

# **Lecture 2**

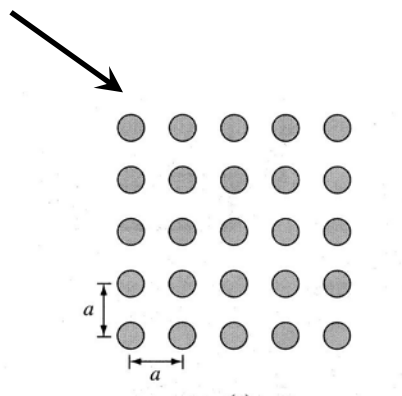
## **Unit Cells and Miller Indexes**

**Reading:**

**(Cont'd) Anderson<sup>2</sup> 1.8, 2.1-2.7**

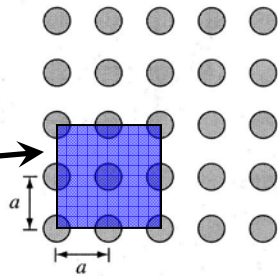
# Unit Cell Concept

The crystal lattice consists of a periodic array of atoms.



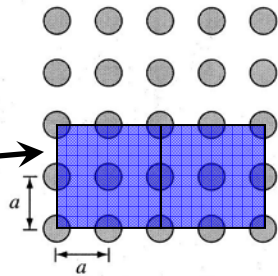
# Unit Cell Concept

A “building block” that can be periodically duplicated to result in the crystal lattice is known as the “unit cell”.



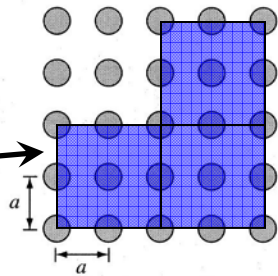
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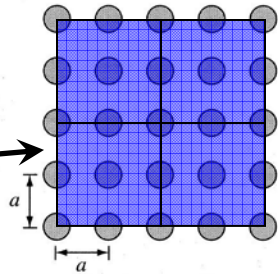
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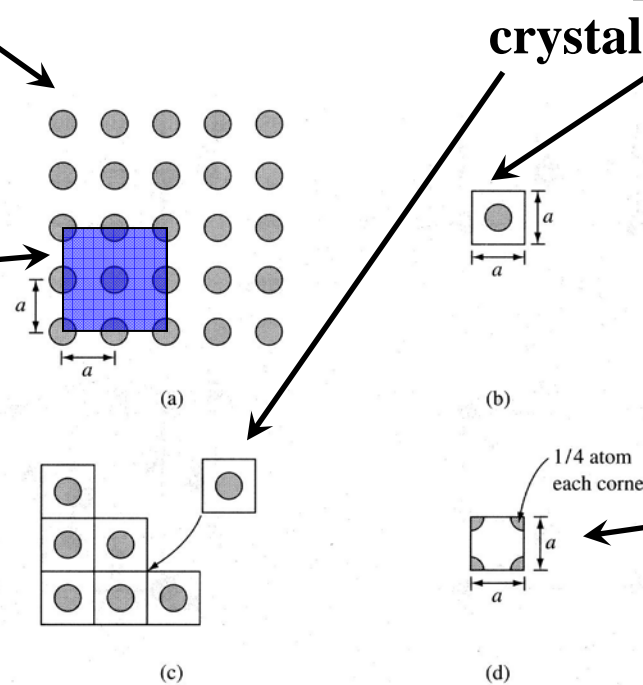
# Unit Cell Concept

The crystal lattice consists of a periodic array of atoms.

A “building block” that can be periodically duplicated to result in the crystal lattice is known as the “unit cell”.

The smallest “building block” that can be periodically duplicated to result in the crystal lattice is known as the “primitive unit cell”.

The unit cell may not be unique.



**Figure 1.2** Introduction to the unit cell method of describing atomic arrangements within crystals. (a) Sample two-dimensional lattice. (b) Unit cell corresponding to the part (a) lattice. (c) Reproduction of the original lattice. (d) An alternative unit cell.

# Unit Cell Concept

Lattice Constant: A length that describes the unit cell. It is normally given in Å, angstroms =  $1e-10$  meters.



QuickTime Movie

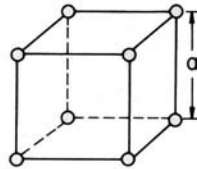


QuickTime Movie

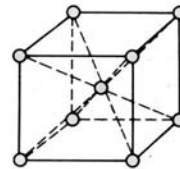


QuickTime Movie

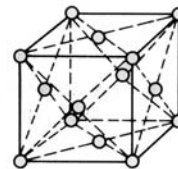
Diamond Structure:  
Constructed by 2  
“inter-penetrating”  
FCC Lattices



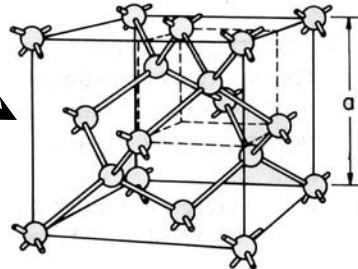
SIMPLE CUBIC  
(P, etc)



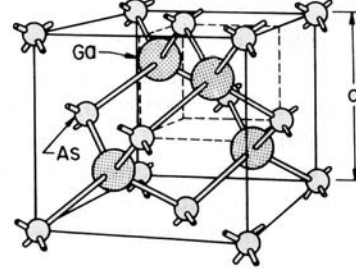
BODY-CENTERED CUBIC  
(Na, W, etc)



FACE-CENTERED CUBIC  
(Al, Au, etc)



DIAMOND  
(C, Ge, Si, etc)



ZINCBLLENDE  
(GaAs, GaP, etc)

Zincblende is a diamond structure with every other atom a different element. Example: Ga only bonds to As and As only bonds to Ga.



QuickTime Movie

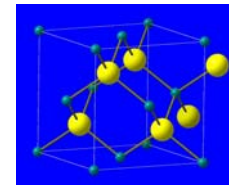
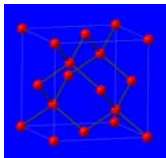
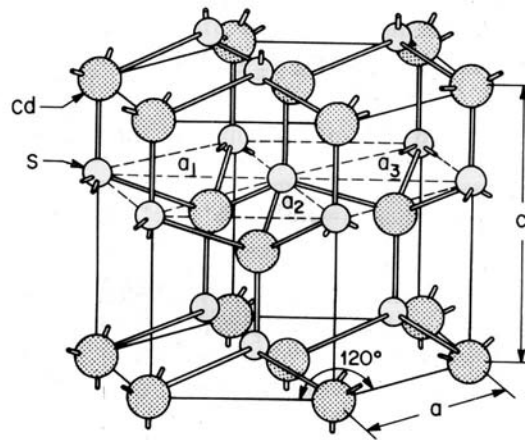


Fig. 1 Some important unit cells (direct lattices) and their representative elements or compounds; a is the lattice constant.

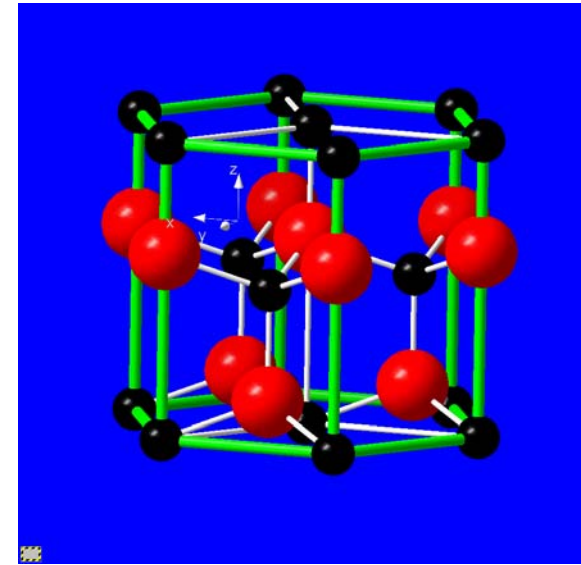


# Unit Cell Concept

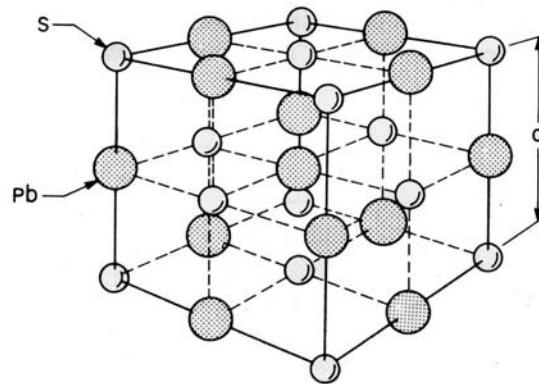
Some unit cells have hexagonal symmetry.



(a)



Rocksalt unit cells are one of the simplest practical unit cells.

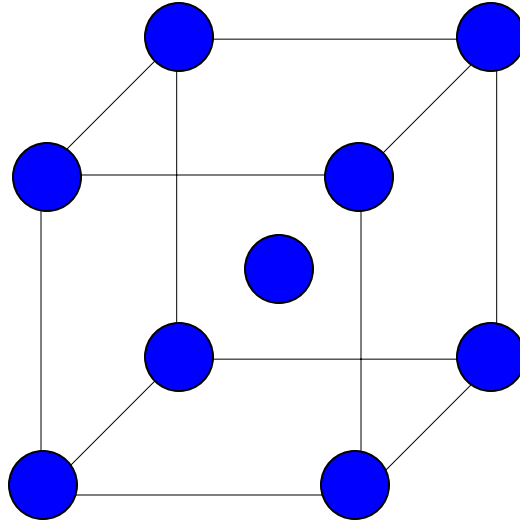


(b)

Fig. 2 Two unit cells of compound semiconductors. (a) Wurtzite lattice (CdS, ZnS, etc.). (b) Rock-salt lattice (PbS, PbTe, etc.).

## Atomic Density

What is the atomic density of a BCC material with lattice constant 5.2 angstroms?

