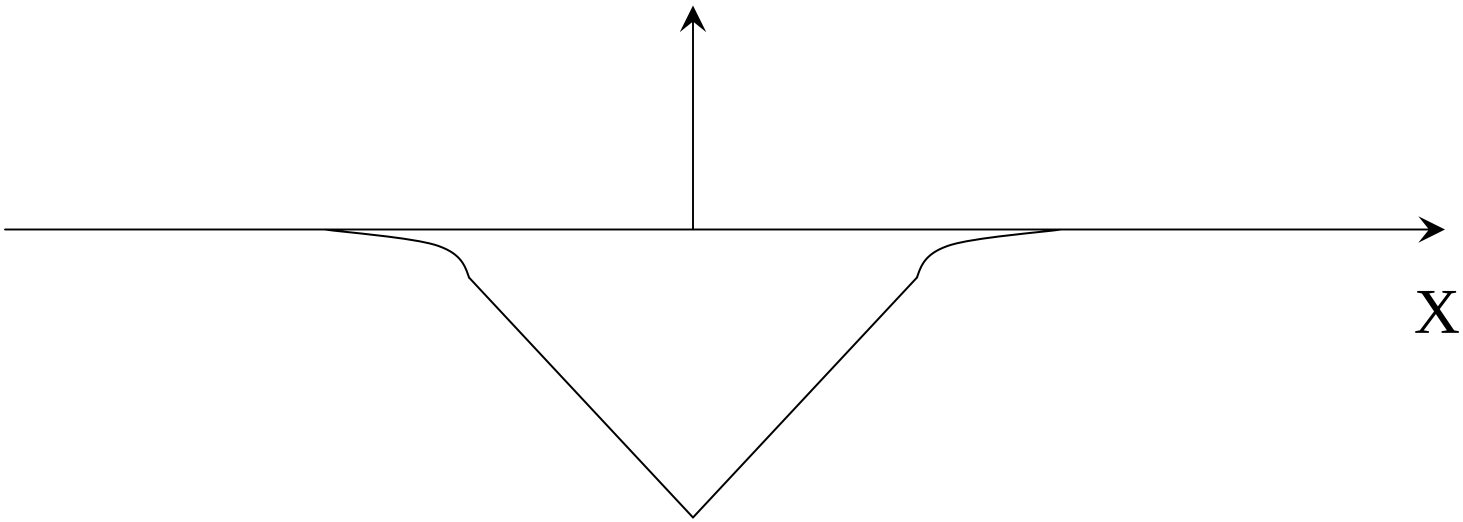
Relative Permittivity of Semiconductor

(previously referred to as ε_R)

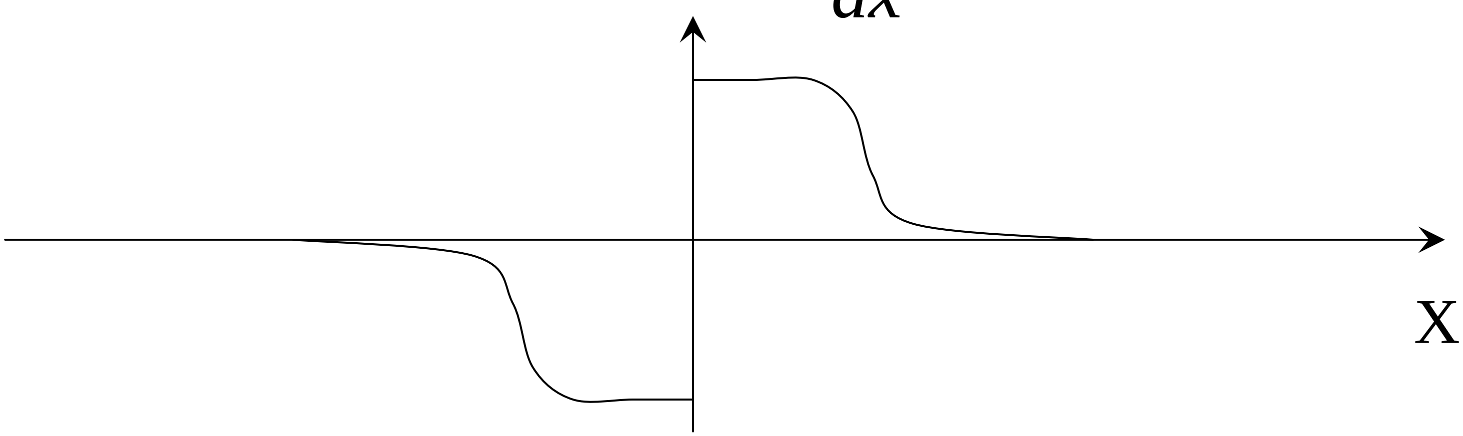
$$\rho = q(p - n + N_D - N_A)$$

Our First Device: p-n Junction Diode

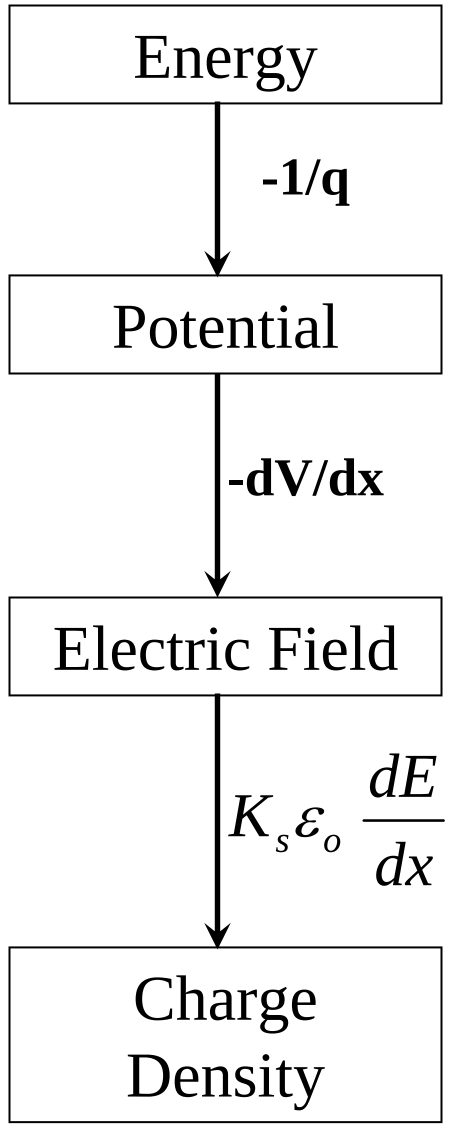
Electric Field, *E*=-dV/dx



$$\rho = K_s \varepsilon_o \frac{dE}{dx}$$



Our First Device: p-n Junction Diode



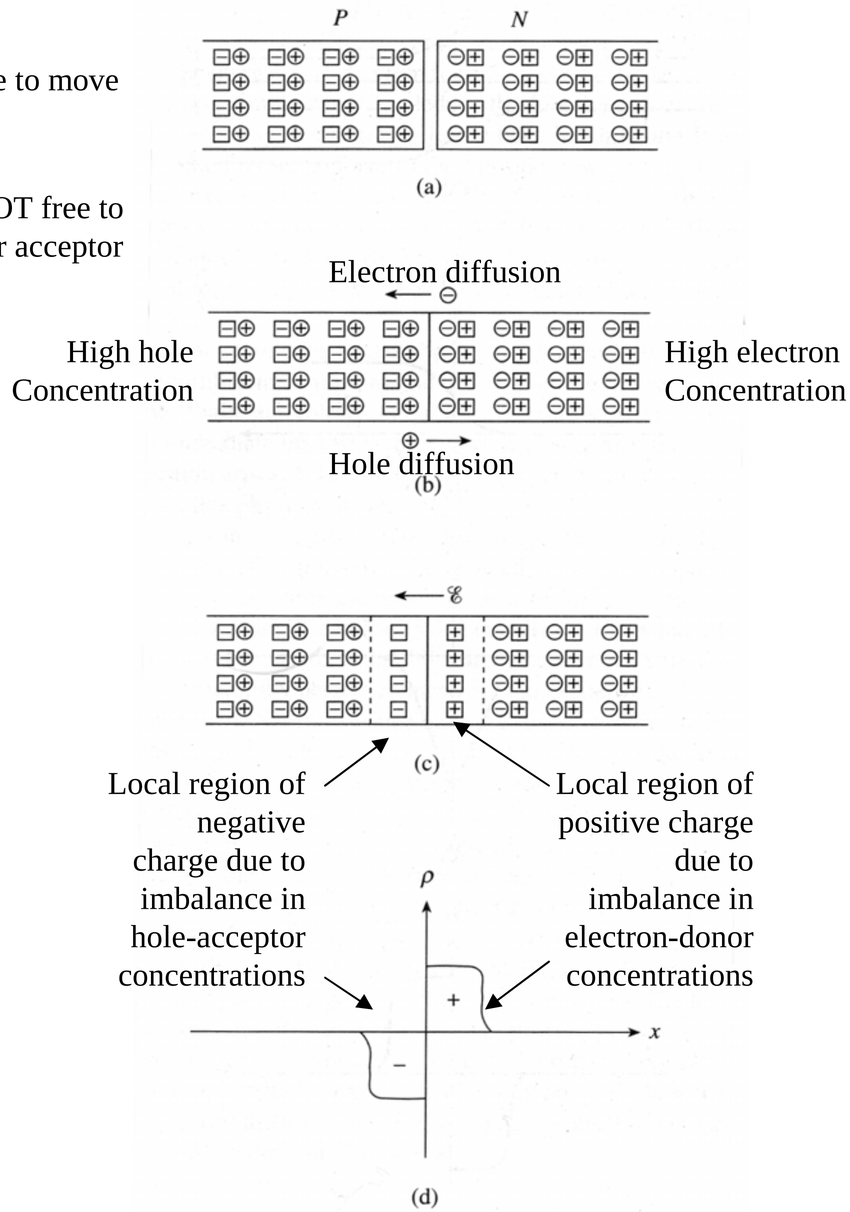
P-N Junction Diodes: Part 2

How do they work? (A little bit of math)

Movement of electrons and holes when forming the junction

Circles are charges free to move (electrons and holes)

Squares are charges NOT free to move (ionized donor or acceptor atoms)



Space Charge or Depletion Region

Movement of electrons and holes when forming the junction

E= - dV/dx

-Edx=dV

$$-\int_{-x_p}^{x_n} E\,dx = \int_{V(-x_p)}^{V(x_n)} dV \quad = \quad V(x_n) - V(-x_p) = V_{bi}$$

but...

$$J_N = q\mu_n nE + qD_N \frac{dn}{dx} = 0$$

← No net current flow in equilibrium

$$E = -\frac{D_N}{\mu_n}\frac{\frac{dn}{dx}}{n} = -\frac{kT}{q}\frac{\frac{dn}{dx}}{n}$$

thus...

