Lecture 5

P-N Junction Diodes

Quantitative Analysis (Math, math and more math)

Quantitative p-n Diode Solution

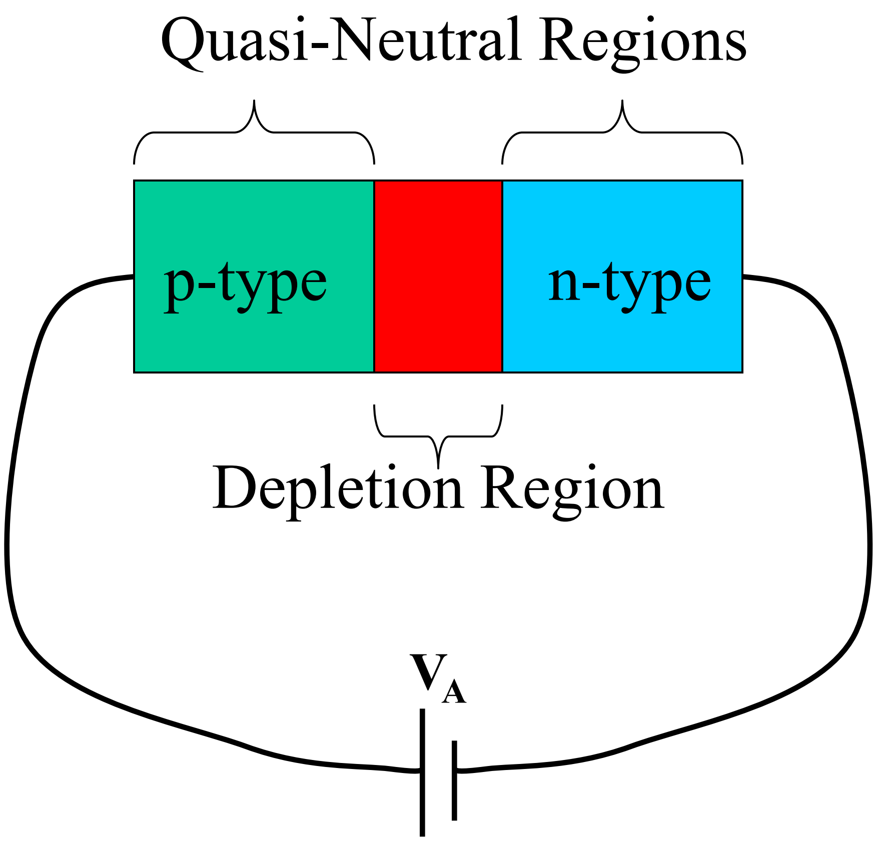
Assumptions:
1) steady state conditions
2) non- degenerate doping
3) one- dimensional analysis
4) low- level injection
5) no light ($G_L = 0$)

Current equations:

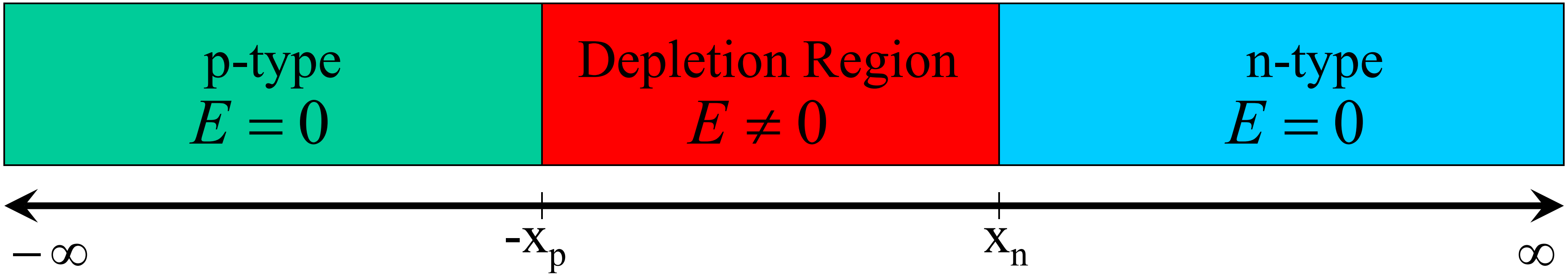
$J = J_p(x) + J_n(x)$

$J_n = q\,\mu_n n E + q D_n (dn/dx)$

$J_p = q\,\mu_p p E - q D_p\,(dp/dx)$



Quantitative p-n Diode Solution



Application of the Minority Carrier Diffusion Equation

$$\frac{\partial(\Delta n_p)}{\partial t} = D_N \frac{\partial^2(\Delta n_p)}{\partial x^2} - \frac{(\Delta n_p)}{\tau_n} + G_L$$

$$0 = D_N \frac{\partial^2(\Delta n_p)}{\partial x^2} - \frac{(\Delta n_p)}{\tau_n}$$

Since electric fields exist in the depletion region, the minority carrier diffusion equation does not apply here.

$$\frac{\partial(\Delta p_n)}{\partial t} = D_P \frac{\partial^2(\Delta p_n)}{\partial x^2} - \frac{(\Delta p_n)}{\tau_p} + G_L$$

$$0 = D_P \frac{\partial^2(\Delta p_n)}{\partial x^2} - \frac{(\Delta p_n)}{\tau_p} + 0$$

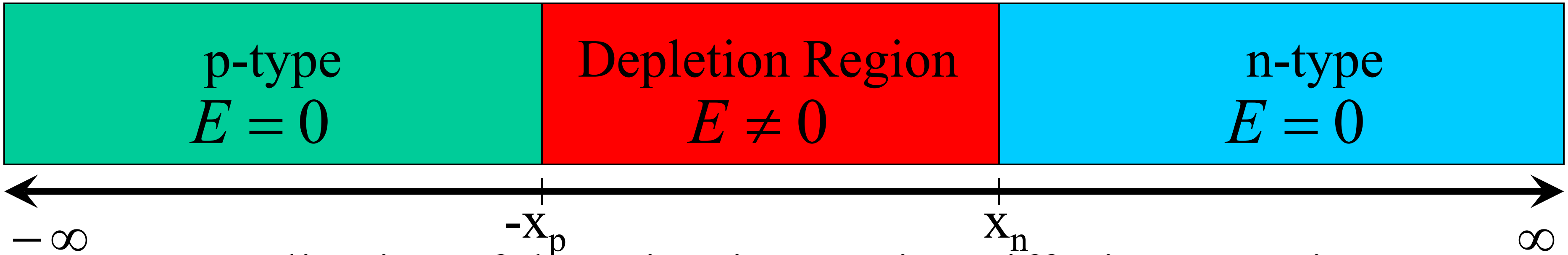
Boundary Condition:
$\Delta n_p(x \rightarrow -\infty) = 0$