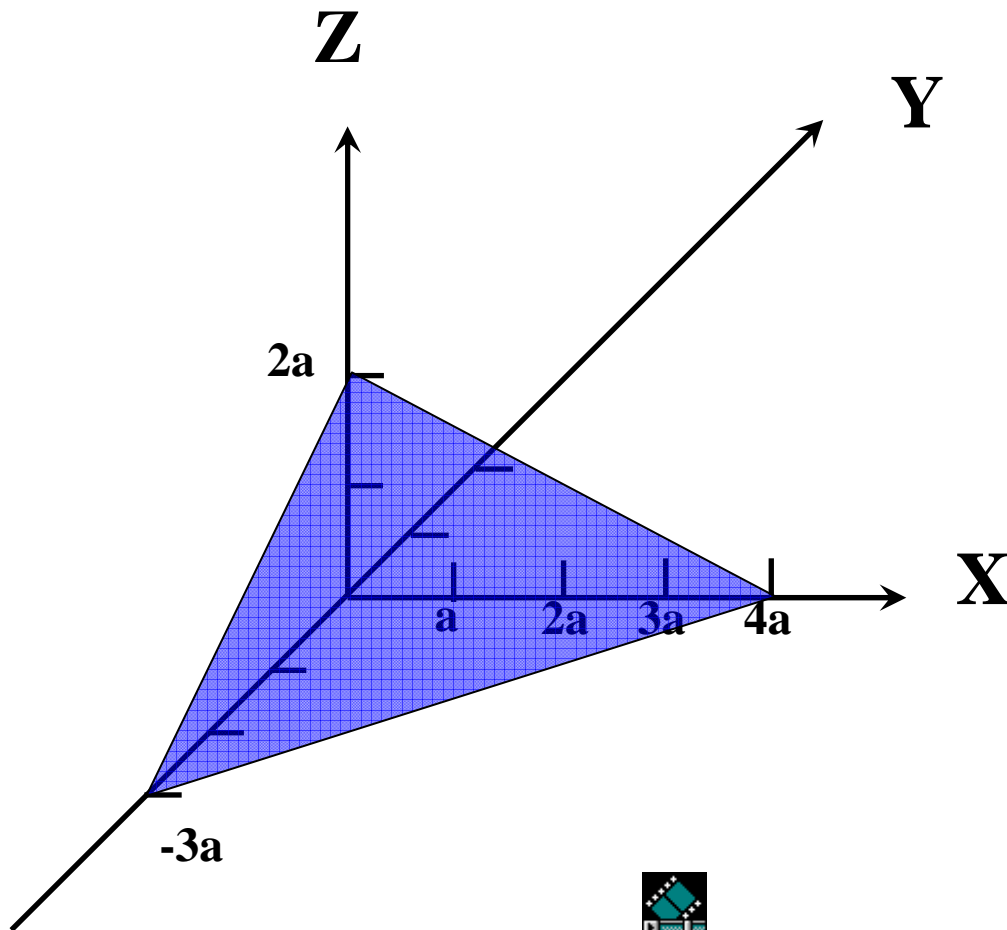


*Number of Atoms per unit cell = 2*

*Volume of unit cell =  $a^3 = (5.2e-8 \text{ cm})^3 = 1.41e-22 \text{ cm}^3$*

*Density =  $\frac{\text{Number of Atoms per unit cell}}{\text{Volume of unit cell}} = 1.4e22 \frac{\text{atoms}}{\text{cm}^3}$*

# Crystalline Planes and Miller Indices



Identify Intercepts in x,y,z  
order =  $4a, -3a, 2a$

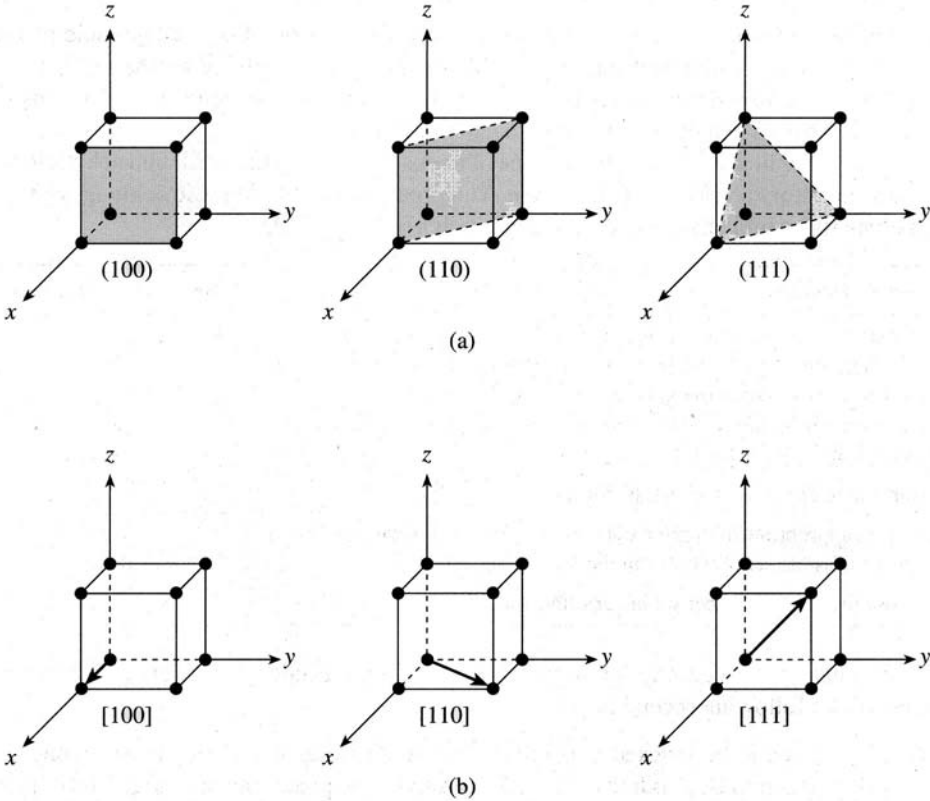
Divide by unit cell length in each  
direction x,y,z order =  $4, -3, 2$

Invert the values =  $1/4, -1/3, 1/2$

Multiply by a number (12 in this  
example) to give smallest whole  
number set =  $3, -4, 6$

Place any minus signs over their  
index and place set in parenthesis  
=  $(\bar{3}46)$

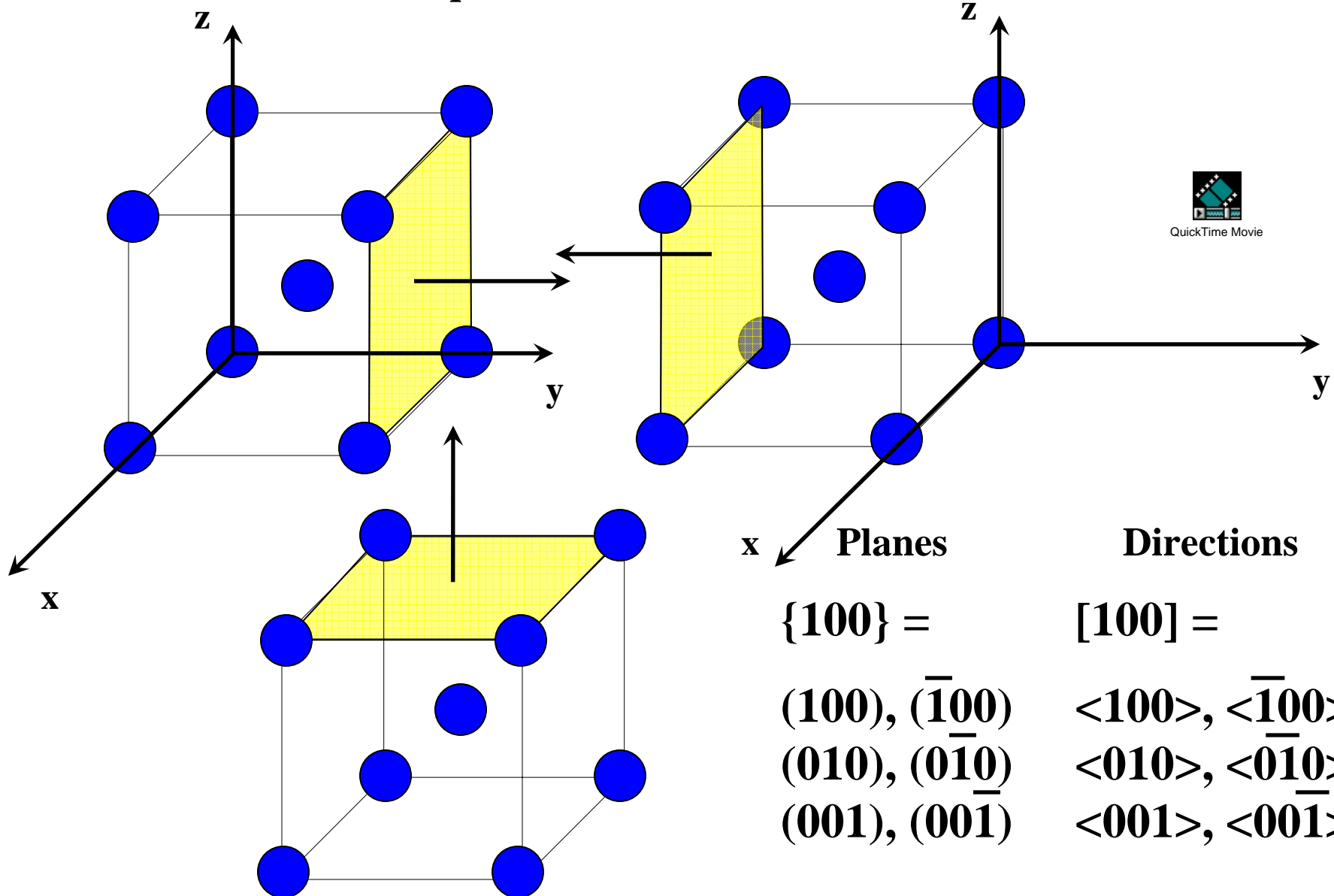
# Crystalline Planes and Miller Indices: Planes and directions



**Figure 1.7** Visualization and Miller indices of commonly encountered (a) crystalline planes and (b) direction vectors.