$$V_{bi} = -\int_{-x_p}^{x_n} E\,dx = \frac{kT}{q}\int_{n(-x_p)}^{n(x_n)} \frac{\frac{dn}{dx}}{n}dx = \frac{kT}{q}\ln\left[\frac{n(x_n)}{n(-x_p)}\right]$$

Movement of electrons and holes when forming the junction

$$V_{bi} = \frac{kT}{q}\ln\left[\frac{n(x_n)}{n(-x_p)}\right] = \frac{kT}{q}\ln\left[\frac{N_D}{n_i^2 / N_A}\right]$$

$$\boxed{V_{bi} = \frac{kT}{q}\ln\left[\frac{N_A N_D}{n_i^2}\right]}$$

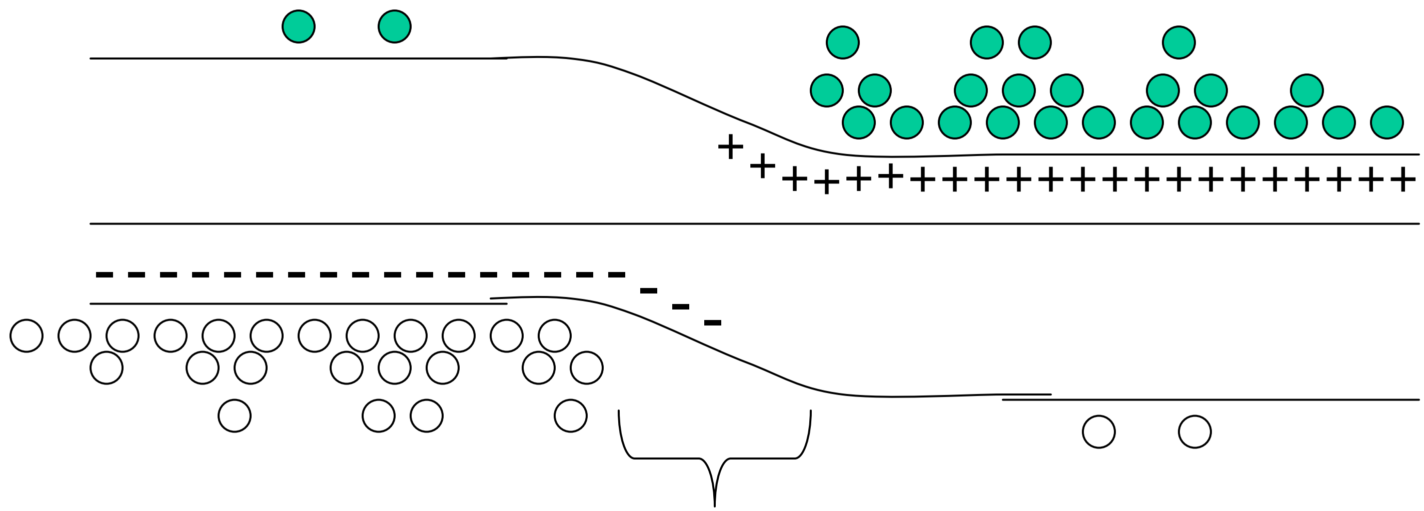
For $N_A=N_D=10^{15}/cm^{-3}$ in silicon at room temperature, V_{bi}~0.6 V*

For a non-degenerate semiconductor, $|-qV_{bi}|<|E_g|$

*Note to those familiar with a diode turn on voltage: This is not the diode turn on voltage! This is the voltage required to reach a flat band diagram and sets an upper limit (typically an overestimate) for the voltage that can be applied to a diode before it burns itself up.

Movement of electrons and holes when forming the junction

Depletion Region Approximation



Depletion Region Approximation states that approximately no free carriers exist in the space charge region and no net charge exists outside of the depletion region (known as the quasi-neutral region). Thus,

$$\frac{dE}{dx} = \frac{\rho}{K_S \varepsilon_o} = \frac{q}{K_S \varepsilon_o}(p - n + N_D - N_A) = 0 \quad \textit{within the quasi-neutral region}$$

becomes...

$$\frac{dE}{dx} = \frac{q}{K_S \varepsilon_o}(N_D - N_A) \quad \textit{within the space charge region}$$

Movement of electrons and holes when forming the junction

Depletion Region Approximation: Step Junction Solution

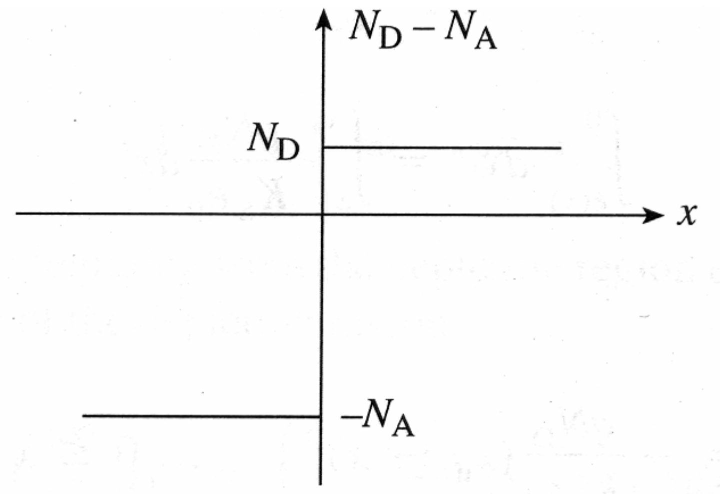
$$\rho = \begin{cases} -qN_A & for\ -x_p \le x \le 0 \\ qN_D & for\ 0 \le x \le x_n \\ 0 & for\ x \le -x_p\ and\ x \ge x_n \end{cases}$$

thus,

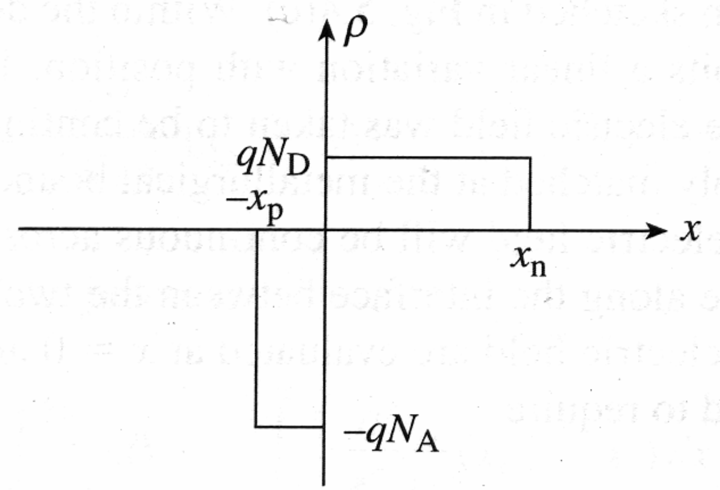
$$\frac{dE}{dx} = \begin{cases} \frac{-qN_A}{K_S \varepsilon_o} & for\ -x_p \le x \le 0 \\ \frac{qN_D}{K_S \varepsilon_o} & for\ 0 \le x \le x_n \\ 0 & for\ x \le -x_p\ and\ x \ge x_n \end{cases}$$

Where we have used:

$$\frac{dE}{dx} = \frac{\rho}{K_S \varepsilon_o}$$



(a)



Movement of electrons and holes when forming the junction

Depletion Region Approximation: Step Junction Solution

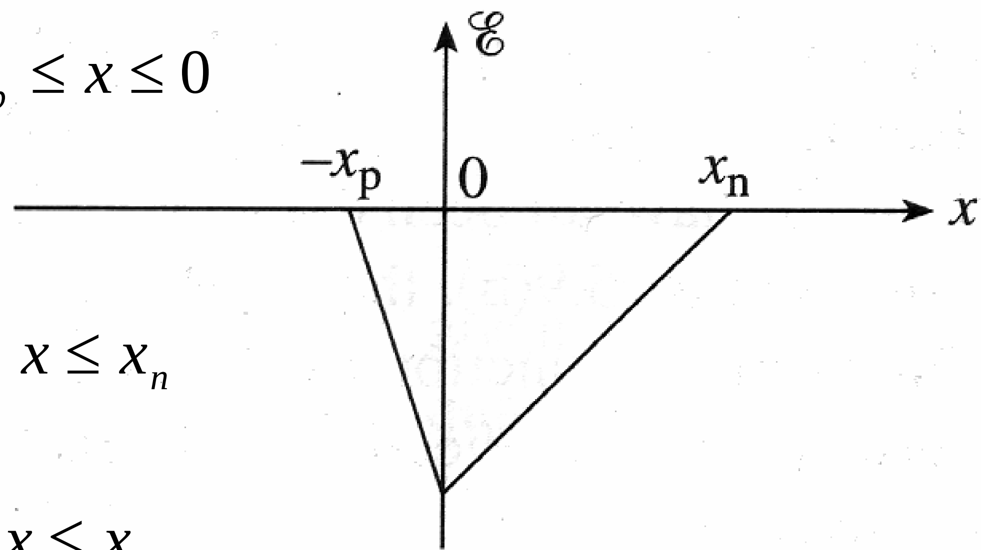
$$\int_0^{E(x)} dE' = \int_{-x_p}^{x} \frac{-qN_A}{K_S \varepsilon_o} dx' \quad for\ -x_p \le x \le 0$$

$$E(x) = \frac{-qN_A}{K_S \varepsilon_o}\left(x + x_p\right) \quad for\ -x_p \le x \le 0$$

and

$$\int_{E(x)}^{0} dE' = \int_{x}^{x_n} \frac{qN_D}{K_S \varepsilon_o} dx' \quad for\ 0 \le x \le x_n$$

$$E(x) = \frac{-qN_D}{K_S \varepsilon_o}\left(x_n - x\right) \quad for\ 0 \le x \le x_n$$