Boundary Condition:
$\Delta n_p(x = -x_p) = ?$

Boundary Condition:
$\Delta p_n(x = x_n) = ?$

Boundary Condition:
$\Delta p_n(x \rightarrow \infty) = 0$

Quantitative p-n Diode Solution



Application of the Minority Carrier Diffusion Equation

Boundary Condition: $\Delta n_p(x=-x_p)=?$

Boundary Condition: $\Delta p_n(x=x_n)=?$

$$n=n_i e^{(F_N-E_i)/kT} \quad \text{and} \quad p=n_i e^{(E_i-F_P)/kT}$$

$$n_p(x=-x_p)\,p_p(x=-x_p)=n_i^2 e^{(F_N-F_P)/kT}$$

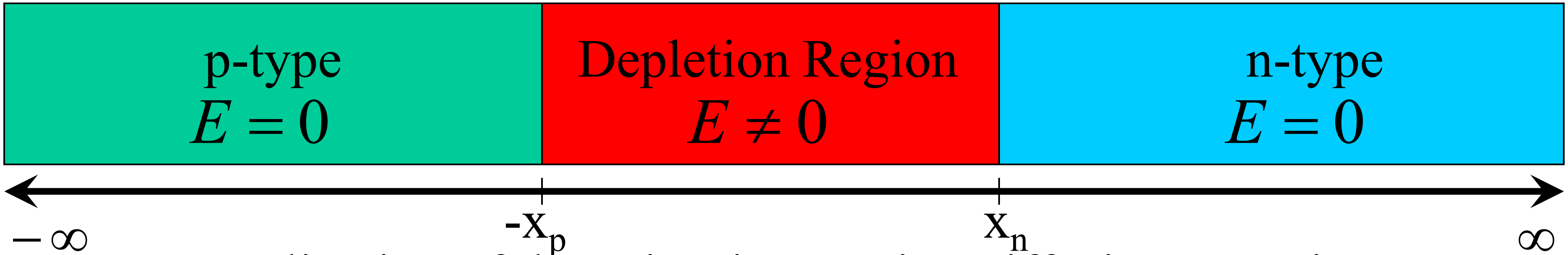
$$n_p(x=-x_p)\,p_p(x=-x_p)=n_p(x=-x_p)N_A=n_i^2 e^{qV_A/kT}$$

$$n_p(x=-x_p)=\frac{n_i^2}{N_A}e^{qV_A/kT}$$

$$\Delta n_p(x=-x_p)=\frac{n_i^2}{N_A}e^{qV_A/kT}-n_o$$

$$\Delta n_p(x=-x_p)=\frac{n_i^2}{N_A}\left(e^{qV_A/kT}-1\right) \quad \text{and similarly at } x=x_n \quad \Delta p_n(x=x_n)=\frac{n_i^2}{N_D}\left(e^{qV_A/kT}-1\right)$$

Quantitative p-n Diode Solution



Application of the Minority Carrier Diffusion Equation

Boundary Condition: $\Delta n_p(x = -x_p) = ?$

Boundary Condition: $\Delta p_n(x = x_n) = ?$

$$n = n_i e^{(F_N - E_i)/kT} \quad and \quad p = n_i e^{(E_i - F_P)/kT}$$

$$n_p(x = -x_p)\, p_p(x = -x_p) = n_i^2 e^{(F_N - F_P)/kT}$$

$$n_p(x = -x_p)\, p_p(x = -x_p) = n_p(x = -x_p) N_A = n_i^2 e^{qV_A/kT}$$

$$n_p(x = -x_p) = \frac{n_i^2}{N_A} e^{qV_A/kT}$$

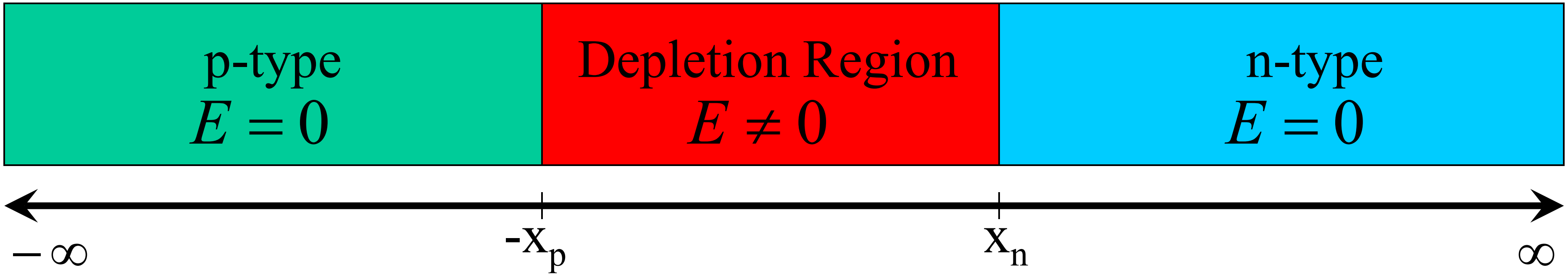
Law of the Junction

$$\Delta n_p(x = -x_p) = \frac{n_i^2}{N_A} e^{qV_A/kT} - n_o$$

$$p_n(x = -x_n) = \frac{n_i^2}{N_D} e^{qV_A/kT}$$

$$\Delta n_p(x = -x_p) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right) \quad and\ similarly\ at\ x = x_n \quad \Delta p_n(x = -x_n) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$$

Quantitative p-n Diode Solution



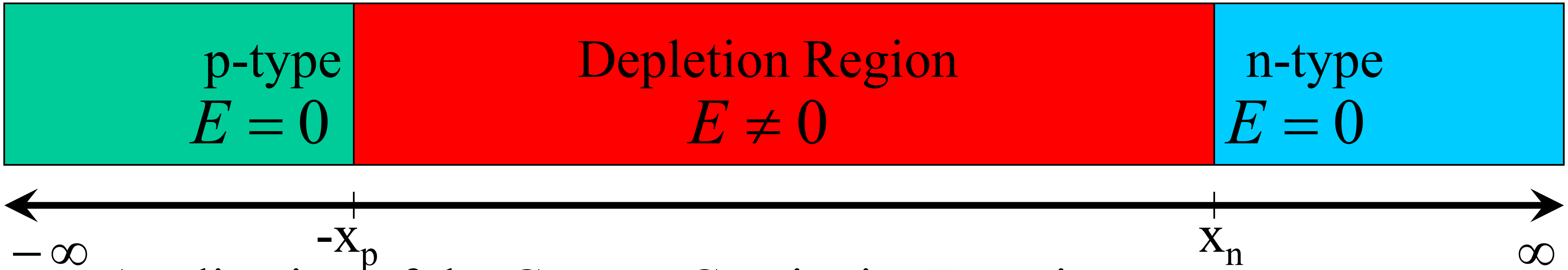
Application of the Current Continuity Equation

$$J_n = q\left(\mu_n nE + D_n \frac{dn}{dx}\right)$$
$$= qD_n \frac{d\left(n_o + \Delta n_p\right)}{dx}$$
$$= qD_n \frac{d\Delta n_p}{dx}$$

?

$$J_p = q\left(\mu_p pE - D_p \frac{dp}{dx}\right)$$
$$= -qD_p \frac{d\left(p_o + \Delta p_n\right)}{dx}$$
$$= -qD_p \frac{d\Delta p_n}{dx}$$

Quantitative p-n Diode Solution



Application of the Current Continuity Equation: Depletion Region

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \cdot J_N + \left.\frac{\partial n}{\partial t}\right|_{Recombination-Generation} + \left.\frac{\partial n}{\partial t}\right|_{\substack{All\ other\ processes \\ such\ as\ light,\ etc...}}$$

$$0 = \frac{1}{q}\nabla \cdot J_N$$

$$0 = \frac{1}{q}\frac{\partial J_N}{\partial x}$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \cdot J_P + \left.\frac{\partial p}{\partial t}\right|_{Recombination-Generation} + \left.\frac{\partial p}{\partial t}\right|_{\substack{All\ other\ processes \\ such\ as\ light,\ etc...}}$$

$$0 = -\frac{1}{q}\nabla \cdot J_P$$

$$0 = -\frac{1}{q}\frac{\partial J_P}{\partial x}$$

No thermal recombination and generation implies J_n and J_p are constant throughout the depletion region. Thus, the total current can be define in terms of only the current at the depletion region edges.