# Crystalline Planes and Miller Indices:

## Equivalent Planes and directions



Planes	Directions
$\{100\} =$	$[100] =$
$(100), (\bar{1}00)$	$\langle 100 \rangle, \langle \bar{1}00 \rangle$
$(010), (0\bar{1}0)$	$\langle 010 \rangle, \langle 0\bar{1}0 \rangle$
$(001), (00\bar{1})$	$\langle 001 \rangle, \langle 00\bar{1} \rangle$

# **Lecture 3**

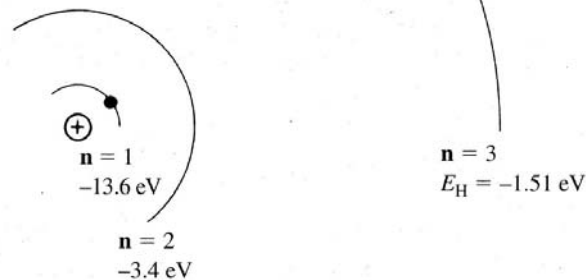
## **Bonding Model and Dopants**

**Reading:**

**(Cont'd) Pierret 1.1, 1.2, 1.4, 2.1-2.6**

# Comparison of the Hydrogen Atom and Silicon Atom

## Hydrogen



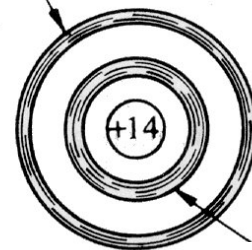
$$\text{Energy}_{\text{Hydrogen electron}} = -\frac{m_o q^4}{2(4\pi \epsilon_0 \hbar n)^2} = -\frac{13.6 \text{ eV}}{n^2}$$

where  $m_o$  = electron mass,  $\hbar = \text{planks constant} / 2\pi = h / 2\pi$   
 $q$  = electron charge, and  $n = 1, 2, 3, \dots$

**Figure 2.1** The hydrogen atom—idealized representation showing the first three allowed electron orbits and the associated energy quantization.

$n=2$ : Complete Shell  
 2 “2s electrons”  
 6 “2p electrons”  
 8 Electrons

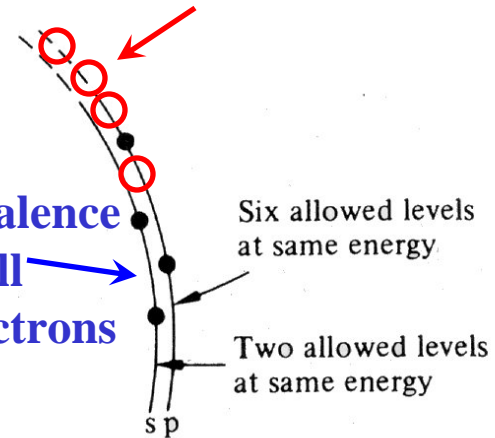
## Silicon



$n=1$ : Complete Shell  
 2 “s electrons”

**4 empty states**

**4 Valence Shell Electrons**

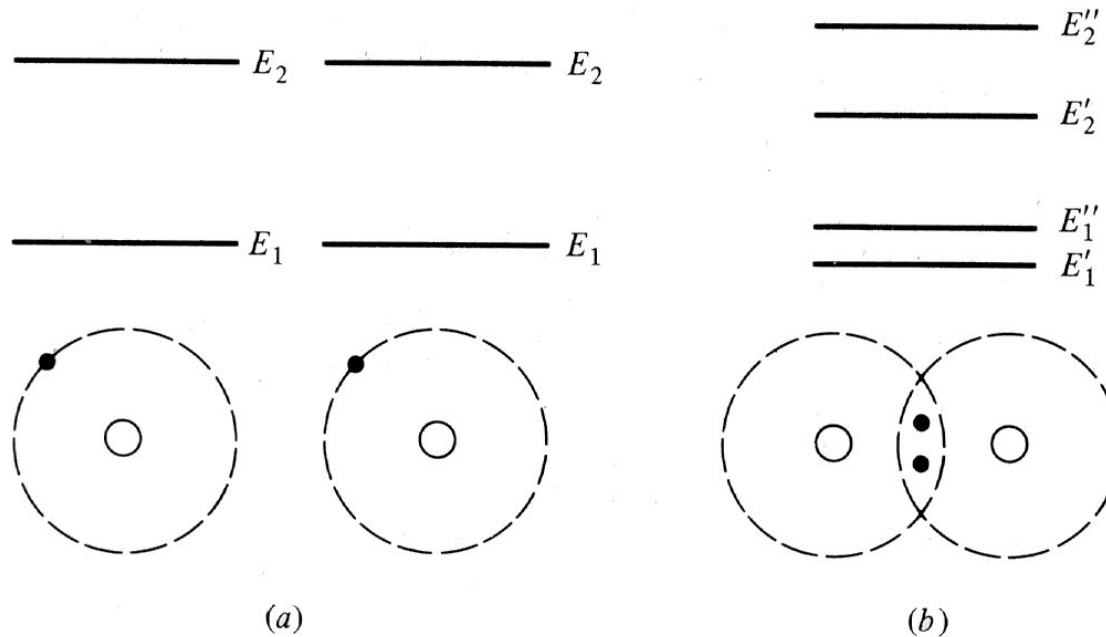


$n=3$ :  
 2 “3s electrons”  
 Only 2 of 6 “3p electrons”

**Figure 2.2** Schematic representation of an isolated Si atom.

# Pauli Exclusion Principle

Only 2 electrons, of spin  $\pm 1/2$ , can occupy the same energy state at the same point in space.

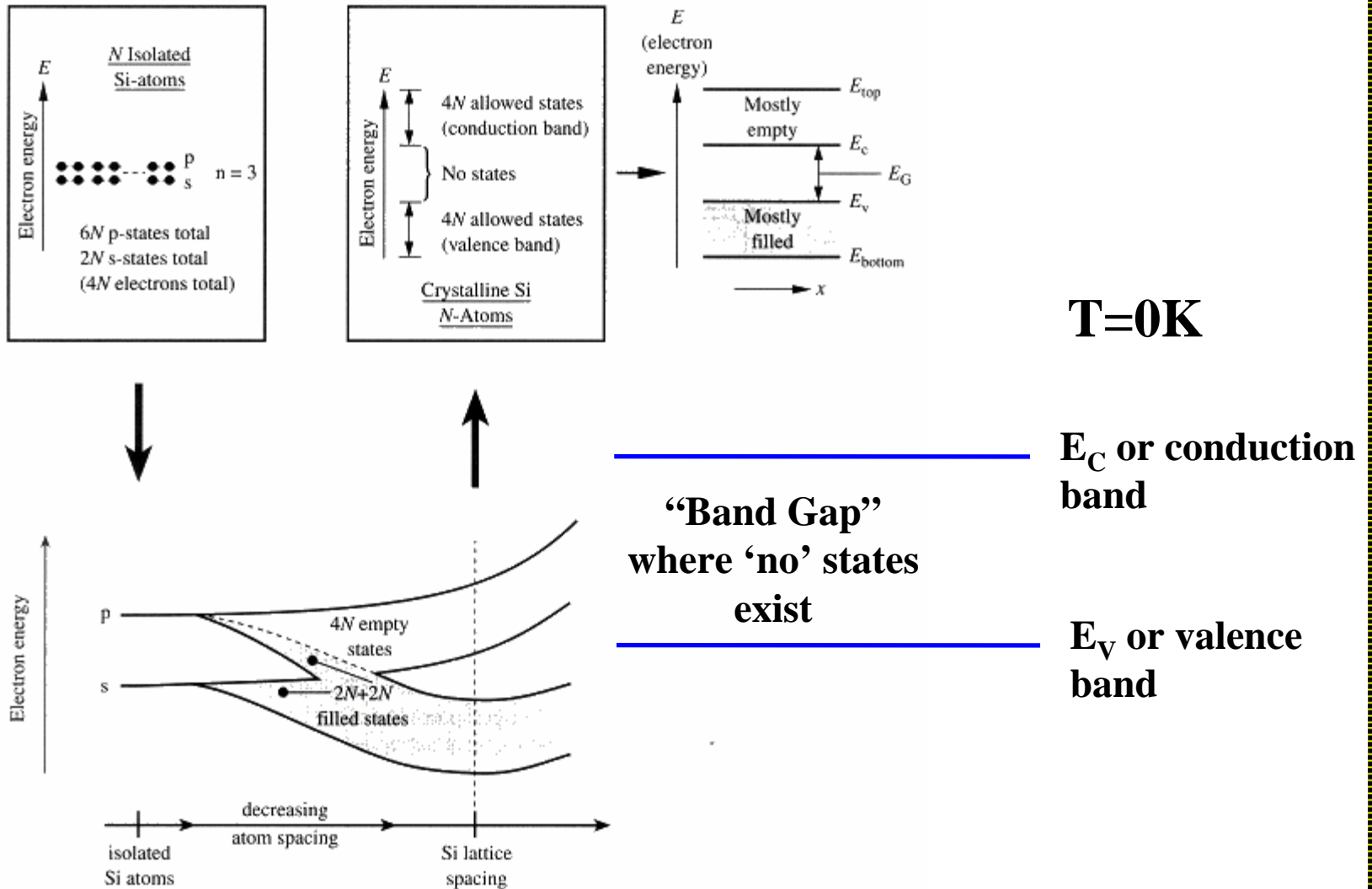


**FIGURE 1-9**

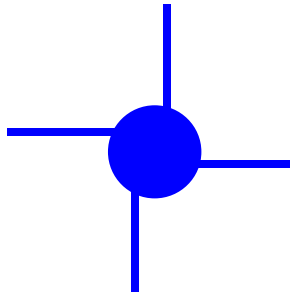
Two hydrogen atoms: (a) noninteracting and (b) interacting. Splitting of energy levels is illustrated for (b).