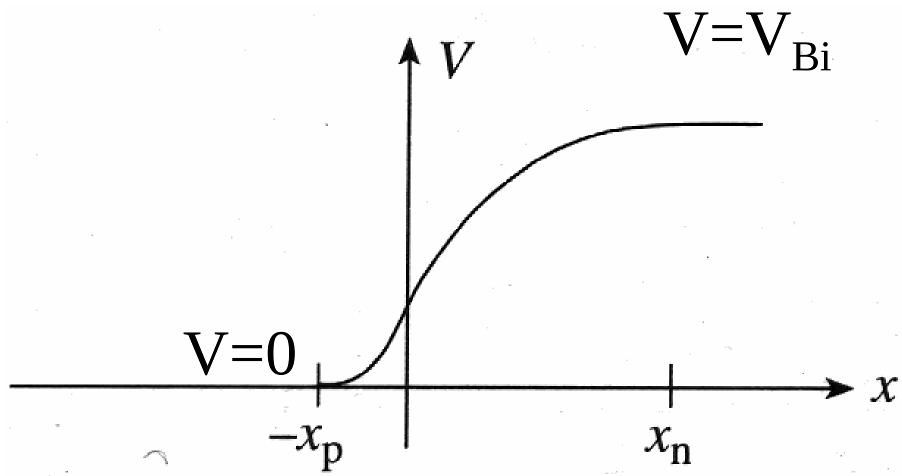
Since $E(x=0^-)=E(x=0^+)$

$$N_A x_p = N_D x_n$$

Movement of electrons and holes when forming the junction

Depletion Region Approximation: Step Junction Solution

$$E = -\frac{dV}{dx}$$



$$\frac{dV}{dx} = \begin{cases} \frac{qN_A}{K_S\varepsilon_o}(x_p + x) & for\ -x_p \le x \le 0 \\ \frac{qN_D}{K_S\varepsilon_o}(x_n - x) & for\ 0 \le x \le x_n \end{cases}$$

or,

$$\int_0^{V(x)} dV' = \int_{-x_p}^{x} \frac{qN_A}{K_S\varepsilon_o}(x_p + x')dx' \quad for\ -x_p \le x \le 0$$

$$\int_{V(x)}^{V_{Bi}} dV' = \int_{x}^{x_n} \frac{qN_D}{K_S\varepsilon_o}(x_n - x')dx' \quad for\ 0 \le x \le x_n$$

$$V(x) = \begin{cases} \frac{qN_A}{2K_S\varepsilon_o}(x_p + x)^2 & for\ -x_p \le x \le 0 \\ V_{bi} - \frac{qN_D}{2K_S\varepsilon_o}(x_n - x)^2 & for\ 0 \le x \le x_n \end{cases}$$

Movement of electrons and holes when forming the junction

Depletion Region Approximation: Step Junction Solution

At x=0,

$$\frac{qN_A}{2K_S\varepsilon_o}\left(x_p\right)^2 = V_{bi} - \frac{qN_D}{2K_S\varepsilon_o}\left(x_n\right)^2$$

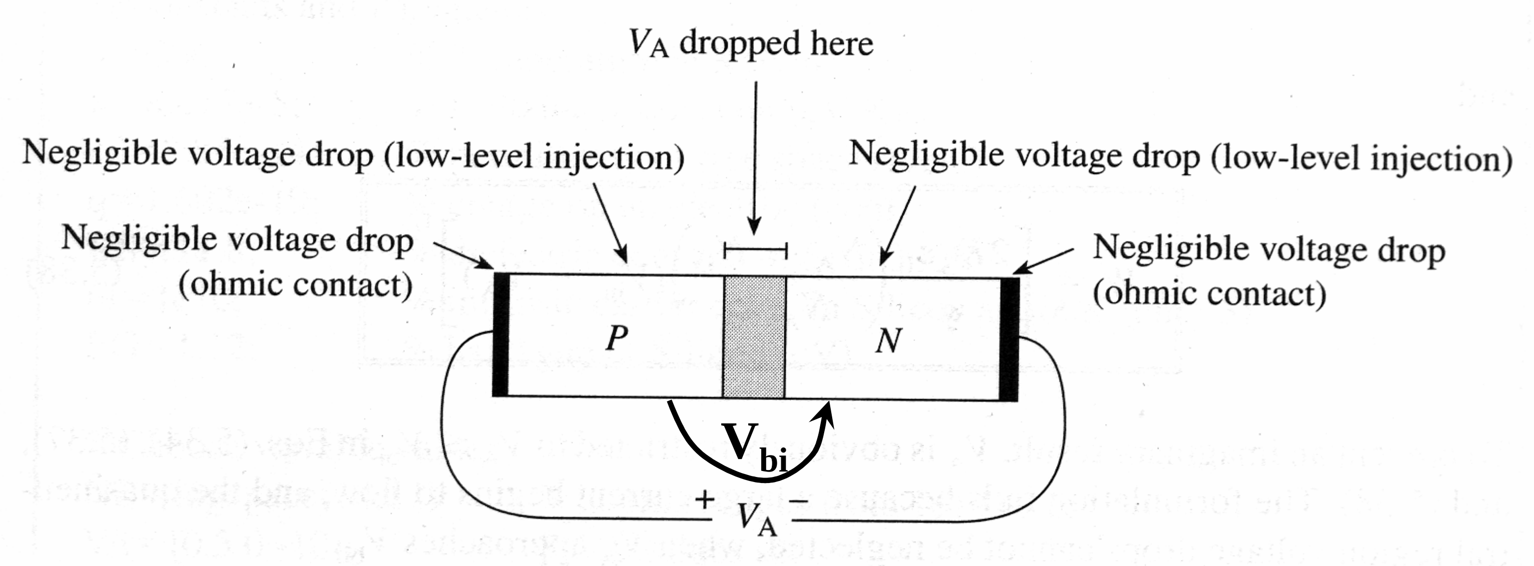
$$Using,\ x_p = \left(x_n N_D\right) / N_A$$

$$x_n = \sqrt{\frac{2K_S\varepsilon_o}{q}\frac{N_A}{N_D\left(N_A+N_D\right)}V_{bi}} \quad and \quad x_p = \sqrt{\frac{2K_S\varepsilon_o}{q}\frac{N_D}{N_A\left(N_A+N_D\right)}V_{bi}}$$

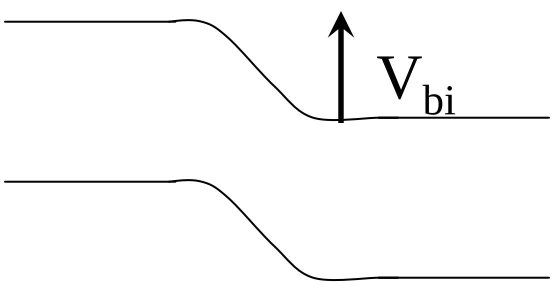
$$W = x_p + x_n = \sqrt{\frac{2K_S\varepsilon_o}{q}\frac{\left(N_A+N_D\right)}{N_A N_D}V_{bi}}$$

Movement of electrons and holes when forming the junction

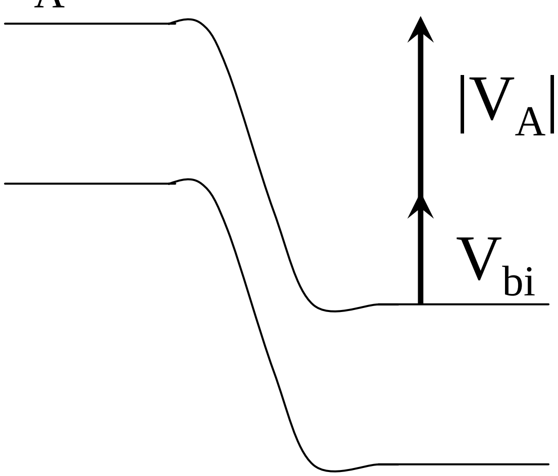
Depletion Region Approximation: Step Junction Solution



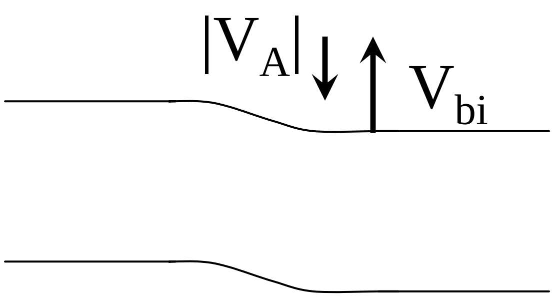
V_A=0 : No Bias



V_A<0 : Reverse Bias



V_A>0 : Forward Bias



Movement of electrons and holes when forming the junction

Depletion Region Approximation: Step Junction Solution

Thus, only the boundary conditions change resulting in direct replacement of V_{bi} with $(V_{bi}-V_A)$

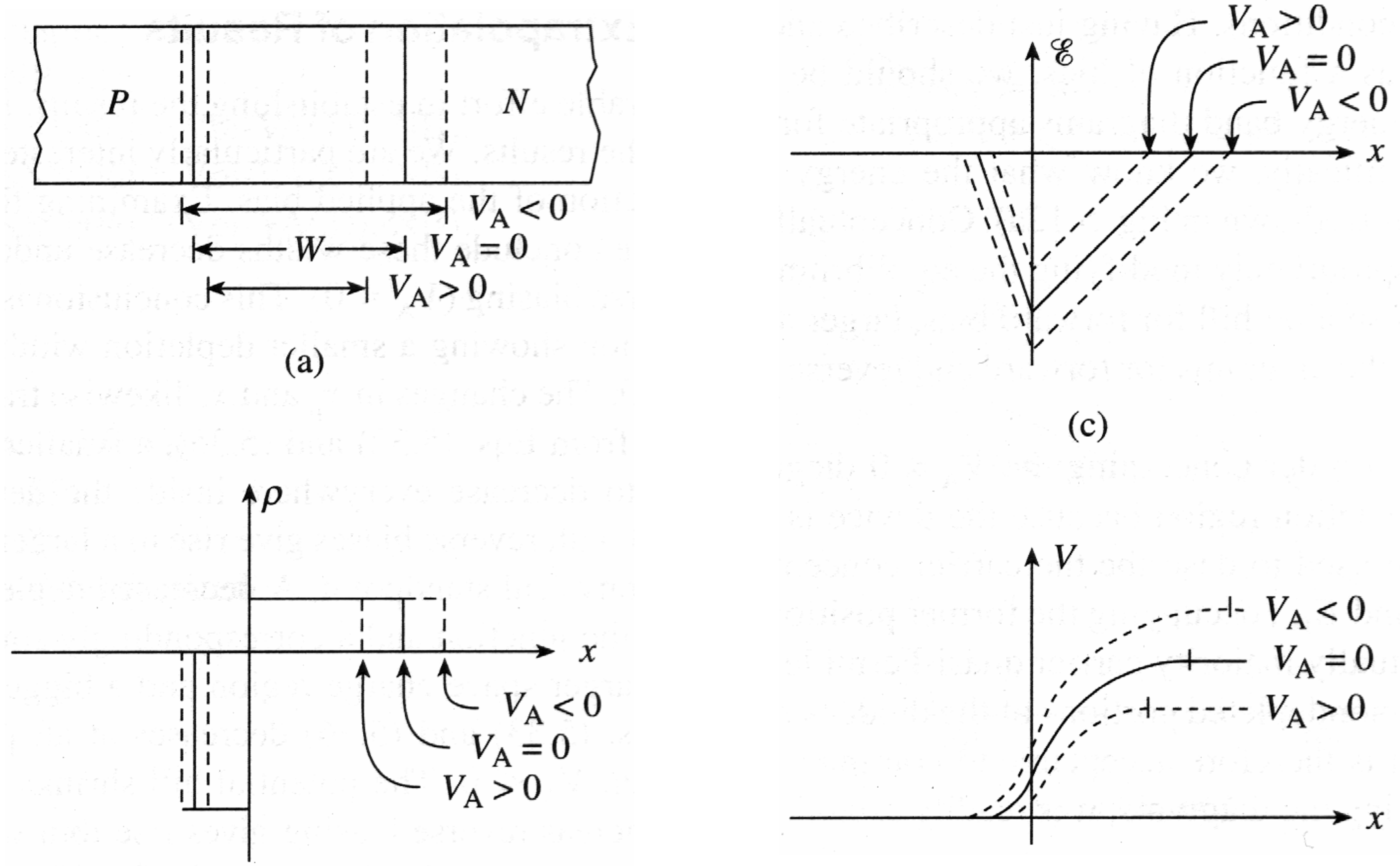
$$x_n = \sqrt{\frac{2K_S\varepsilon_o}{q}\frac{N_A}{N_D\left(N_A+N_D\right)}\left(V_{bi}-V_A\right)} \quad and \quad x_p = \sqrt{\frac{2K_S\varepsilon_o}{q}\frac{N_D}{N_A\left(N_A+N_D\right)}\left(V_{bi}-V_A\right)}$$

$$W = x_p + x_n = \sqrt{\frac{2K_S\varepsilon_o}{q}\frac{\left(N_A+N_D\right)}{N_A N_D}\left(V_{bi}-V_A\right)}$$