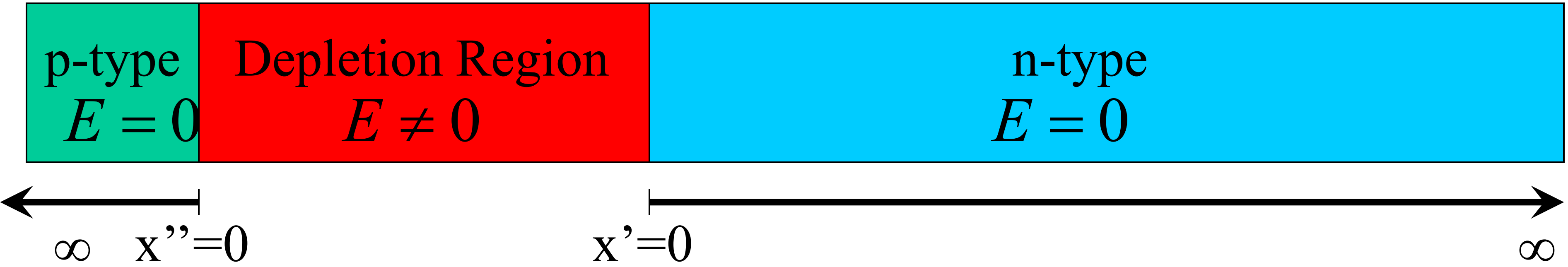
$$J = J_n(-x_p) + J_p(x_n)$$

Quantitative p-n Diode Solution

Approach:

- Solve minority carrier diffusion equation in quasi-neutral regions
- Determine minority carrier currents from continuity equation
- Evaluate currents at the depletion region edges
- Add these together and multiply by area to determine the total current through the device.
- Use translated axes, x→x' and -x→x'' in our solution.

Quantitative p-n Diode Solution



$$0 = D_P \frac{\partial^2(\Delta p_n)}{\partial x'^2} - \frac{(\Delta p_n)}{\tau_p}$$

$$\Delta p_n(x') = Ae^{(-x'/L_P)} + Be^{(+x'/L_P)} \quad where \quad L_P \equiv \sqrt{D_p \tau_p}$$

Boundary Conditions:

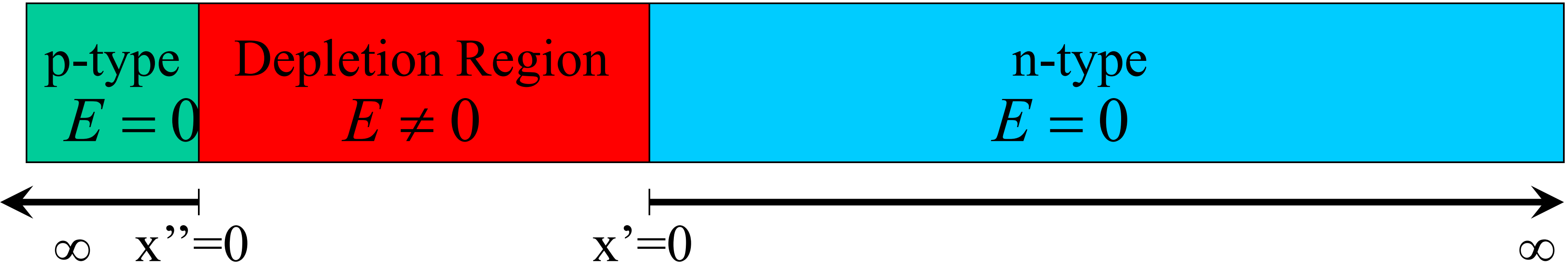
$$\Delta p_n(x' \rightarrow \infty) = 0$$

$$\Delta p_n(x'=0) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$$

$$B = 0 \quad and \quad A = \Delta p_n(x'=0) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$$

$$\boxed{\Delta p_n(x') = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)e^{(-x'/L_P)} \quad for\ x' \geq 0}$$

Quantitative p-n Diode Solution

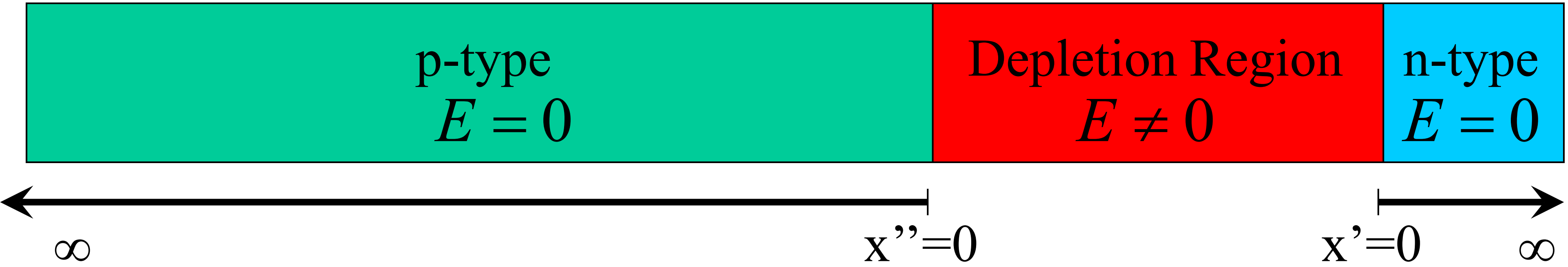


$$\Delta p_n(x') = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)e^{(-x'/L_P)} \quad \text{for } x' \geq 0$$

$$J_p = -qD_p \frac{d\Delta p_n}{dx}$$

$$\boxed{J_p = q\frac{D_p n_i^2}{L_p N_D}\left(e^{qV_A/kT} - 1\right)e^{(-x'/L_P)} \quad \text{for } x' \geq 0}$$