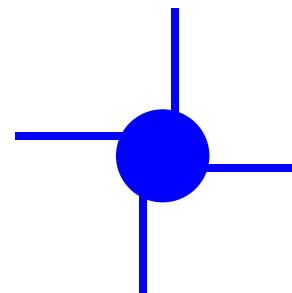
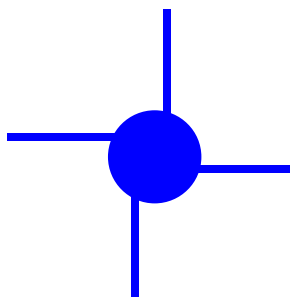
# Banding of Discrete states and the Simplified Model

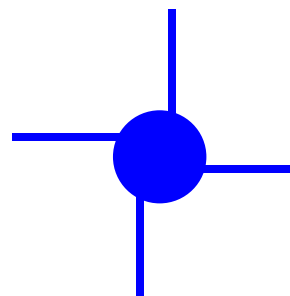
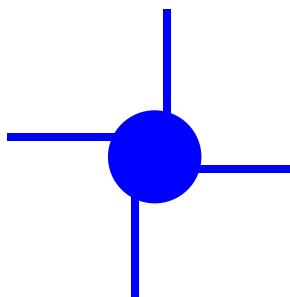


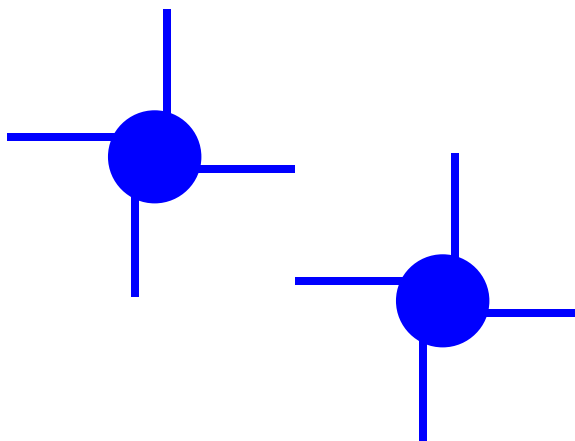
**Figure 2.5** Conceptual development of the energy band model starting with  $N$  isolated Si atoms on the top left and concluding with a “dressed-up” version of the energy band model on the top right.

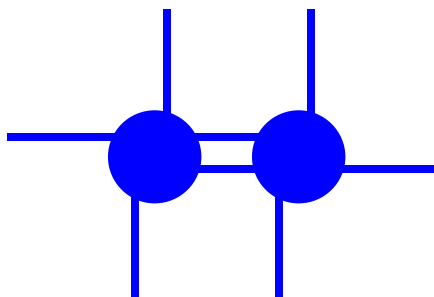


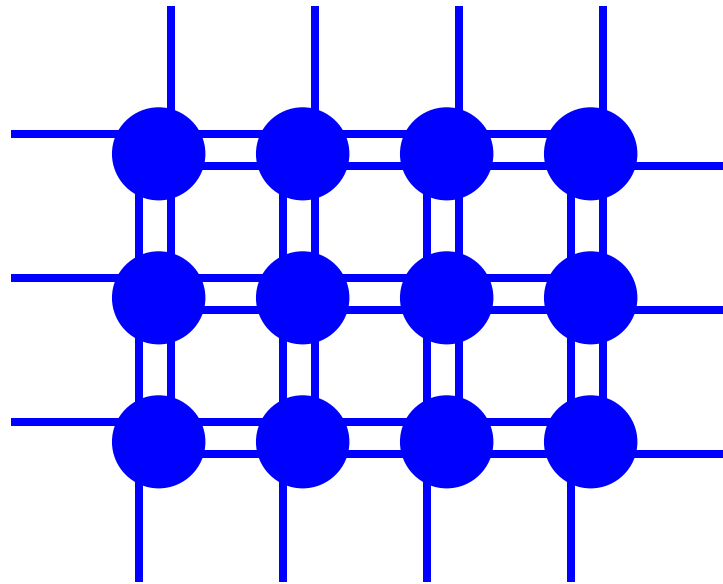
**4 electrons available for sharing  
(covalent bonding) in outer shell  
of atoms**