Movement of electrons and holes when forming the junction

Step Junction Solution: What does it mean?

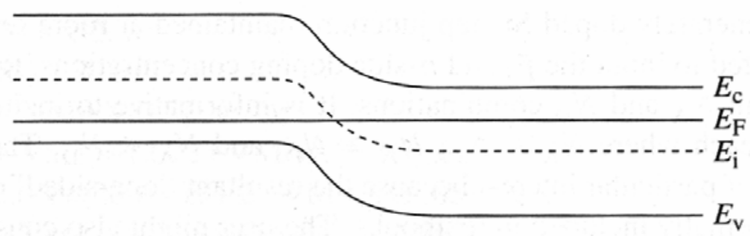
Consider a p^+ -n junction (heavily doped p-side, normal or lightly doped n side).



Movement of electrons and holes when forming the junction

Step Junction Solution: What does it mean?

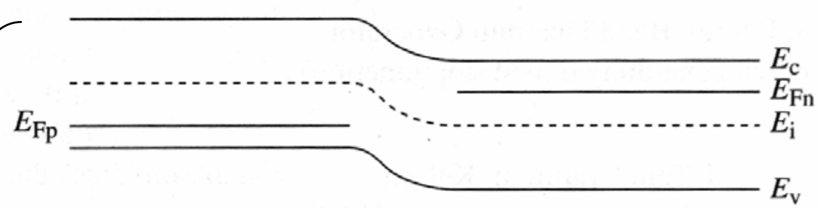
Fermi-level only applies to equilibrium (no current flowing)



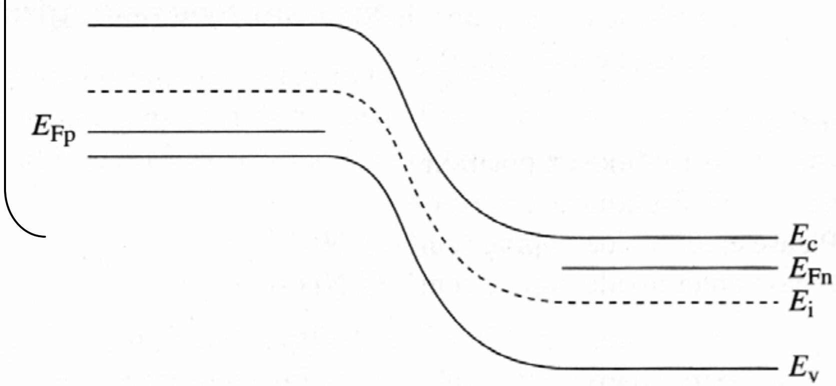
(a) Equilibrium ($V_A = 0$)

Majority carrier Quasi-fermi levels

$$E_{fp}-E_{fn}=-qV_A$$



(b) Forward bias ($V_A > 0$)



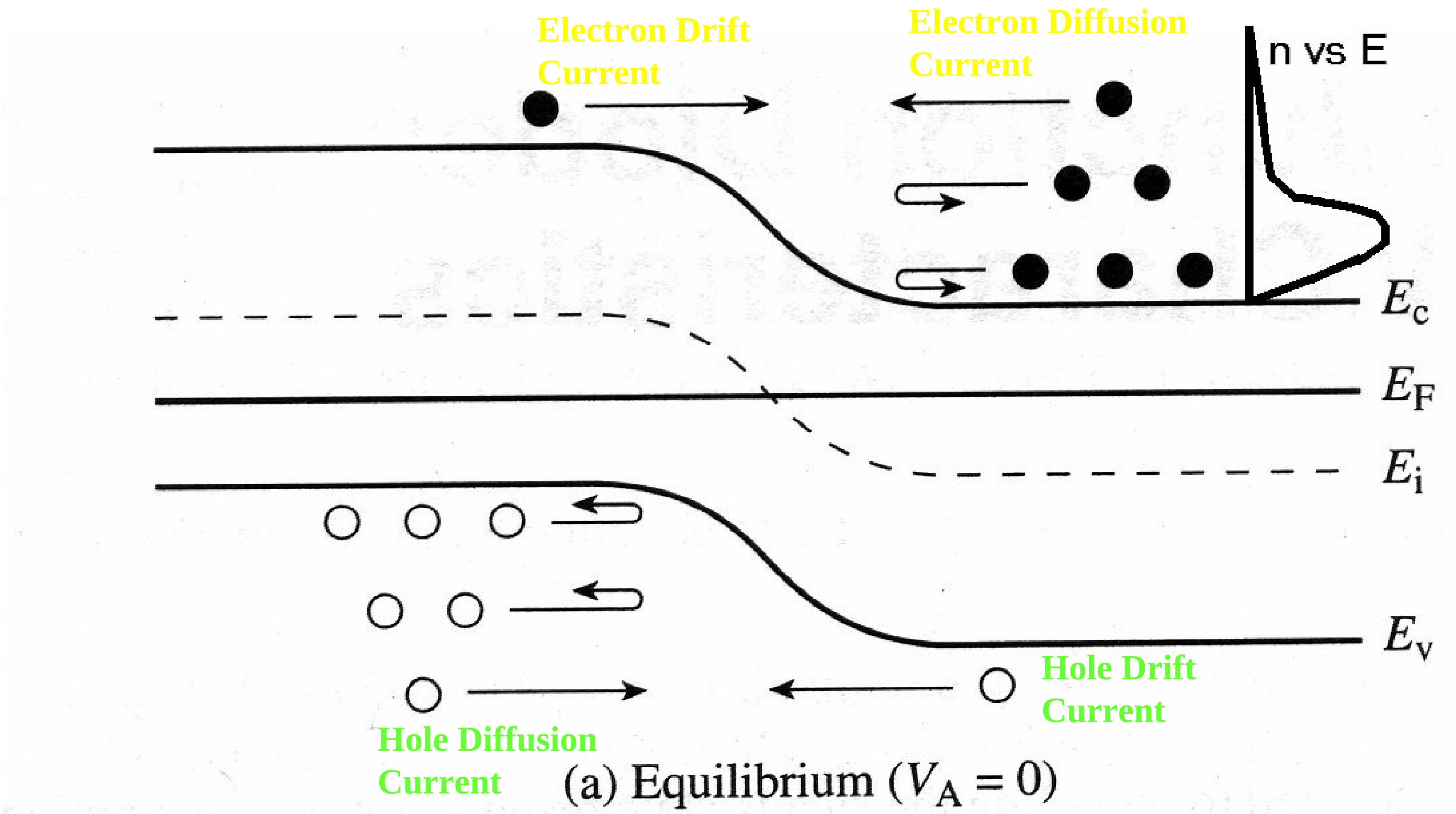
(c) Reverse bias ($V_A < 0$)

P-N Junction Diodes: Part 3

Current Flowing through a Diode

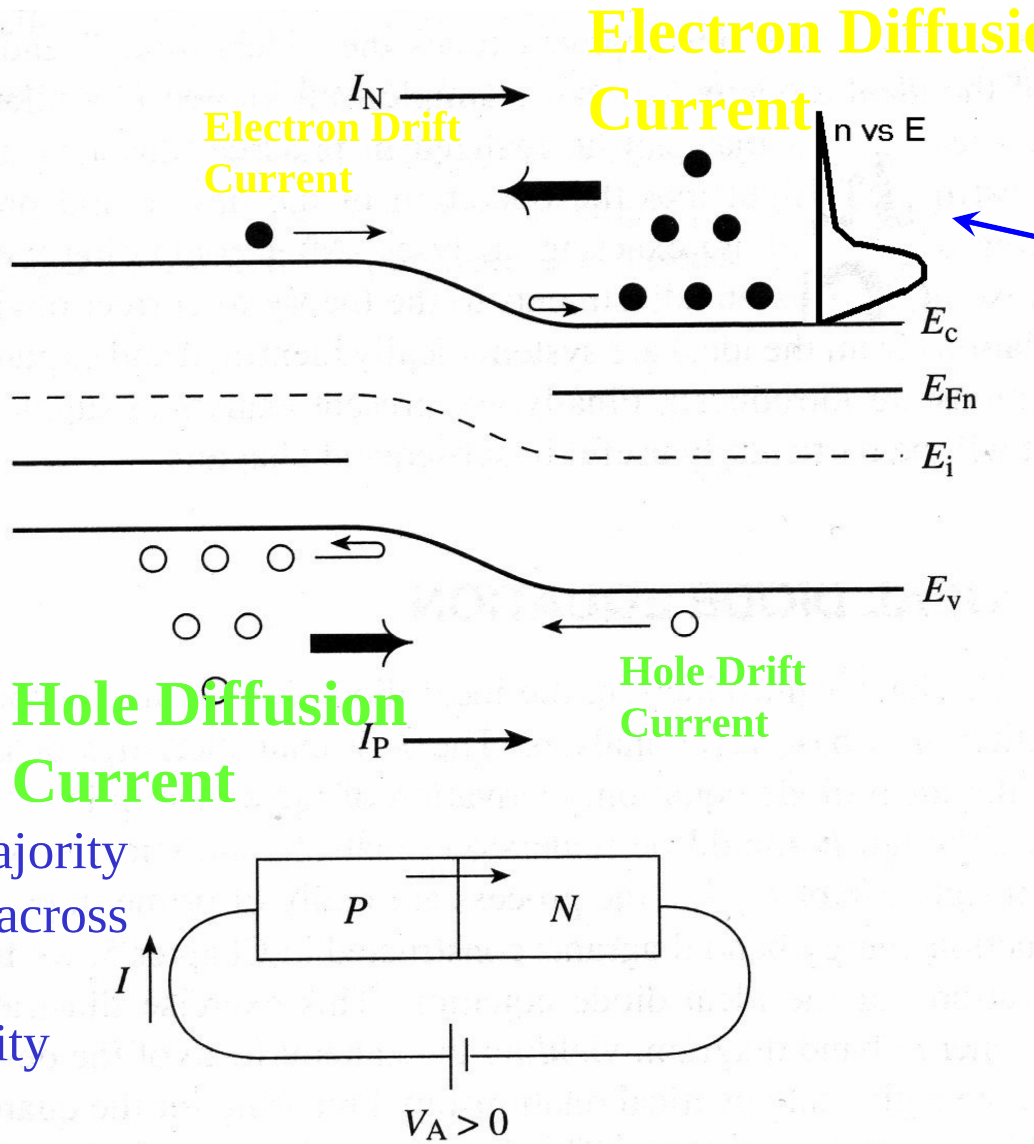
P-n Junction I-V Characteristics

In Equilibrium, the Total current balances due to the sum of the individual components