Quantitative p-n Diode Solution



Similarly for electrons on the p-side…

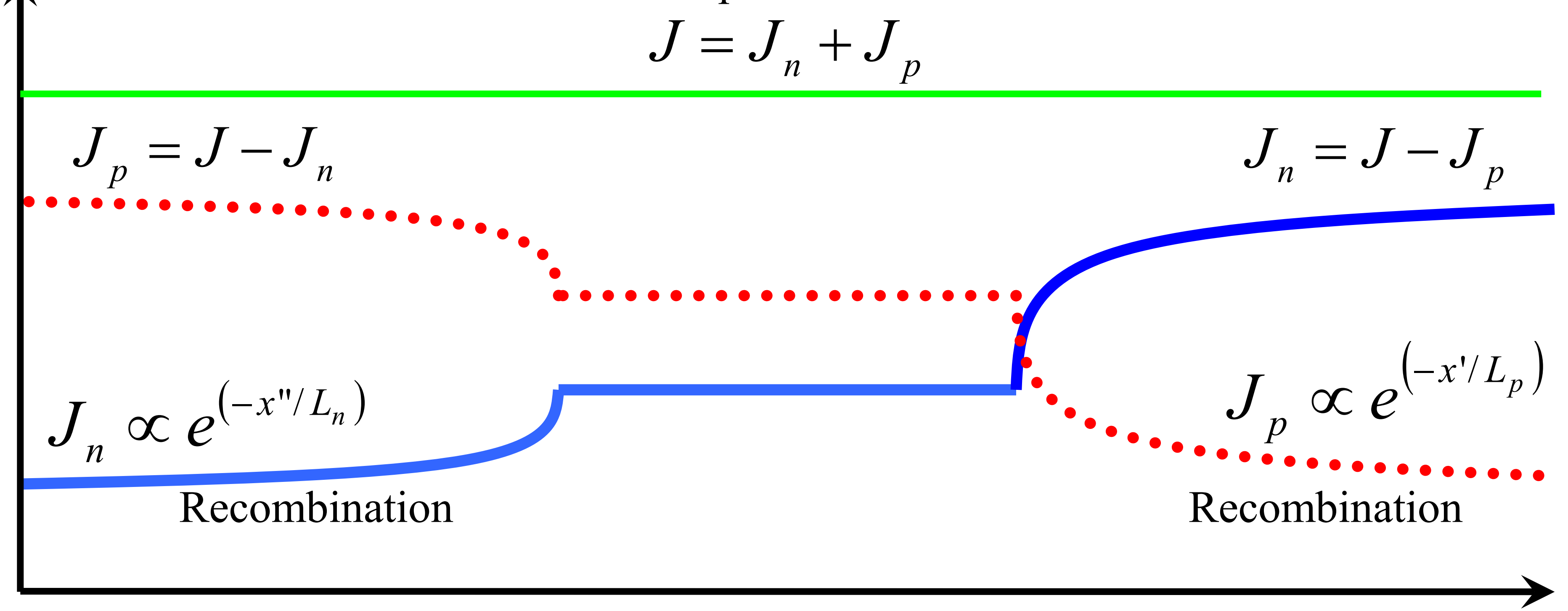
$$\Delta n_p(x'') = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right)e^{(-x''/L_n)} \quad for\ x'' \geq 0$$

$$\mathrm{J_n} = -\mathrm{qD_n}\frac{\mathrm{d}\Delta n_\mathrm{p}}{dx}$$

$$\mathrm{J_n} = q\frac{D_n n_i^2}{L_n N_A}\left(e^{qV_A/kT} - 1\right)e^{(-x''/L_n)} \quad for\ x'' \geq 0$$

Quantitative p-n Diode Solution

Total on current is constant throughout the device. Thus, we can characterize the current flow components as…



Quantitative p-n Diode Solution

Thus, evaluating the current components at the depletion region edges, we have…

$$J = J_n(x''=0) + J_p(x'=0) = J_n(x''=0) + J_p(x''=0) = J_n(x'=0) + J_p(x'=0)$$

$$J = q\left(\frac{D_n n_i^2}{L_n N_A} + \frac{D_p n_i^2}{L_p N_D}\right)\left(e^{qV_A/kT} - 1\right) \quad for\ all\ x$$

J_{ob} and J_{oe}

or

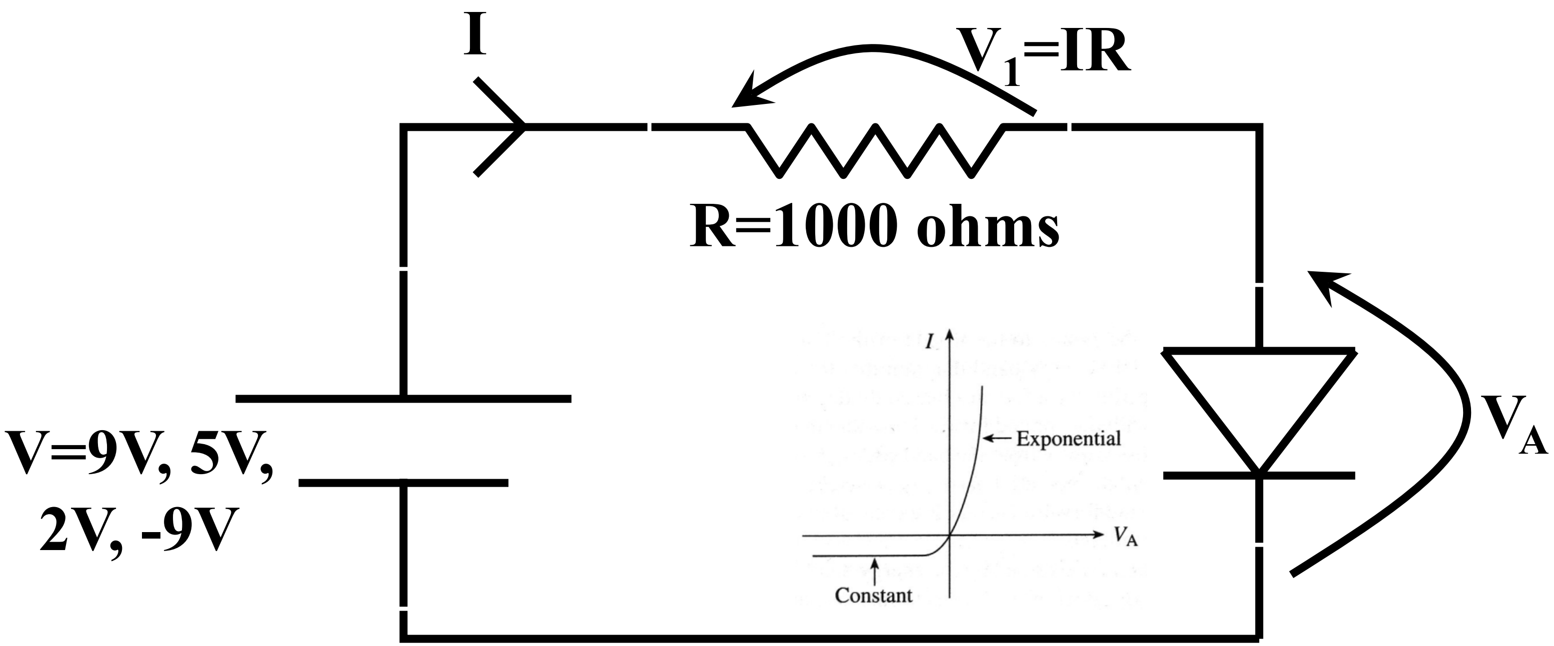
$$I = I_o\left(e^{qV_A/kT} - 1\right) \quad where\ I_o = qA\left(\frac{D_n n_i^2}{L_n N_A} + \frac{D_p n_i^2}{L_p N_D}\right)$$

I_o *is the "reverse saturation current"*

In solar cells, sometimes the two parts of I_o (or J_o) are broken up into J_{oe} and J_{ob} representing the leakage components from the emitter and base respectively.