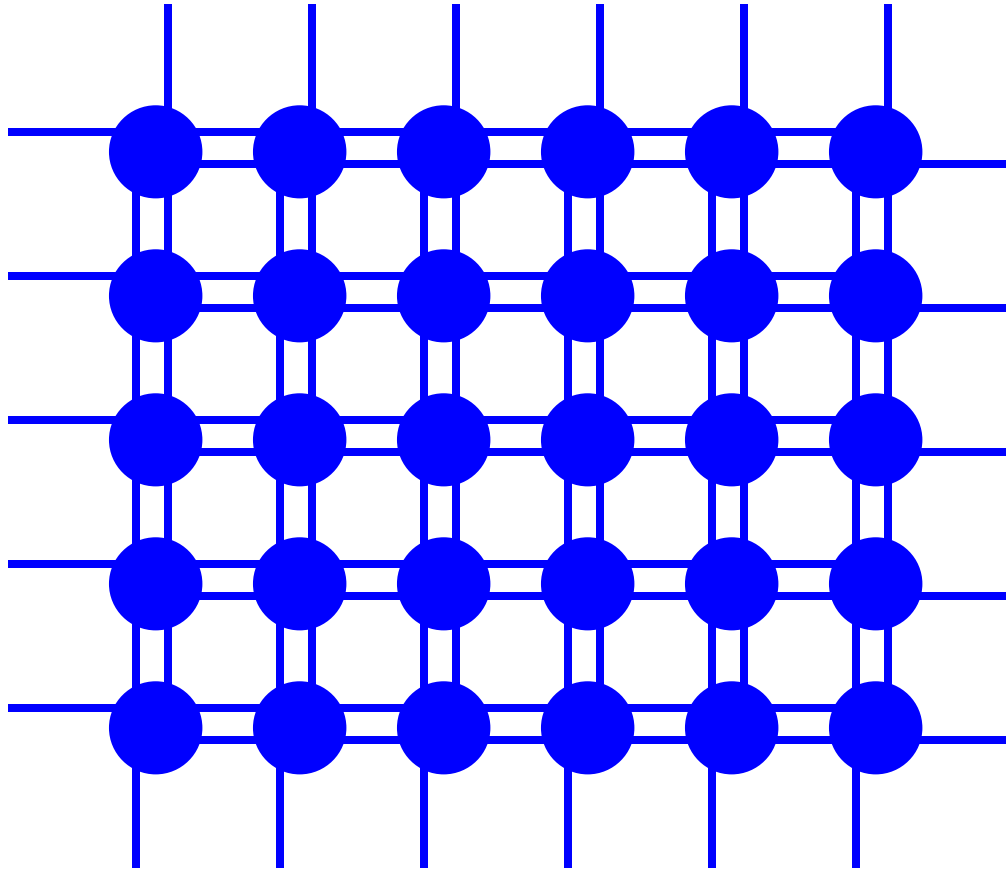




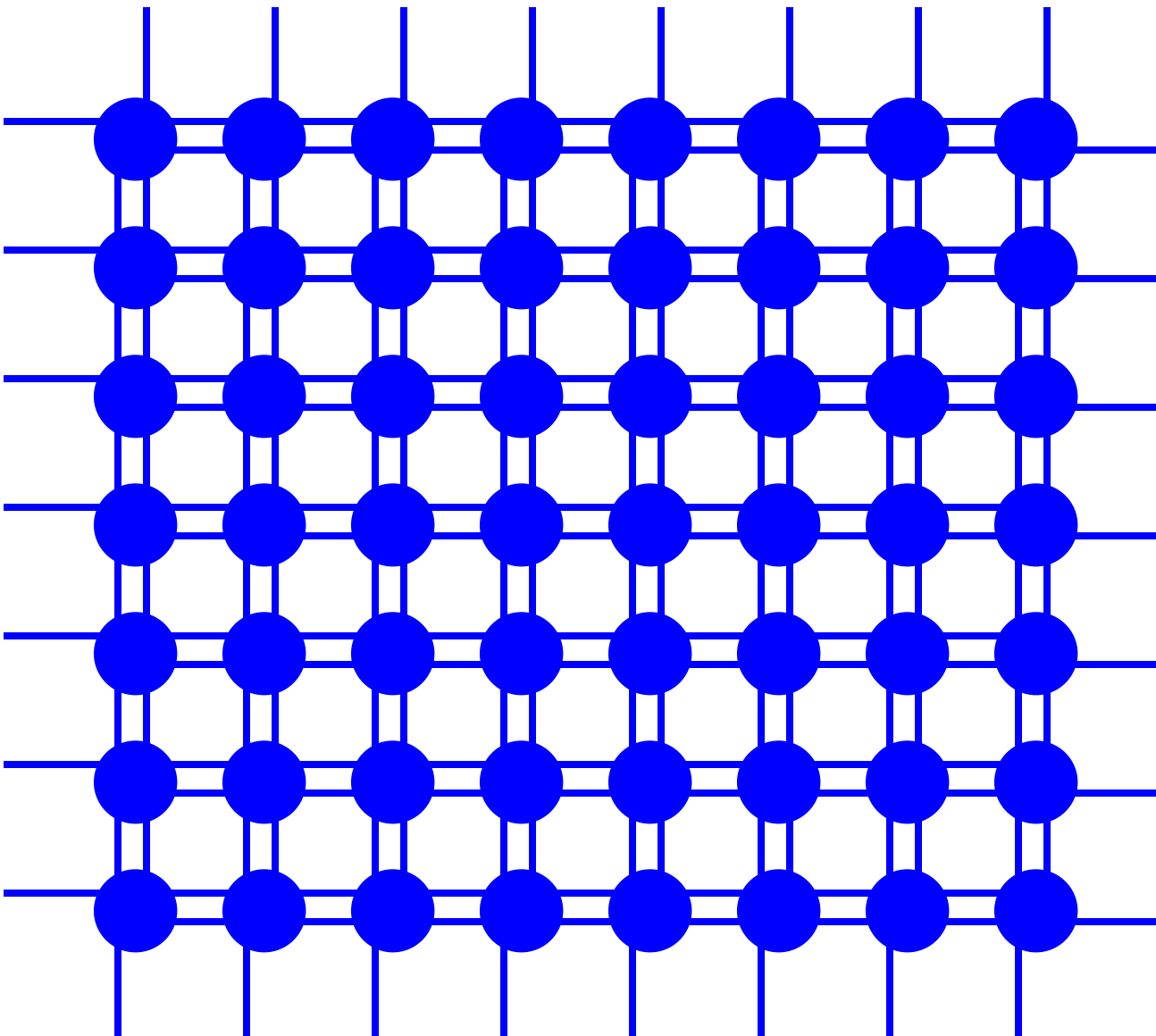






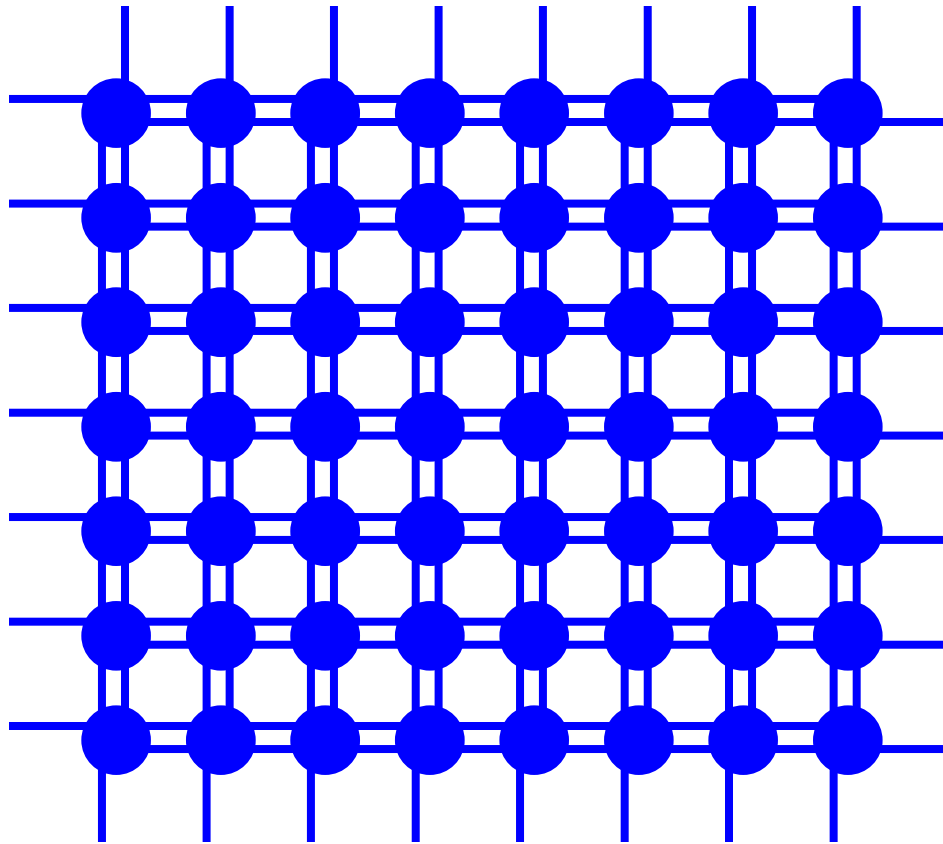






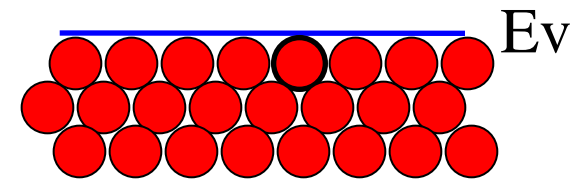
# Band Occupation at Low Temperature (0 Kelvin)

For  $(E_{\text{thermal}}=kT)=0$



No electrons in conduction band means no electron conduction is possible

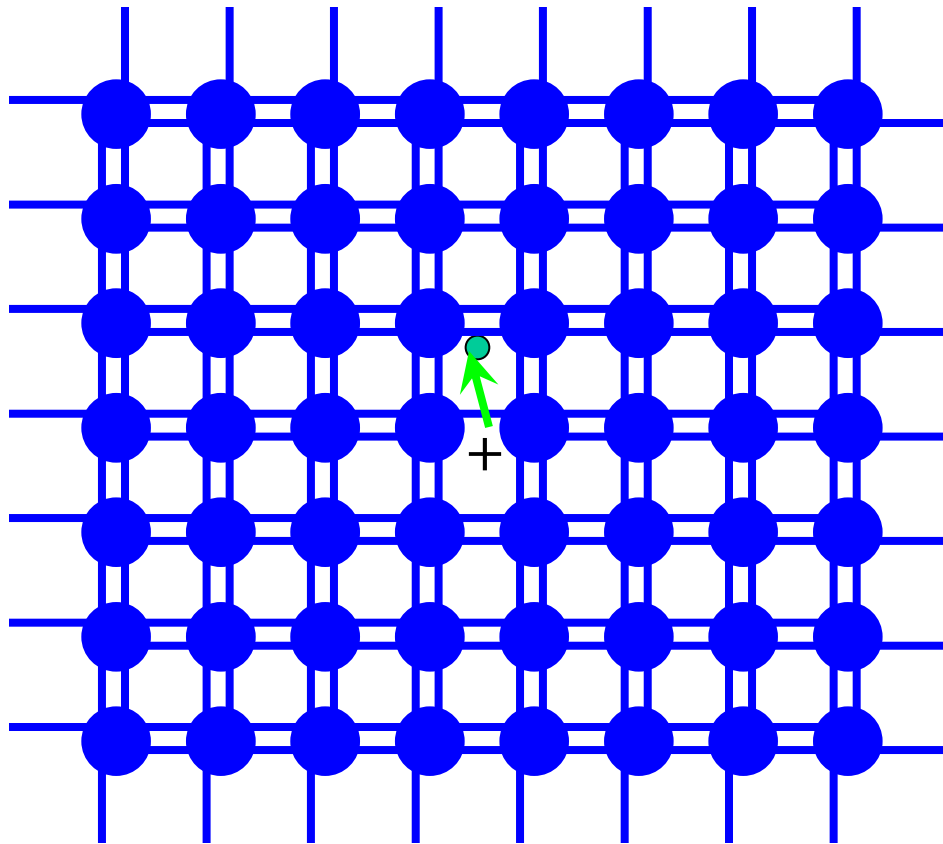
—————  $E_c$



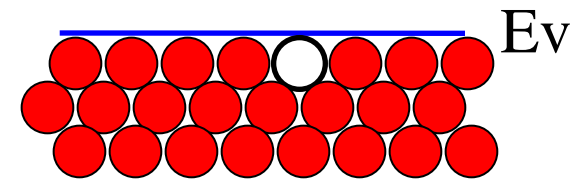
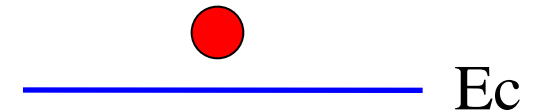
No "Holes" valence band means no "hole" conduction is possible

# Band Occupation at Higher Temperature ( $T > 0$ Kelvin)

For  $(E_{\text{thermal}} = kT) > 0$



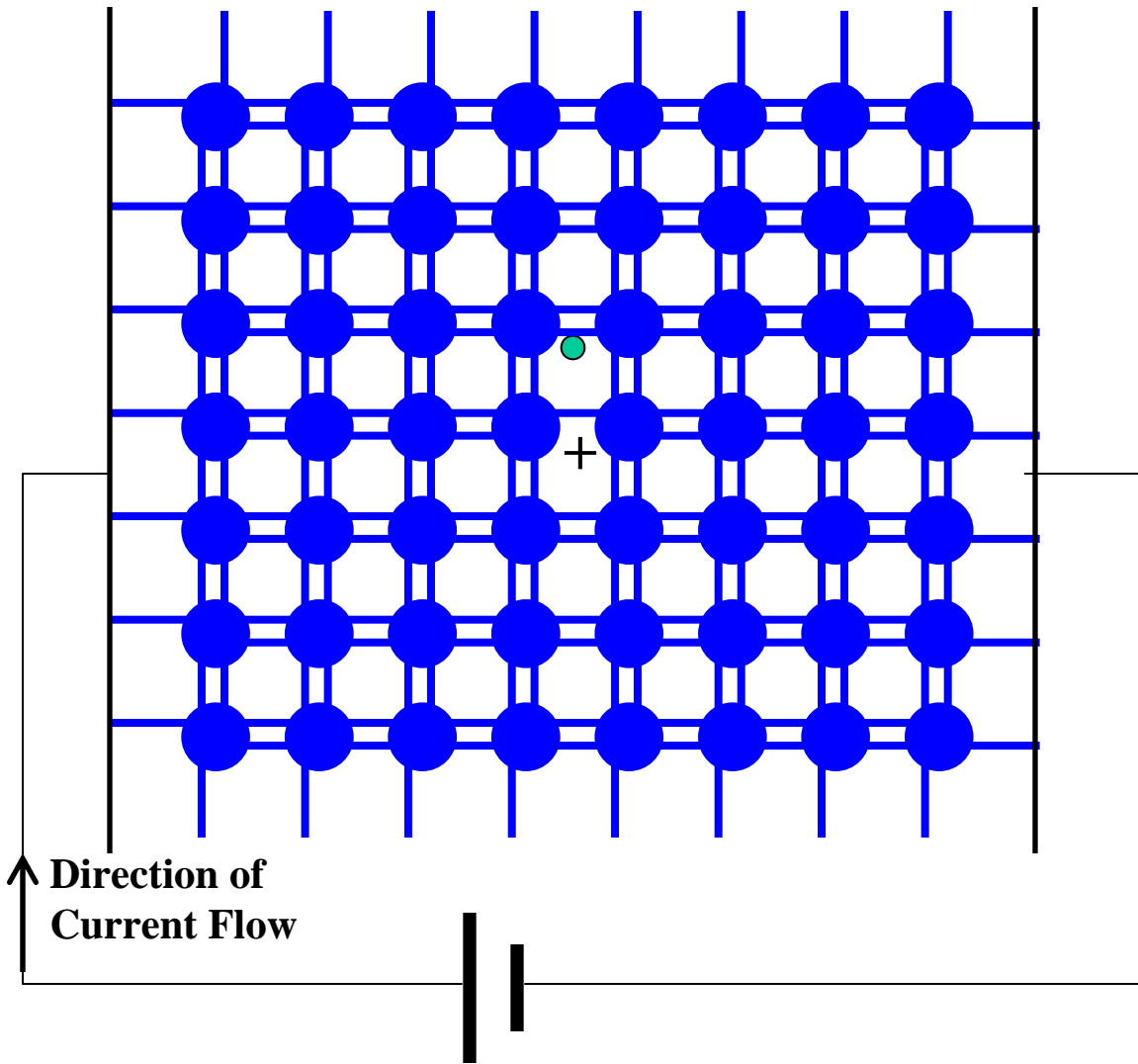
Electron free to move  
in conduction band