(a) Equilibrium ($V_A = 0$)

P-n Junction I-V Characteristics



(b) Forward bias ($V_A > 0$)

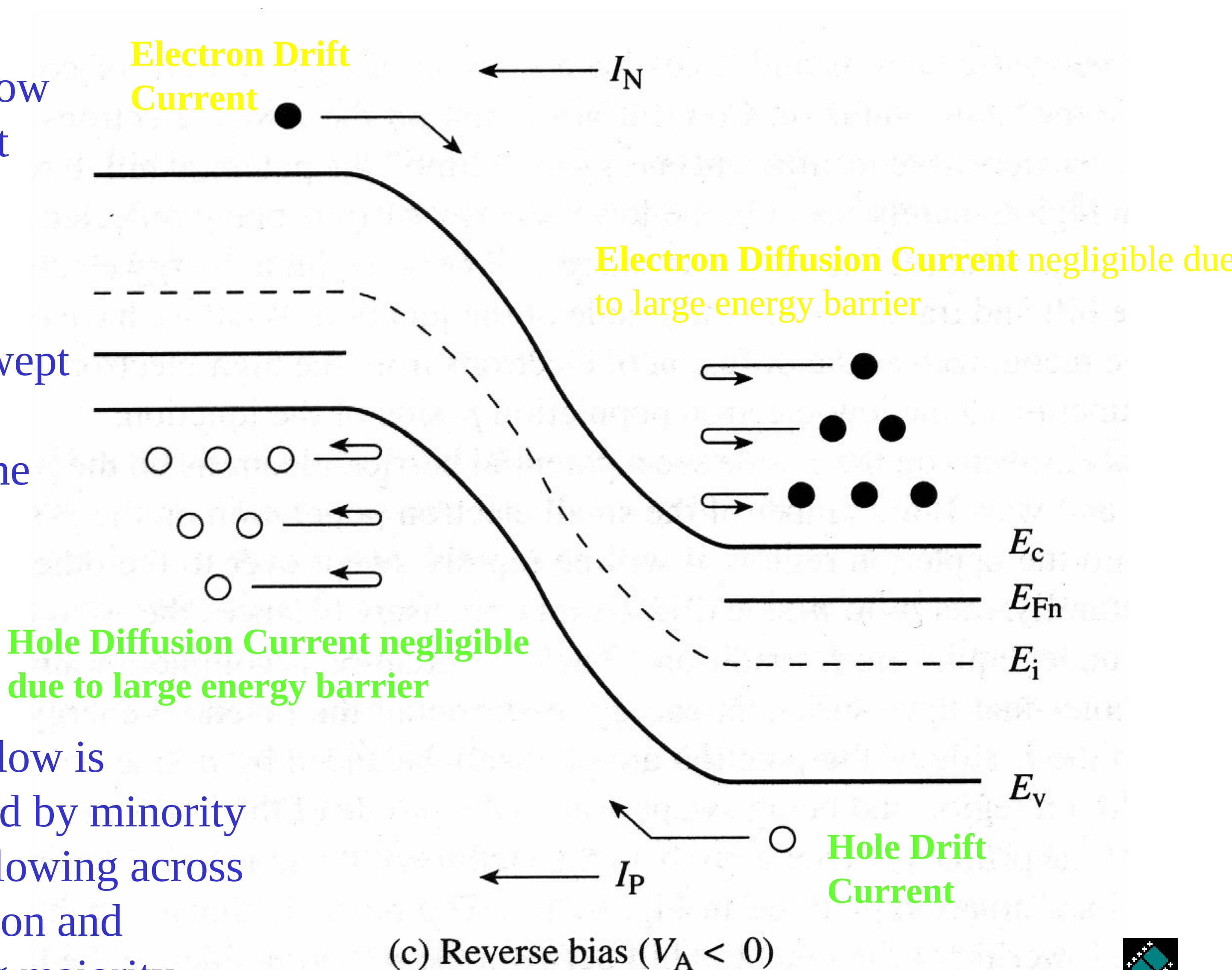
Current flow is proportional to $e^{(Va/Vref)}$ due to the exponential decay of carriers into the majority carrier bands

Current flow is dominated by majority carriers flowing across the junction and becoming minority carriers

QuickTime Movie

P-n Junction I-V Characteristics

Current flow is constant due to thermally generated carriers swept out by E-fields in the depletion region

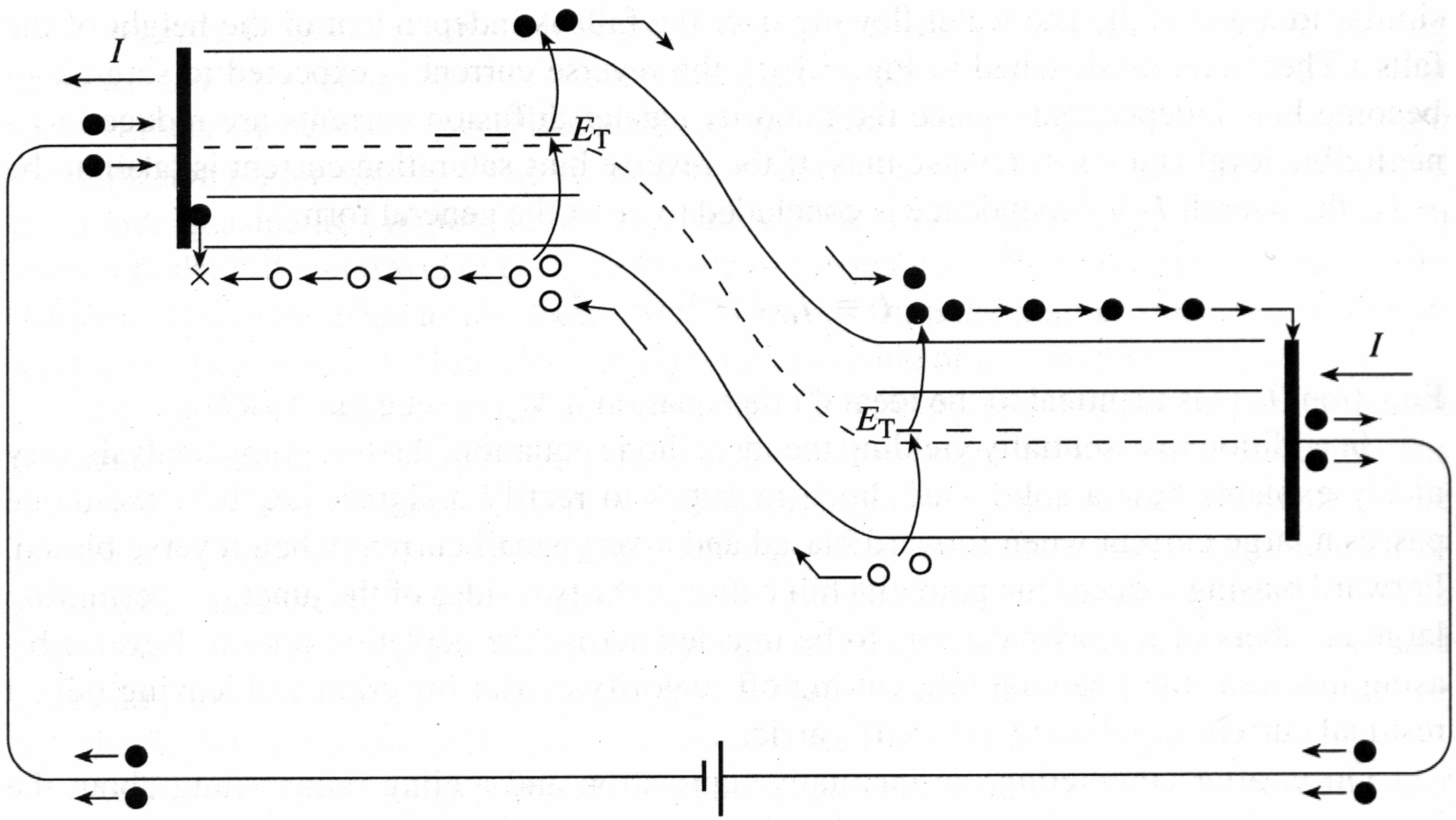


Current flow is dominated by minority carriers flowing across the junction and becoming majority carriers