Note: Vref from our previous qualitative analysis equation is the thermal voltage, kT/q

Quantitative p-n Diode Solution
Examples: Diode in a circuit acts to "clamp voltages"



$$9V = I(1000) + V_A$$

$$I = I_o\left(e^{qV_A/kT} - 1\right) \quad where\ I_o = 1\ pA$$

$$I = 1e-12\left(e^{V_A/0.0259V} - 1\right)$$

or

$$9V = \left[1e-12\left(e^{V_A/0.0259V} - 1\right)\right](1000) + V_A$$

$$9V = 1e-9\left(e^{V_A/0.0259V} - 1\right) + V_A$$

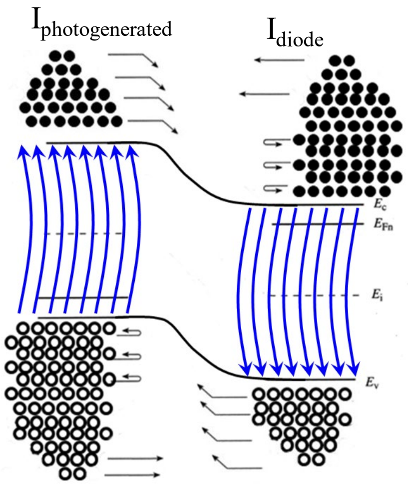
Solutions

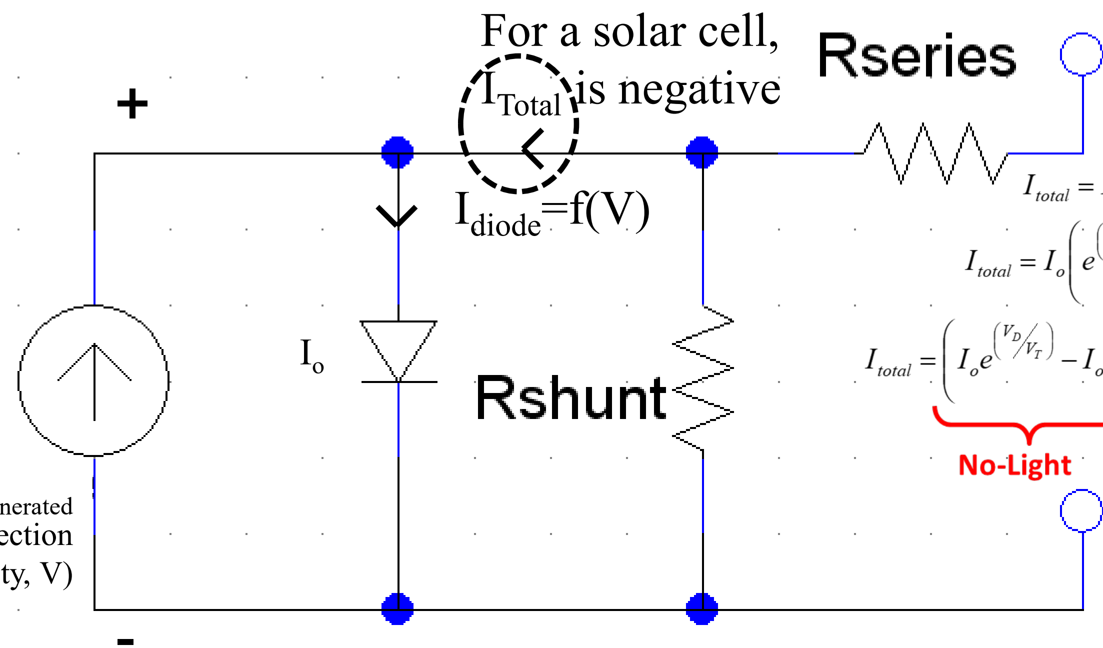
V	V_A	I
9V	0.59V	8.4 mA
5V	0.58V	4.4 mA
2V	0.55V	1.5 mA
-9V	-9.0V	-1 pA

In forward bias (V_A>0) the V_A is ~constant for large differences in current

In reverse bias (V_A<0) the current is ~constant (=saturation current)

Electrical Model of a Solar Cell





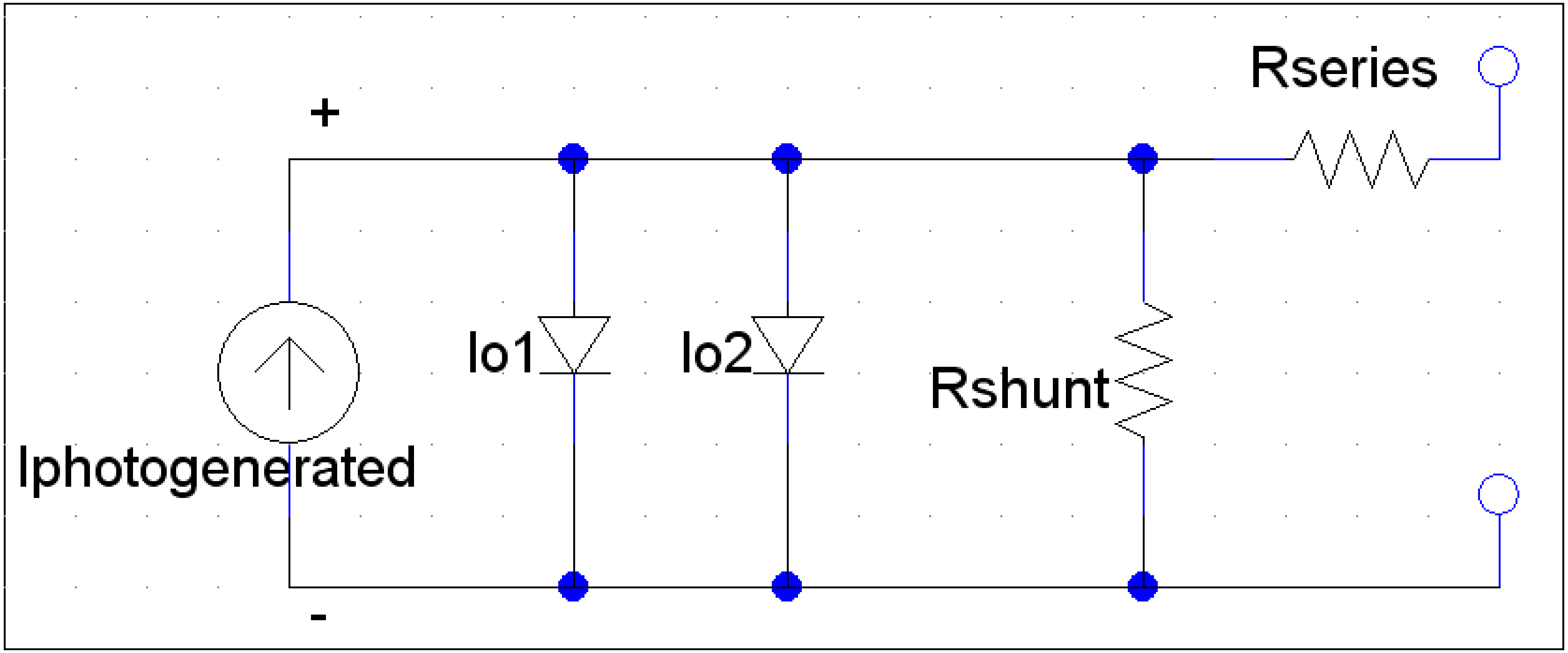
$$I_{total} = I_{dark} + I_{Due\,to\,Light}$$

$$I_{total} = I_o\left(e^{\left(V_D/V_T\right)} - 1\right) + I_{Due\,to\,Light}$$

$$I_{total} = \underbrace{\left(I_o e^{\left(V_D/V_T\right)} - I_o\right)}_{\text{No-Light}} + \underbrace{(-qA)(L_N + W + L_P)G_L}_{\text{Light}}$$

- Without light, the solar cell is just a diode (with non-ideal series and shunt resistors included in it's model).
- With light, an internal voltage is generated that drives current out to the external terminals through R_{series}.
- The Diode and R_{shunt} act to "clamp" the developed voltage, in a sense, fighting against the creation of the voltage.
- V_{turnon} of the diode ($<V_{BI}$) represents the highest possible voltage produced
- $I_{photo\ generated}$ is due to the collection of minority carriers by the junction resulting in a forward bias which in turn tries to drive those collected carriers back across the junction (diode in the model) they were just collected by.