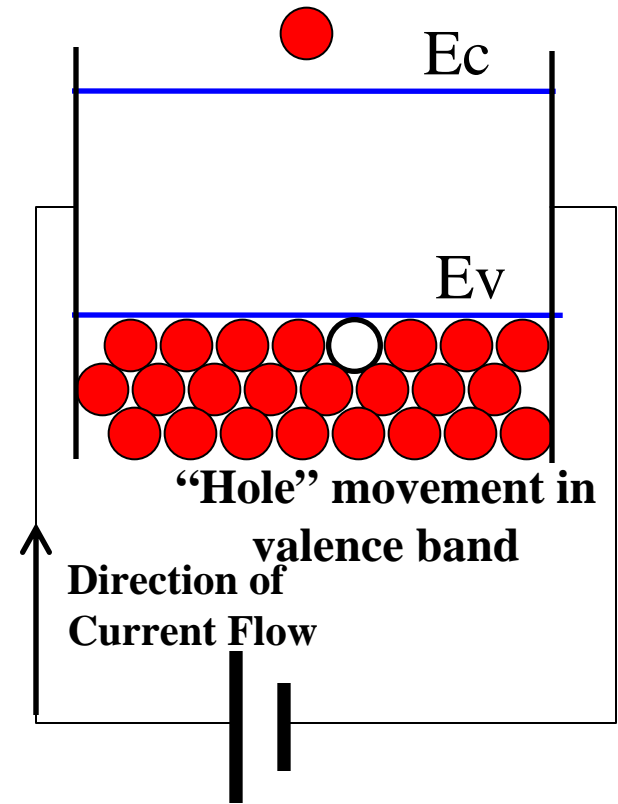
“Hole” free to move in  
valence band

# Carrier Movement Under Bias

For  $(E_{\text{thermal}}=kT) > 0$

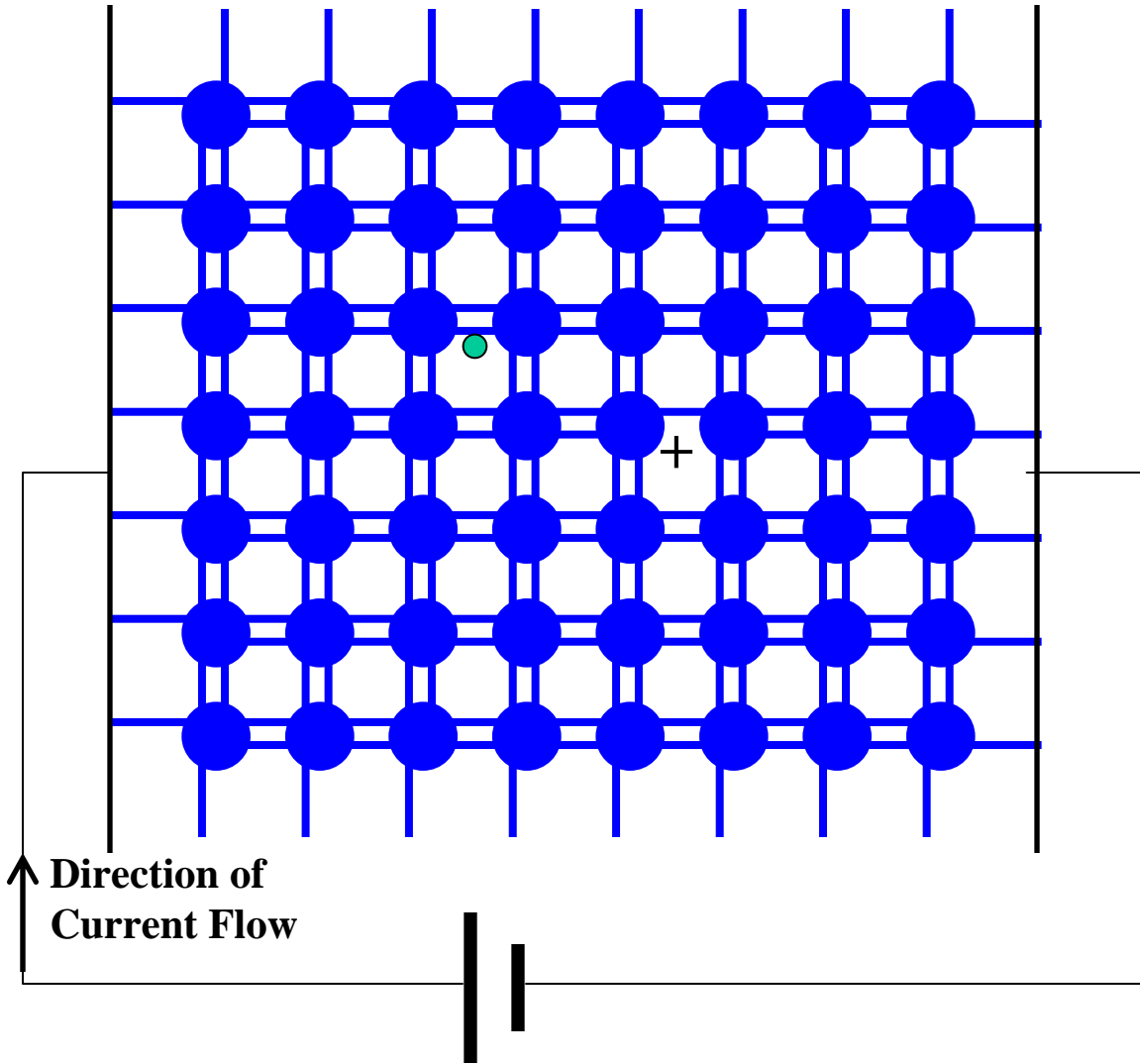


Electron free to move in conduction band

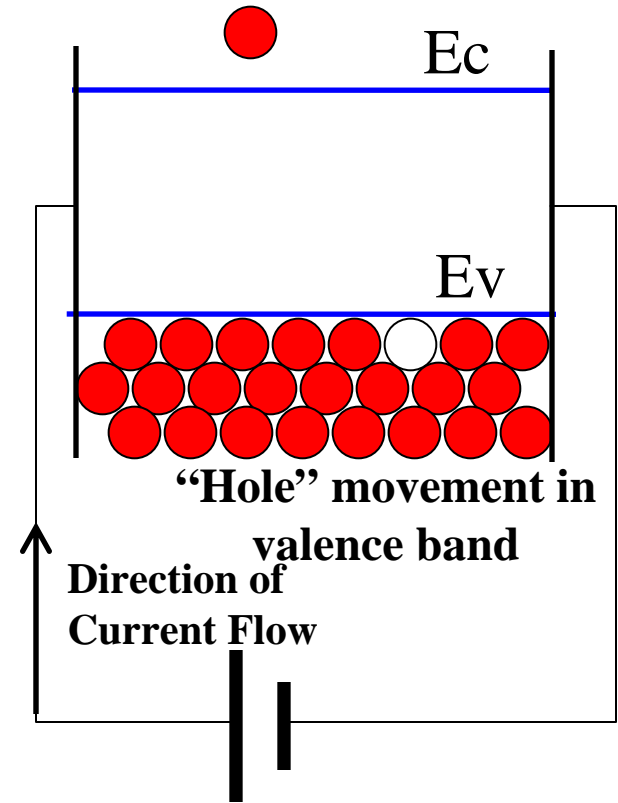


# Carrier Movement Under Bias

For  $(E_{\text{thermal}}=kT) > 0$

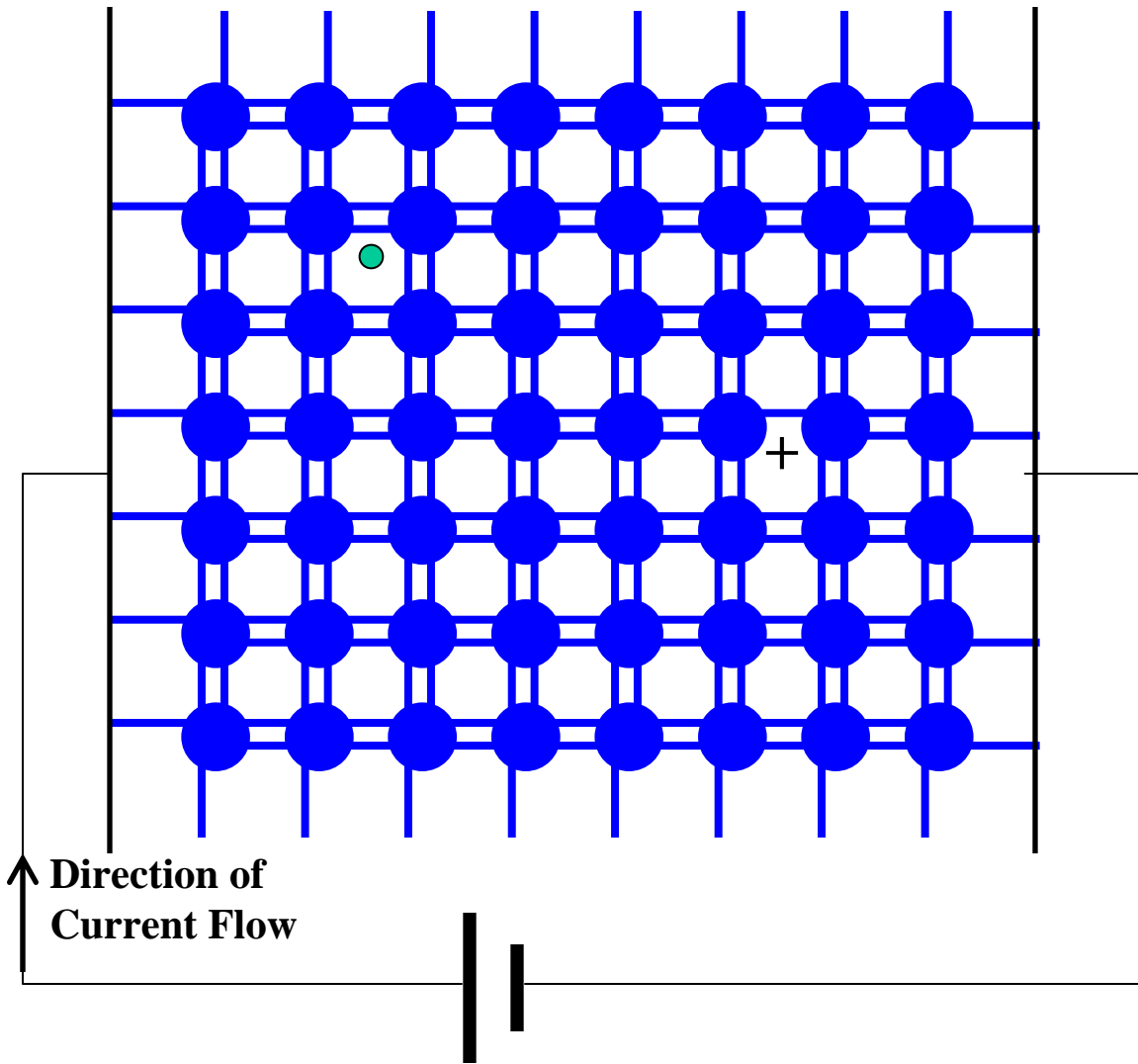


Electron free to move in conduction band

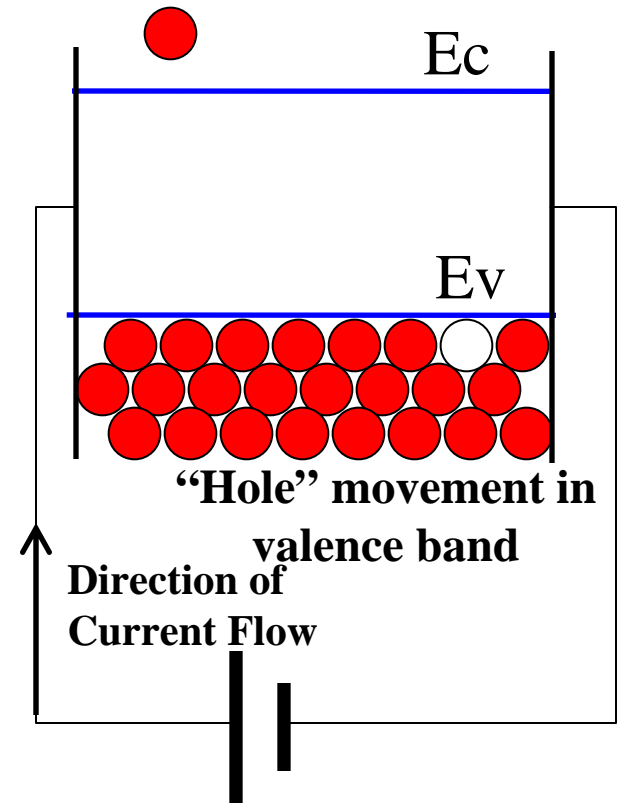


# Carrier Movement Under Bias

For  $(E_{\text{thermal}}=kT) > 0$