QuickTime Movie

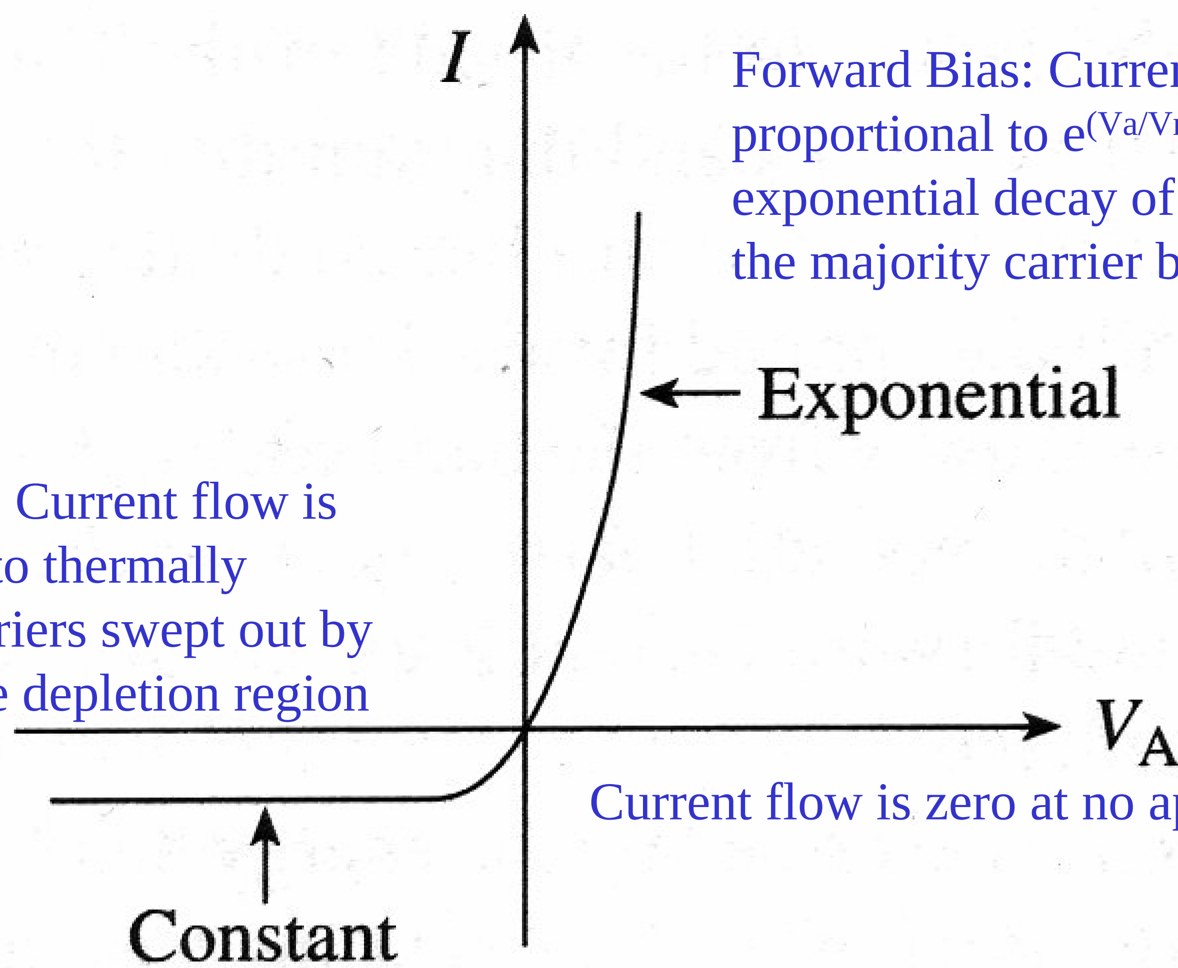
P-n Junction I-V Characteristics

Where does the reverse bias current come from? Generation near the depletion region edges "replenishes" the current source.



P-n Junction I-V Characteristics
Putting it all together

Forward Bias: Current flow is proportional to $e^{(Va/Vref)}$ due to the exponential decay of carriers into the majority carrier bands



Reverse Bias: Current flow is constant due to thermally generated carriers swept out by E-fields in the depletion region

Current flow is zero at no applied voltage

$$I=I_o(e^{Va/Vref} - 1)$$

P-N Junction Diodes: Part 3

Quantitative Analysis (Math, math and more math)

Quantitative p-n Diode Solution

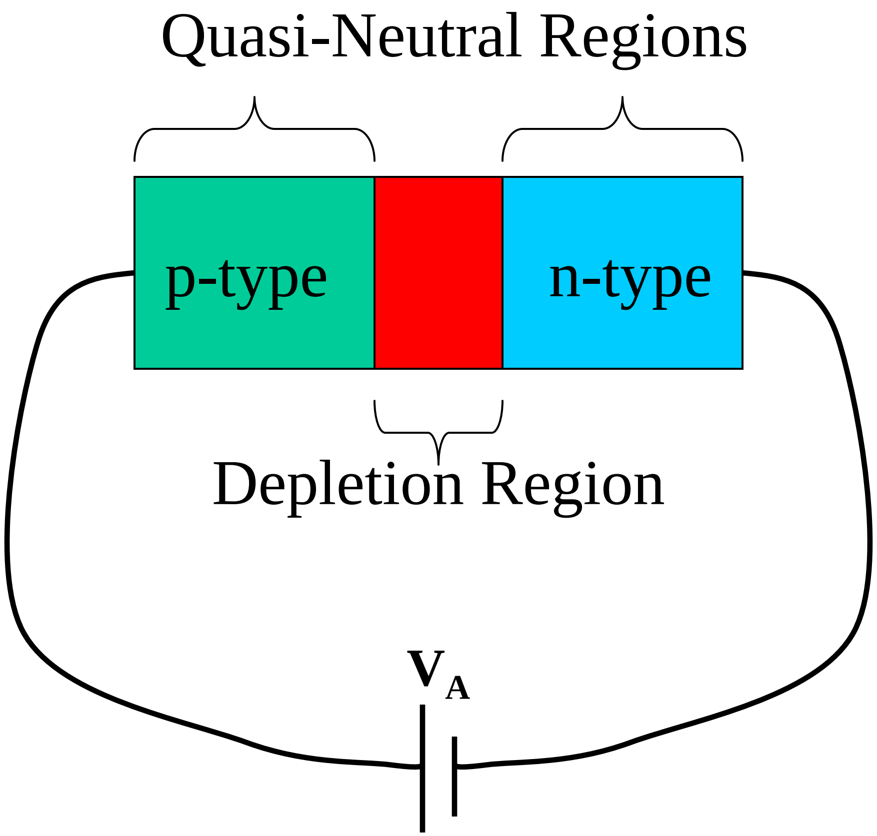
Assumptions:
1) steady state conditions
2) non- degenerate doping
3) one- dimensional analysis
4) low- level injection
5) no light ($G_L = 0$)

Current equations:

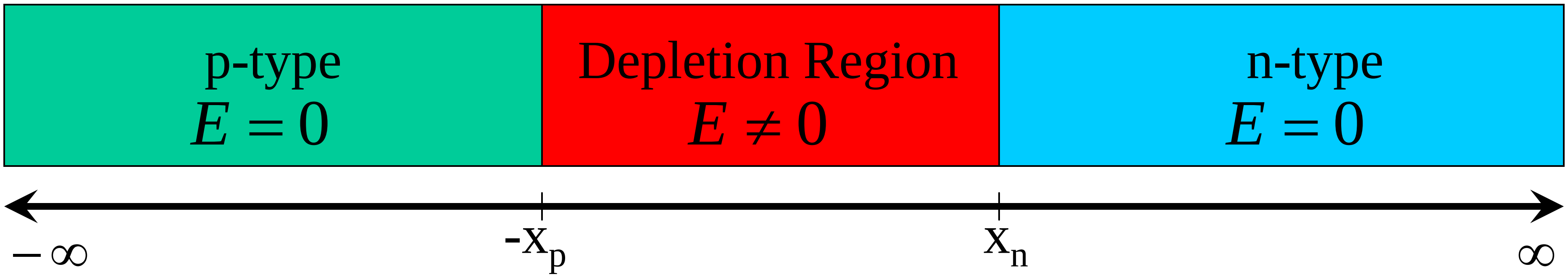
$J = J_p(x) + J_n(x)$

$J_n = q\,\mu_n n E + qD_n (dn/dx)$

$J_p = q\,\mu_p p E - qD_p\,(dp/dx)$



Quantitative p-n Diode Solution



Application of the Minority Carrier Diffusion Equation

$$\frac{\partial(\Delta n_p)}{\partial t} = D_N \frac{\partial^2(\Delta n_p)}{\partial x^2} - \frac{(\Delta n_p)}{\tau_n} + G_L$$

$$0 = D_N \frac{\partial^2(\Delta n_p)}{\partial x^2} - \frac{(\Delta n_p)}{\tau_n}$$

Since electric fields exist in the depletion region, the minority carrier diffusion equation does not apply here.

$$\frac{\partial(\Delta p_n)}{\partial t} = D_P \frac{\partial^2(\Delta p_n)}{\partial x^2} - \frac{(\Delta p_n)}{\tau_p} + G_L$$

$$0 = D_P \frac{\partial^2(\Delta p_n)}{\partial x^2} - \frac{(\Delta p_n)}{\tau_p} + 0$$

Boundary Condition :

$$\Delta n_p(x \to -\infty) = 0$$

Boundary Condition :

$$\Delta n_p(x = -x_p) = ?$$

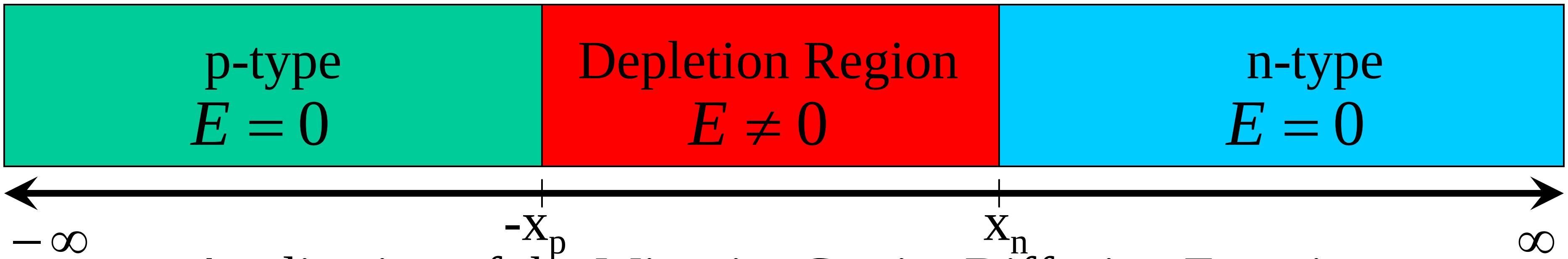
Boundary Condition :

$$\Delta p_n(x = x_n) = ?$$

Boundary Condition :

$$\Delta p_n(x \to \infty) = 0$$

Quantitative p-n Diode Solution



Application of the Minority Carrier Diffusion Equation

Boundary Condition: $\Delta n_p(x = -x_p) = ?$

Boundary Condition: $\Delta p_n(x = x_n) = ?$

$$n = n_i e^{(F_N - E_i)/kT} \quad and \quad p = n_i e^{(E_i - F_P)/kT}$$

$$n_p(x = -x_p)\, p_p(x = -x_p) = n_i^2 e^{(F_N - F_P)/kT}$$

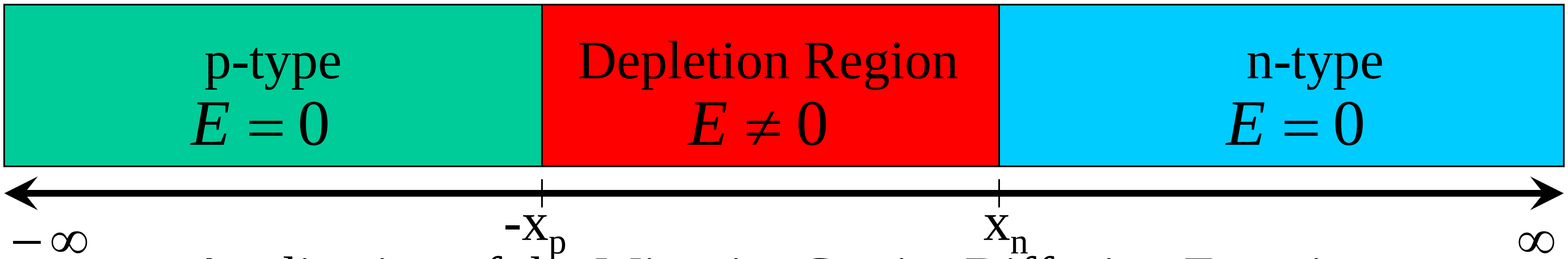
$$n_p(x = -x_p)\, p_p(x = -x_p) = n_p(x = -x_p) N_A = n_i^2 e^{qV_A/kT}$$

$$n_p(x = -x_p) = \frac{n_i^2}{N_A} e^{qV_A/kT}$$

$$\Delta n_p(x = -x_p) = \frac{n_i^2}{N_A} e^{qV_A/kT} - n_o$$

$$\Delta n_p(x = -x_p) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right) \quad and\ similarly\ at\ x = x_n \quad \Delta p_n(x = x_n) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$$