Electrical Model of a Solar Cell

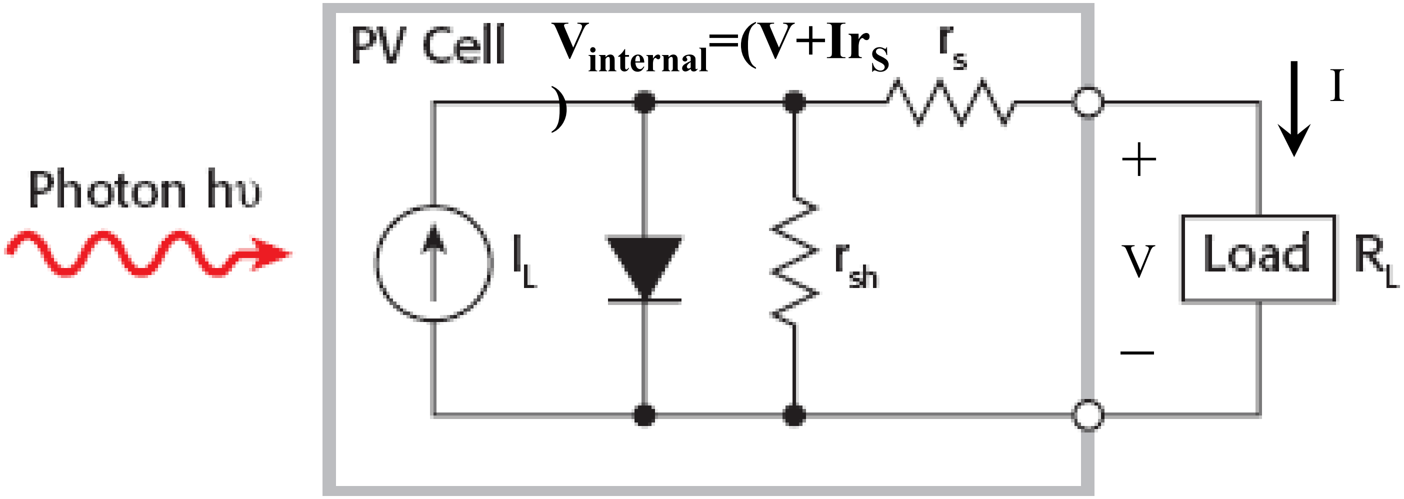


•Some detailed models may add an additional diode. In this case:

- •I_{o1} is a perfect diode with ideality factor, n = 1 and a leakage current I_{o1}
- •I_{o2} is a non-perfect diode with ideality factor, n > 1 and a leakage current I_{o2} . This diode may represent effects such as depletion region recombination (n=2), or tunneling assisted leakage (n>2) or any other host of non-ideal effects.

•Since the actual shunt and series resistances, scale with cell area, they are often quoted as normalized resistances in Ohm-cm^2 (i.e. V/J not V/I) to allow easy comparisons

Solar Cell Equivalent Circuit



- Using the diode equation: $\mathbf{I = I_O(e^{\{qV/nkT\}} - 1)}$
- $\mathbf{I = I_L - I_O(e^{\{[V+Ir_S]/nV_T\}} - 1) - (\{V + Ir_S\}/r_{shunt})}$
- I_L is the light induced current and is the short circuit current **(I_{SC})** when r_s is negligible
- $\mathbf{V_{OC} = kT/q\ (\ln \{[I_L/I_{OC}] + 1\})}$
- $\mathbf{r_S}$ is the series resistance due to bulk material resistance and metal contact resistances.
- $\mathbf{r_{Sh}}$ is the shunt resistance due to lattice defects in the depletion region and leakage current on the edges of the cell, etc....
- $\mathbf{V_T = kT/q}$
- ***n* –** non ideality factor, = 1 for an ideal diode

IV Curves

- V_m and I_m – the operating point yielding the maximum power output
- FF – fill factor – measure of how "square" the output characteristics are and used to determine efficiency.

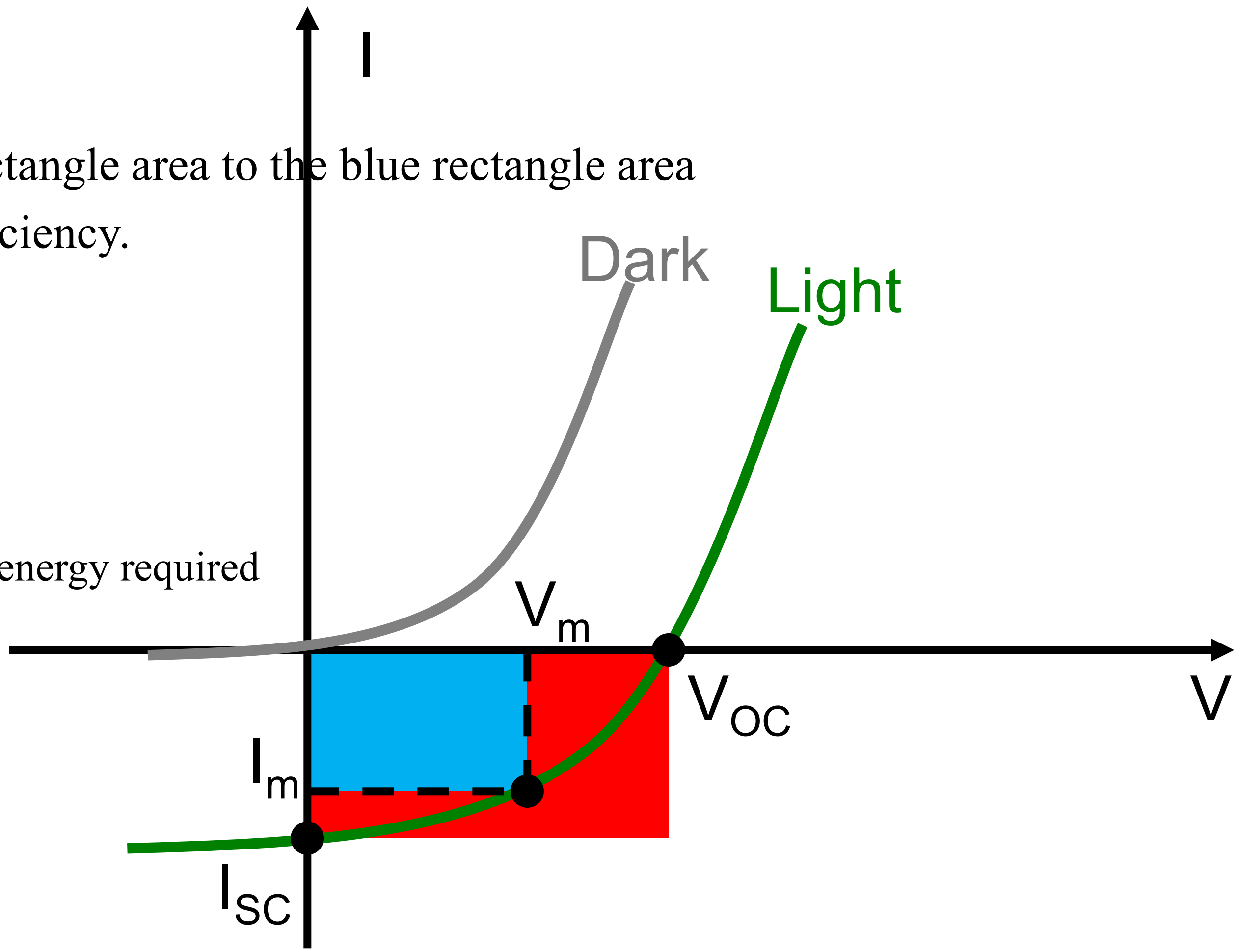
  $FF = V_m I_m / V_{OC} I_{SC}$

  Is the ratio of the red rectangle area to the blue rectangle area
- η - power conversion efficiency.