Electron free to move in conduction band



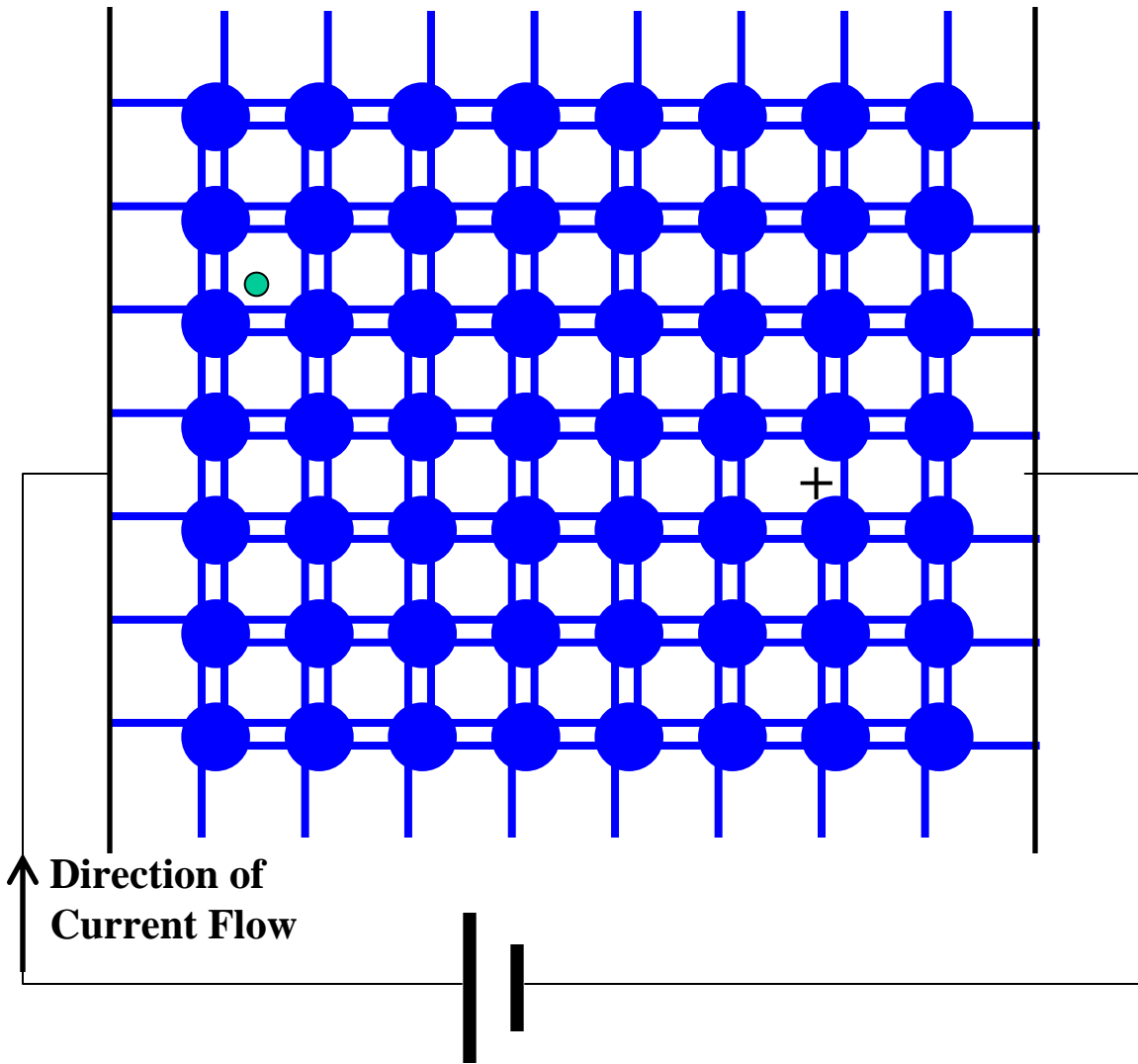
"Hole" movement in valence band

# Carrier Movement Under Bias

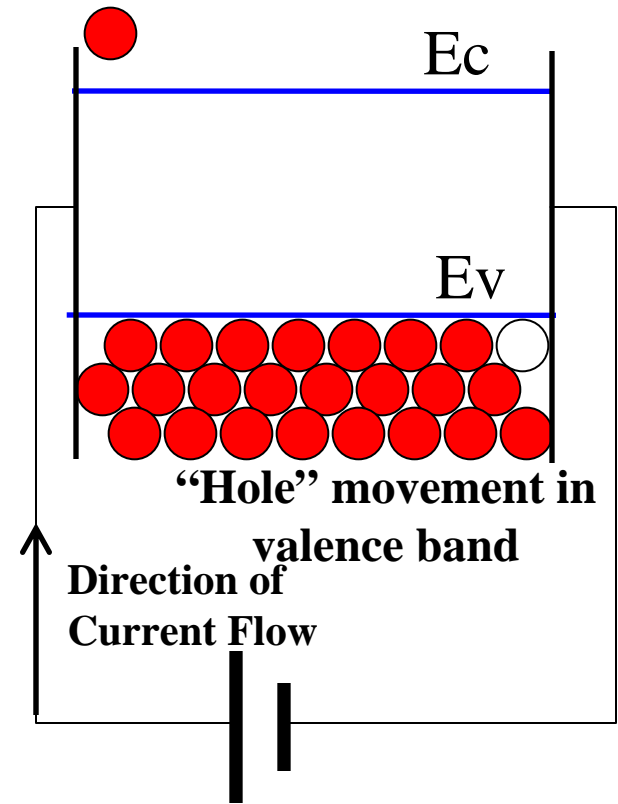


QuickTime Movie

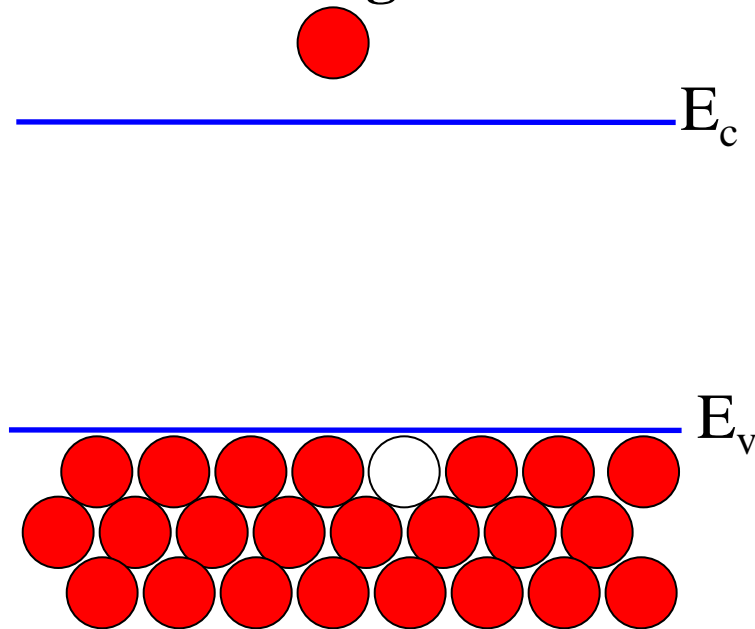
For  $(E_{\text{thermal}}=kT) > 0$



Electron free to move in conduction band



# Clarification of confusing issues: “Holes” and Electrons



The valence band may have  $\sim 4 \times 10^{22} \text{ cm}^{-3}$  valence electrons “participating in the bonding processes holding the crystal together.”