Quantitative p-n Diode Solution



Application of the Minority Carrier Diffusion Equation

Boundary Condition:

$$\Delta n_p(x=-x_p)=?$$

Boundary Condition:

$$\Delta p_n(x=x_n)=?$$

$$n = n_i e^{(F_N - E_i)/kT} \quad \text{and} \quad p = n_i e^{(E_i - F_P)/kT}$$

$$n_p(x=-x_p)\,p_p(x=-x_p) = n_i^2 e^{(F_N - F_P)/kT}$$

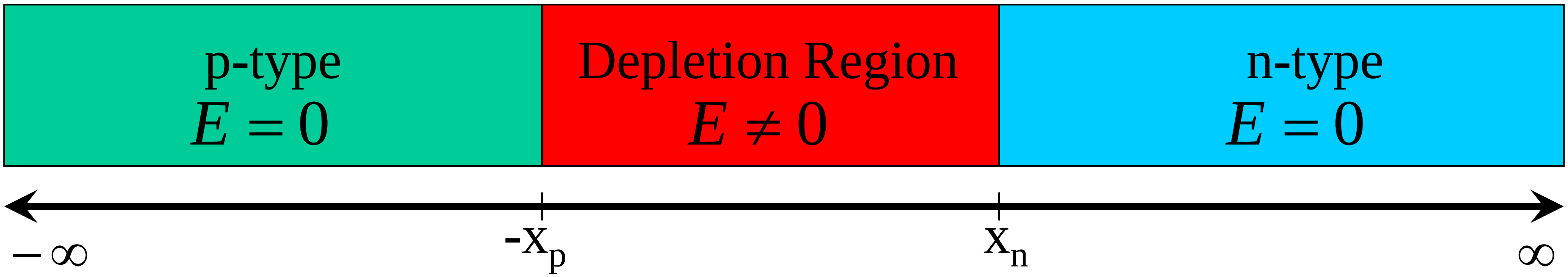
$$n_p(x=-x_p)\,p_p(x=-x_p) = n_p(x=-x_p)\,N_A = n_i^2 e^{qV_A/kT}$$

$$n_p(x=-x_p) = \frac{n_i^2}{N_A} e^{qV_A/kT}$$

$$\Delta n_p(x=-x_p) = \frac{n_i^2}{N_A} e^{qV_A/kT} - n_o$$

$$\Delta n_p(x=-x_p) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right) \quad \text{and similarly at } x = x_n \quad \Delta p_n(x=x_n) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$$

Quantitative p-n Diode Solution



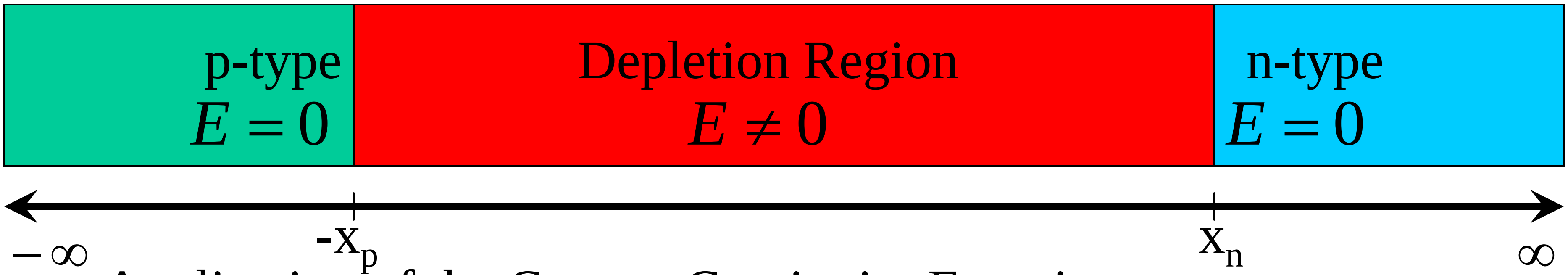
Application of the Current Continuity Equation

$$J_n = q\left(\mu_n nE + D_n \frac{dn}{dx}\right)$$
$$= qD_n \frac{d(n_o + \Delta n_p)}{dx}$$
$$= qD_n \frac{d\Delta n_p}{dx}$$

?

$$J_p = q\left(\mu_p pE - D_p \frac{dp}{dx}\right)$$
$$= -qD_p \frac{d(p_o + \Delta p_n)}{dx}$$
$$= -qD_p \frac{d\Delta p_n}{dx}$$

Quantitative p-n Diode Solution



Application of the Current Continuity Equation: Depletion Region

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \cdot J_N + \left.\frac{\partial n}{\partial t}\right|_{Recombination-Generation} + \left.\frac{\partial n}{\partial t}\right|_{\substack{All\ other\ processes \\ such\ as\ light,\ etc...}}$$

$$0 = \frac{1}{q}\nabla \cdot J_N$$

$$0 = \frac{1}{q}\frac{\partial J_N}{\partial x}$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \cdot J_P + \left.\frac{\partial p}{\partial t}\right|_{Recombination-Generation} + \left.\frac{\partial p}{\partial t}\right|_{\substack{All\ other\ processes \\ such\ as\ light,\ etc...}}$$

$$0 = -\frac{1}{q}\nabla \cdot J_P$$

$$0 = -\frac{1}{q}\frac{\partial J_P}{\partial x}$$

No thermal recombination and generation implies J_n and J_p are constant throughout the depletion region. Thus, the total current can be define in terms of only the current at the depletion region edges.

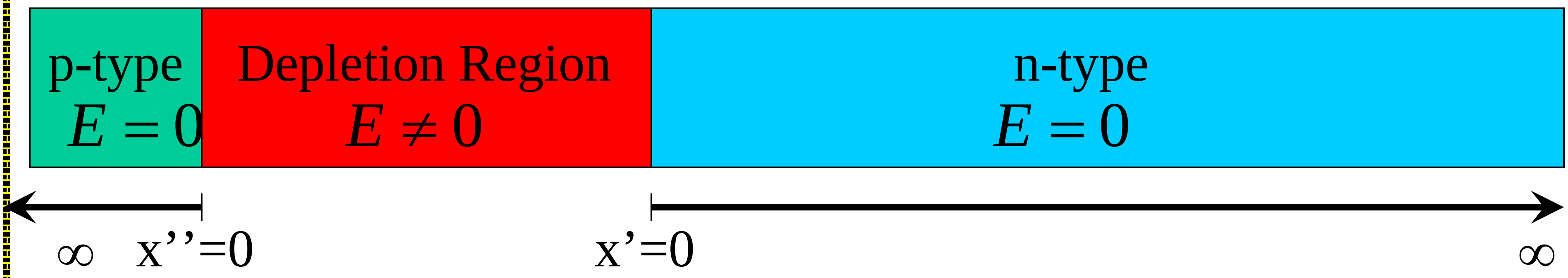
$$J = J_n(-x_p) + J_p(x_n)$$

Quantitative p-n Diode Solution

Approach:

- Solve minority carrier diffusion equation in quasi-neutral regions
- Determine minority carrier currents from continuity equation
- Evaluate currents at the depletion region edges
- Add these together and multiply by area to determine the total current through the device.
- Use translated axes, $x \rightarrow x'$ and $-x \rightarrow x''$ in our solution.

Quantitative p-n Diode Solution



$$0 = D_P \frac{\partial^2(\Delta p_n)}{\partial x'^2} - \frac{(\Delta p_n)}{\tau_p}$$

$$\Delta p_n(x') = Ae^{(-x'/L_P)} + Be^{(+x'/L_P)} \quad where \quad L_P \equiv \sqrt{D_p \tau_p}$$

Boundary Conditions:

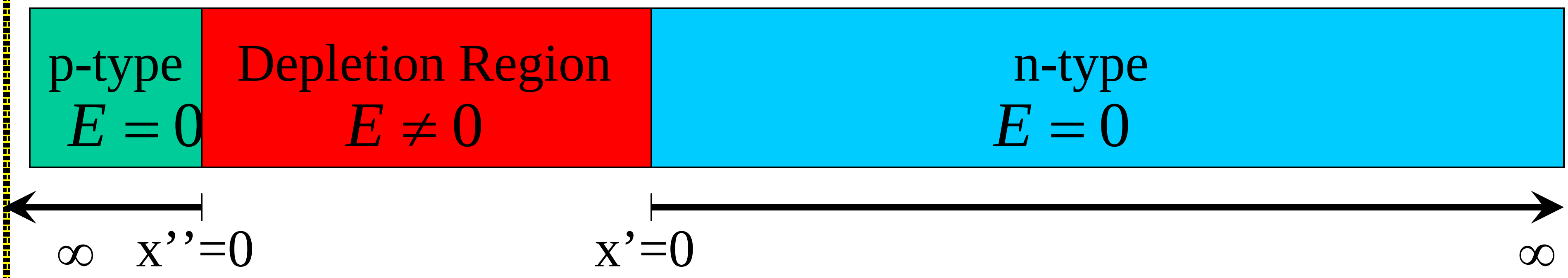
$$\Delta p_n(x' \to \infty) = 0$$

$$\Delta p_n(x'=0) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$$

$$B = 0 \quad and \quad A = \Delta p_n(x'=0) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$$

$$\boxed{\Delta p_n(x') = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)e^{(-x'/L_P)} \quad for\ x' \geq 0}$$