  $\eta = P_{max} / P_{in}$

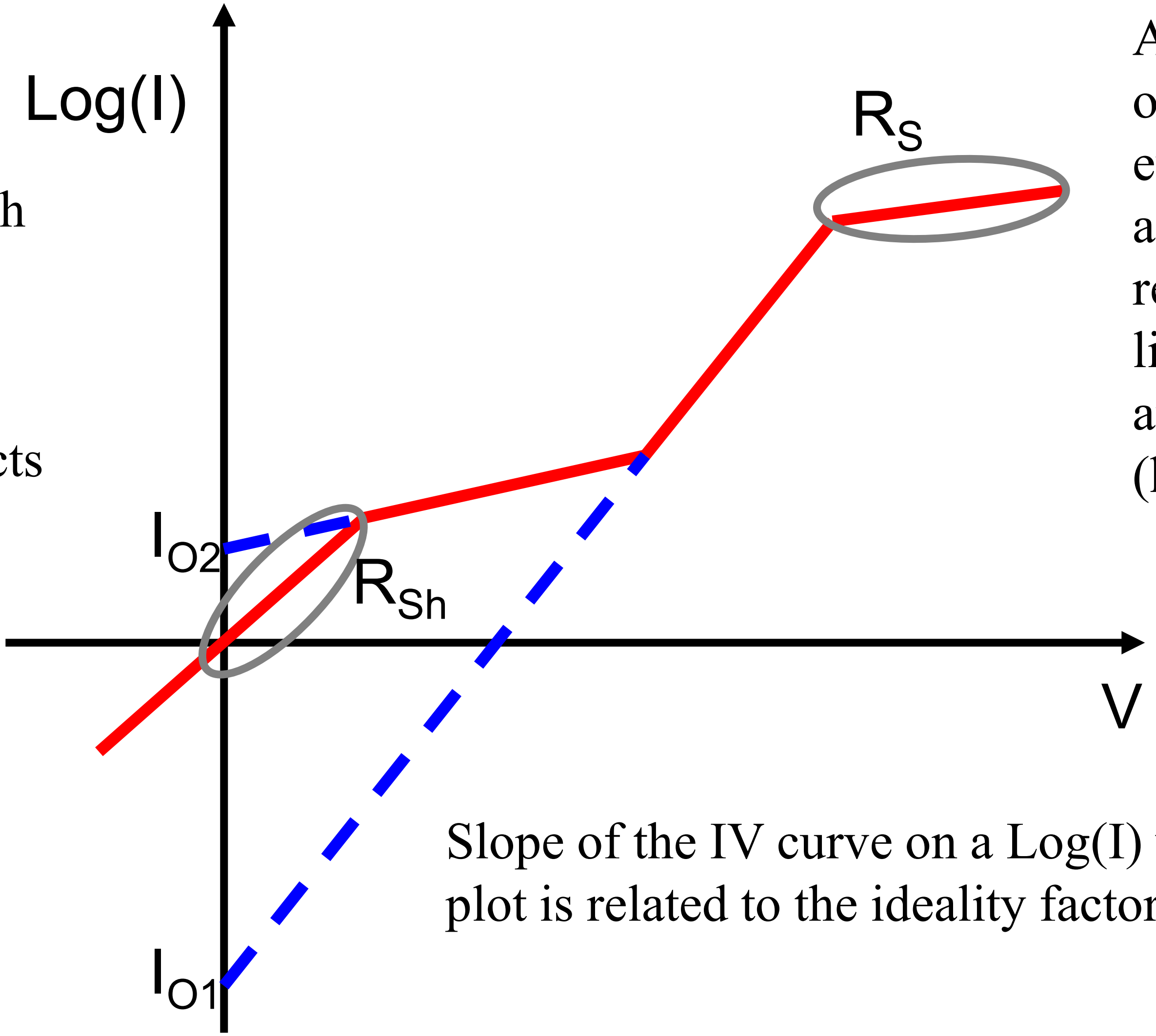
  $= V_m I_m / P_{in}$

  $= FF V_{OC} I_{SC} / P_{in}$
- If $E_G \downarrow$ then:
  - More photons have the energy required to create an EHP
  - $I_{SC} \uparrow$ and $V_{OC} \downarrow$
- Large R_S and low R_{Sh} reduces V_{OC} and I_{SC}



IC Curves – Dark measurement

Good cells with really low I_{o1} will be dominated by non-ideal effects (R_{sh} and I_{o2}) at low voltages.

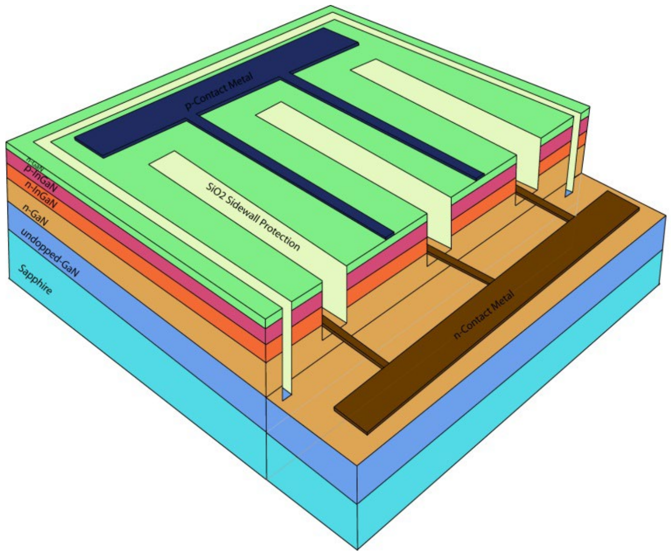


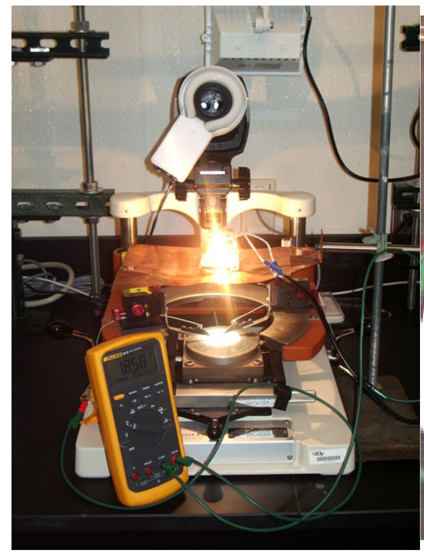
All cells (good or bad) will eventually reach a series resistance limited regime at high current (high voltages).