The valence band might only have  $\sim 10^6$  to  $10^{19} \text{ cm}^{-3}$  “holes” in the valence band (missing valence electrons). Thus, it is easier to account for the influence of the holes by counting the holes directly as opposed to counting very small changes in the valence electron concentrations.

**Example:** If there are  $10^{22} \text{ cm}^{-3}$  atoms in a crystal with each atom having 4 valence electrons. What is the difference in valence electron concentration for  $10^{12}$  holes versus  $10^{13} \text{ cm}^{-3}$  holes?

$$\text{Answer: } 4 \times 10^{22} \text{ cm}^{-3} - 10^{12} \text{ cm}^{-3} = 3.999999999999 \times 10^{22} \text{ cm}^{-3} \text{ verses}$$

$$4 \times 10^{22} \text{ cm}^{-3} - 10^{13} \text{ cm}^{-3} = 3.999999999999 \times 10^{22} \text{ cm}^{-3}$$

**For “accounting reasons” keeping track of holes is easier!**



# Clarification of confusing issues: “Holes” and Electrons

## Terminology

Only these  
“particles”  
carry  
electricity.  
Thus, we  
call these  
“carriers”

Electrons: Sometimes referred to as conduction electrons: The electrons in the conduction band that are free to move throughout the crystal.

Holes: Missing electrons normally found in the valence band (or empty states in the valence band that would normally be filled).

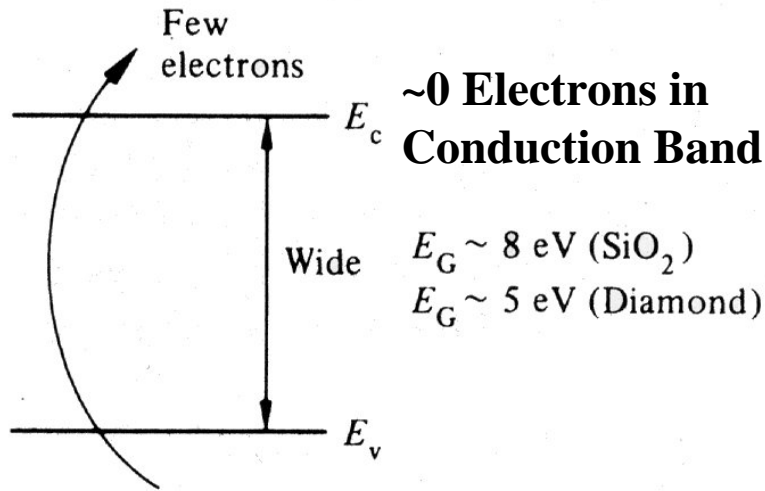
If we talk about empty states in the conduction band, we DO NOT call them holes! This would be confusing. The conduction band has mostly empty states and a few electrons.

If we talk about filled states in the valence band, we DO NOT call them electrons! This would be confusing. We can call them Valence Electrons to indicate they are bond to atoms (in the valence shells of atoms). The valence band has mostly filled states and a few holes.

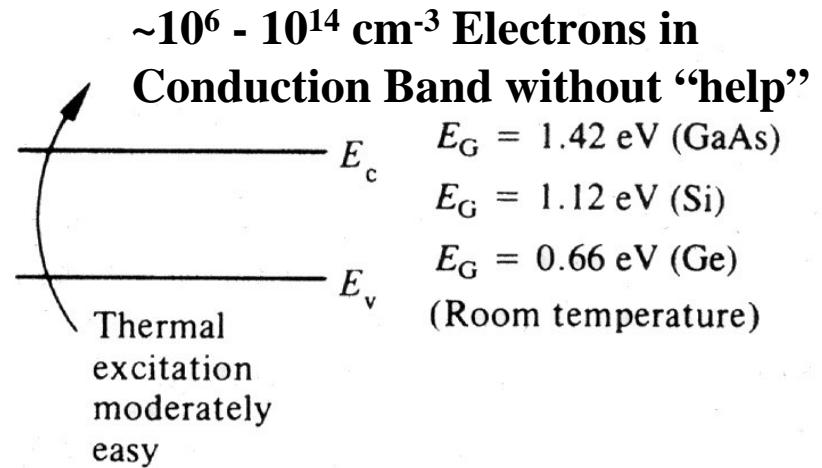
For the vast majority of this class we only talk about electrons (conduction band electrons) and holes (empty states in the valence band)!

# Material Classification based on Size of Bandgap:

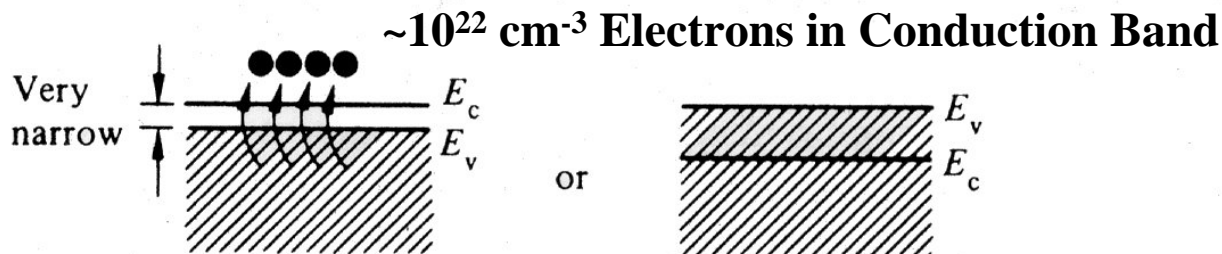
Ease of achieving thermal population of conduction band determines whether a material is an insulator, semiconductor, or metal



(a) Insulator



(b) Semiconductor



(c) Metal

**Figure 2.8** Explanation of the distinction between (a) insulators, (b) semiconductors, and (c) metals using the energy band model.

# Intrinsic Carrier Concentration

- For each electron promoted to the conduction band, one hole is left in the valence band. Thus, the number of electrons in the conduction band is equal to the number of holes in the valence band unless there is “help” to change the relative populations in each band.
- Intrinsic carrier concentration is the number of electron (=holes) per cubic centimeter populating the conduction band (or valence band) is called the intrinsic carrier concentration,  $n_i$
- $n_i = f(T)$  that increases with increasing  $T$  (more thermal energy)

## At Room Temperature ( $T=300$ K)

$n_i \sim 2 \times 10^6 \text{ cm}^{-3}$  for GaAs with  $E_g = 1.42 \text{ eV}$ ,

$n_i \sim 1 \times 10^{10} \text{ cm}^{-3}$  for Si with  $E_g = 1.1 \text{ eV}$ ,

$n_i \sim 2 \times 10^{13} \text{ cm}^{-3}$  for Ge with  $E_g = 0.66 \text{ eV}$ ,

$n_i \sim 1 \times 10^{-14} \text{ cm}^{-3}$  for GaN with  $E_g = 3.4 \text{ eV}$

# Carrier Movement in Free Space

## Newton's second law

$$F = -qE = m_o \frac{dv}{dt}$$

$F \equiv$  force,  $v \equiv$  velocity,  $t \equiv$  time,

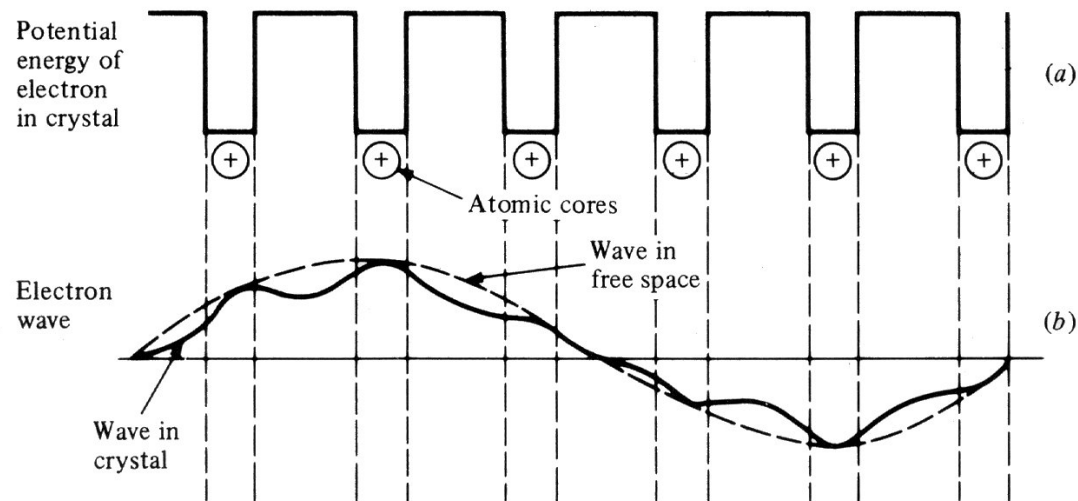
$q \equiv$  electronic charge,  $m_o \equiv$  electron mass

## Carrier Movement Within the Crystal

- Electron is a quasi-particle that behaves as a “wave” due to quantum mechanical effects.
- The electron “wavelength” is perturbed by the crystal's periodic potential.



QuickTime Movie



**FIGURE 1-11**  
Representation of motion of electron wave in crystal potential. (After Wolfendale [3].)

## Carrier Movement Within the Crystal

$$F = -qE = m_n^* \frac{dv}{dt}$$

$F \equiv$  force,  $v \equiv$  velocity,  $t \equiv$  time,

$q \equiv$  electronic charge,

$m_n^* \equiv$  electron effective mass

$$F = qE = m_p^* \frac{dv}{dt}$$

$F \equiv$  force,  $v \equiv$  velocity,  $t \equiv$  time,

$q \equiv$  electronic charge,

$m_p^* \equiv$  hole effective mass

**Table 2.1** Density of States Effective Masses at 300 K.

Material	$m_n^*/m_0$	$m_p^*/m_0$
Si	1.18	0.81
Ge	0.55	0.36
GaAs	0.066	0.52

Ge and GaAs have “lighter electrons” than Si  
which results in faster devices