Quantitative p-n Diode Solution

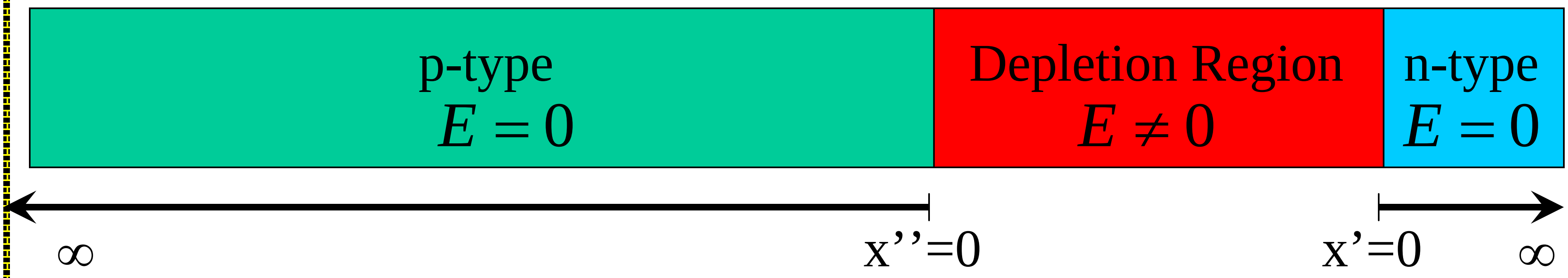


$$\Delta p_n(x') = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)e^{(-x'/L_P)} \quad for\ x' \geq 0$$

$$\mathrm{J_p} = -\mathrm{qD_p}\frac{\mathrm{d}\Delta p_\mathrm{n}}{dx}$$

$$\boxed{\mathrm{J_p} = q\frac{D_p n_i^2}{L_p N_D}\left(e^{qV_A/kT} - 1\right)e^{(-x'/L_P)} \quad for\ x' \geq 0}$$

Quantitative p-n Diode Solution



Similarly for electrons on the p-side…

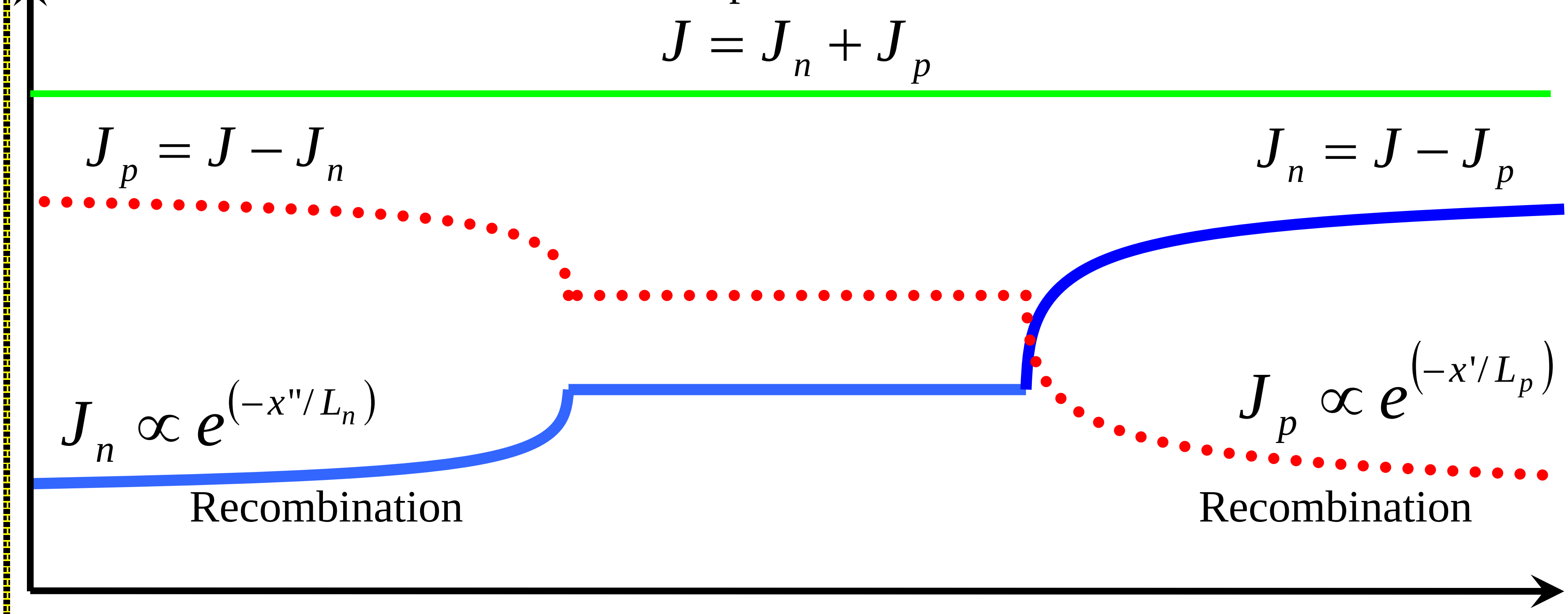
$$\Delta n_p(x'') = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)e^{(-x''/L_n)} \quad for\ x'' \geq 0$$

$$\mathrm{J_n} = -\mathrm{q}\mathrm{D_n}\frac{\mathrm{d}\Delta n_{\mathrm{p}}}{dx}$$

$$\mathrm{J_n} = q\frac{D_n n_i^2}{L_n N_A}\left(e^{qV_A/kT} - 1\right)e^{(-x''/L_n)} \quad for\ x'' \geq 0$$

Quantitative p-n Diode Solution

Total on current is constant throughout the device. Thus, we can characterize the current flow components as…



Quantitative p-n Diode Solution

Thus, evaluating the current components at the depletion region edges, we have…

$$J = J_n(x''=0) + J_p(x'=0) = J_n(x''=0) + J_p(x''=0) = J_n(x'=0) + J_p(x'=0)$$

$$J = q\left(\frac{D_n n_i^2}{L_n N_A} + \frac{D_p n_i^2}{L_p N_D}\right)\left(e^{qV_A/kT} - 1\right) \quad \textit{for all } x$$

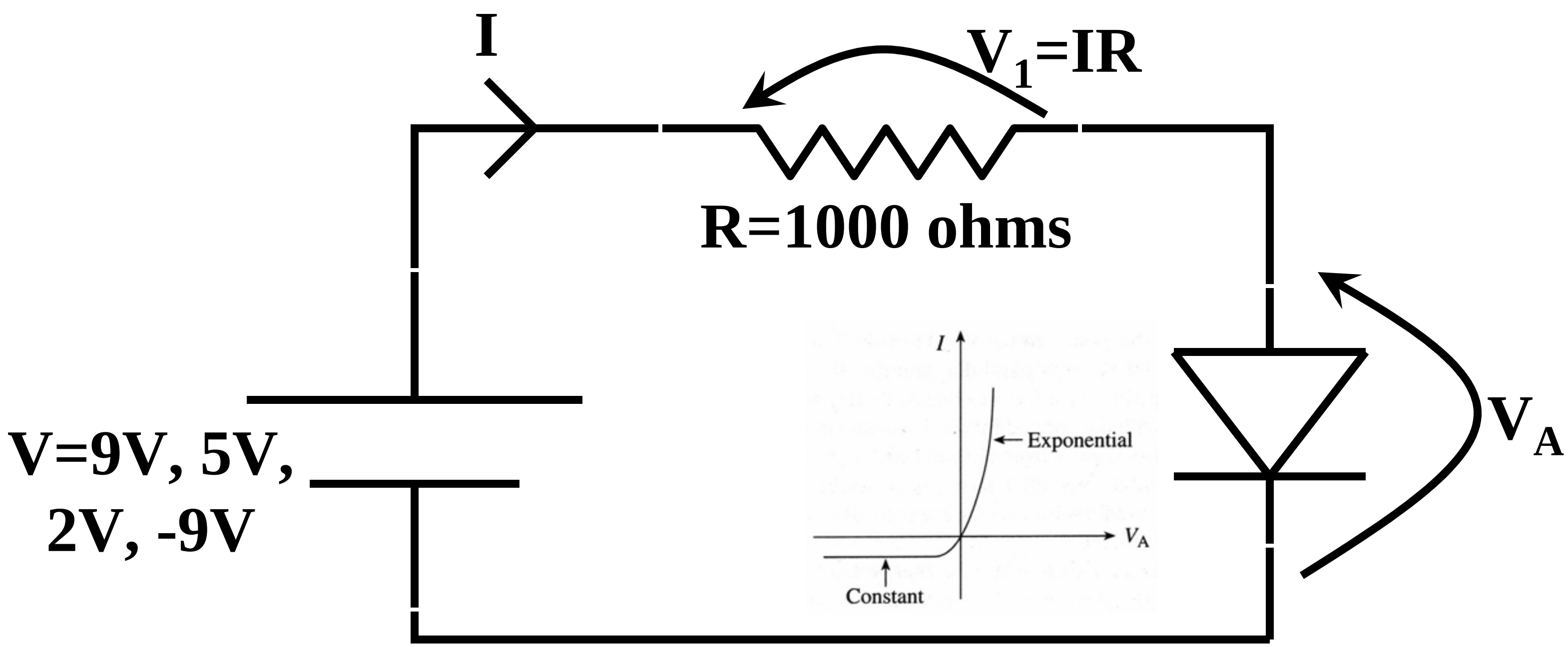
or

$$I = I_o\left(e^{qV_A/kT} - 1\right) \quad \textit{where } I_o = qA\left(\frac{D_n n_i^2}{L_n N_A} + \frac{D_p n_i^2}{L_p N_D}\right)$$

I_o *is the "reverse saturation current"*

Note: Vref from our previous qualitative analysis equation is the thermal voltage, kT/q

Quantitative p-n Diode Solution
Examples: Diode in a circuit



$$9\text{V} = \text{I}(1000) + \text{V}_\text{A}$$

$$\text{I} = 1e-12\left(e^{V_A/0.0259V} - 1\right)$$

or

$$9\text{V} = \left[1e-12\left(e^{V_A/0.0259V} - 1\right)\right](1000) + \text{V}_\text{A}$$

$$9\text{V} = 1e-9\left(e^{V_A/0.0259V} - 1\right) + \text{V}_\text{A}$$

$$\text{I} = I_o\left(e^{qV_A/kT} - 1\right) \quad where\ I_o = 1\ pA$$

Solutions

V	V_A	I
9V	0.59V	8.4 mA
5V	0.58V	4.4 mA
2V	0.55V	1.5 mA
-9V	-9.0V	-1 pA

In forward bias (V_A>0) the V_A is ~constant for large differences in current

In reverse bias (V_A<0) the current is ~constant (=saturation current)