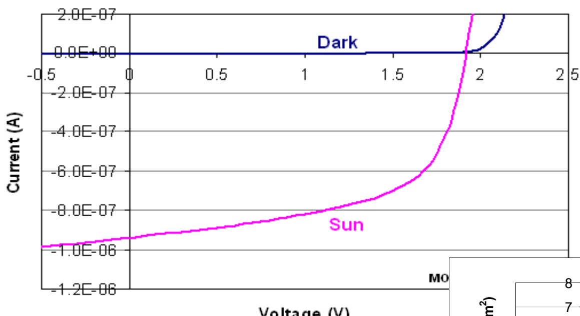
Slope of the IV curve on a Log(I) vs V plot is related to the ideality factor.

Ga Tech Record Device Results

Record device performance: Highest single junction open circuit voltage (2.4 V) and very high Voc for VHESC relevant material (2.0 eV).

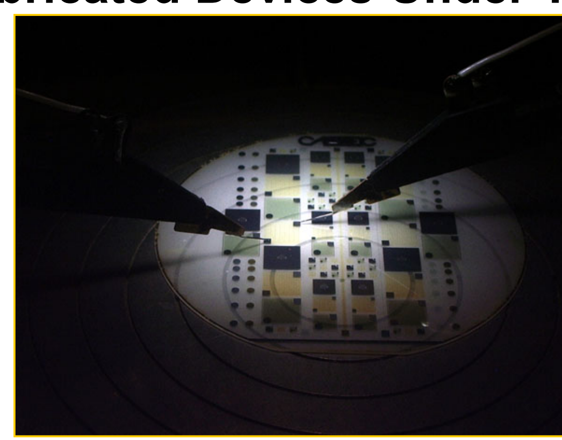






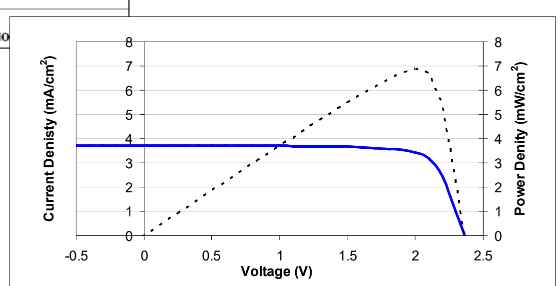
2.4- 2.8 eV Material
Voc = 1.95V
FF = 57.3%

Fabricated Devices Under Test



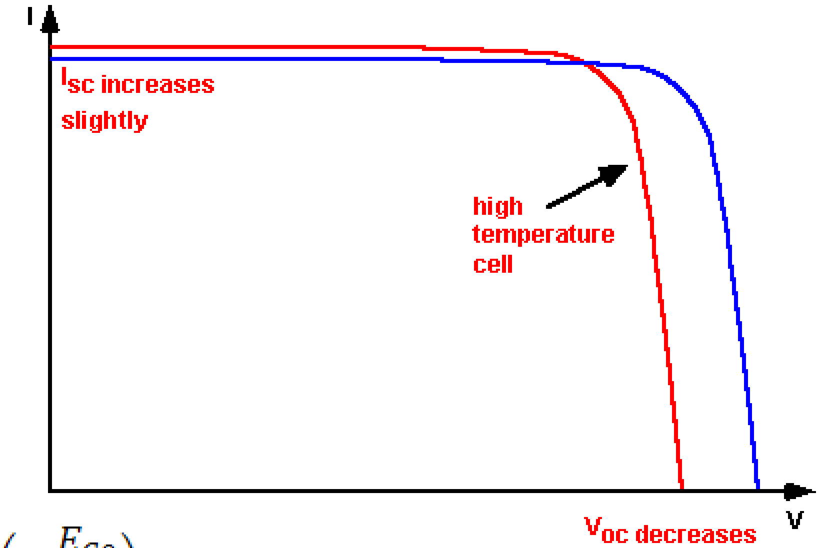
Note: Multi-meter shown for effect. Actual measurements use the most precise instrumentation currently available

Eg>2.8 eV Material
Voc = 2.4V



Effect of Temperature

- Changes in temperature change the band gap of a semiconductor.
- Increasing temperature:
  - Decrease in the band gap
    - Very slight increase in photocurrent
    - Strong decrease in photovoltage



$$I_0 = qA\frac{Dn_i^2}{LN_D}$$

γ

$$n_i^2 = 4\left(\frac{2\pi kT}{h^2}\right)^3 (m_e^* m_h^*)^{3/2}\exp\left(-\frac{E_{G0}}{kT}\right) = BT^3\exp\left(-\frac{E_{G0}}{kT}\right)$$

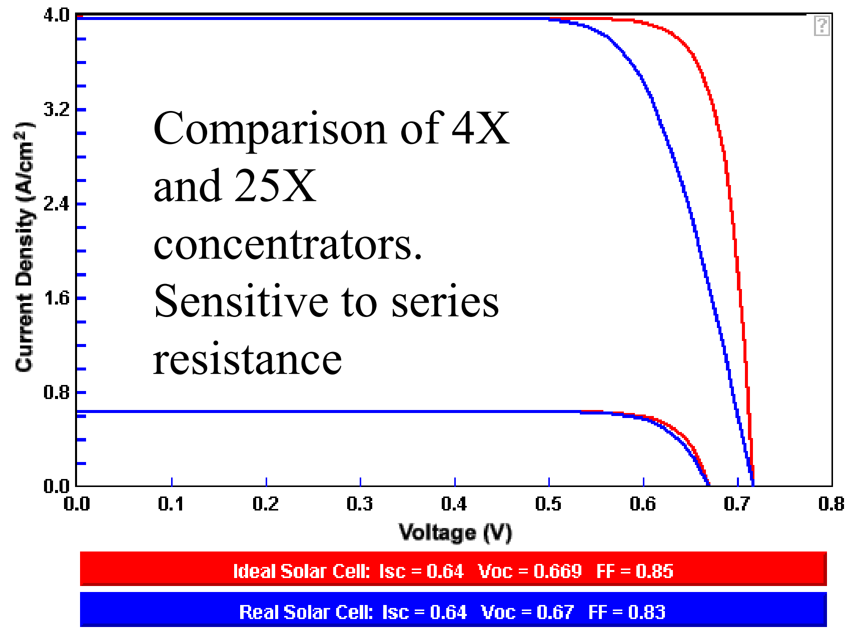
$$I_0 = qA\frac{D}{LN_D}BT^3\exp\left(-\frac{E_{G0}}{kT}\right) \approx B'T^\gamma\exp\left(-\frac{E_{G0}}{kT}\right)$$

$$V_{OC} = \frac{kT}{q}\ln\left(\frac{I_{SC}}{I_0}\right) = \frac{kT}{q}[\ln I_{SC} - \ln I_0] = \frac{kT}{q}\ln I_{SC} - \frac{kT}{q}\ln\left[B'T^\gamma exp\left(-\frac{qV_{G0}}{kT}\right)\right]$$

$$= \frac{kT}{q}\left(\ln I_{SC} - \ln B' - \gamma\ln T + \frac{qV_{G0}}{kT}\right)$$

$$\frac{dV_{OC}}{dT} = -\frac{V_{G0} - V_{OC} + \gamma\frac{kT}{q}}{T} \approx -2.2mV\ per\ {}^oC\ for\ Si$$

Effect of Solar Concentration



A concentrator solar cell is designed to operate under illumination greater than 1 sun. Concentrators have several potential advantages:

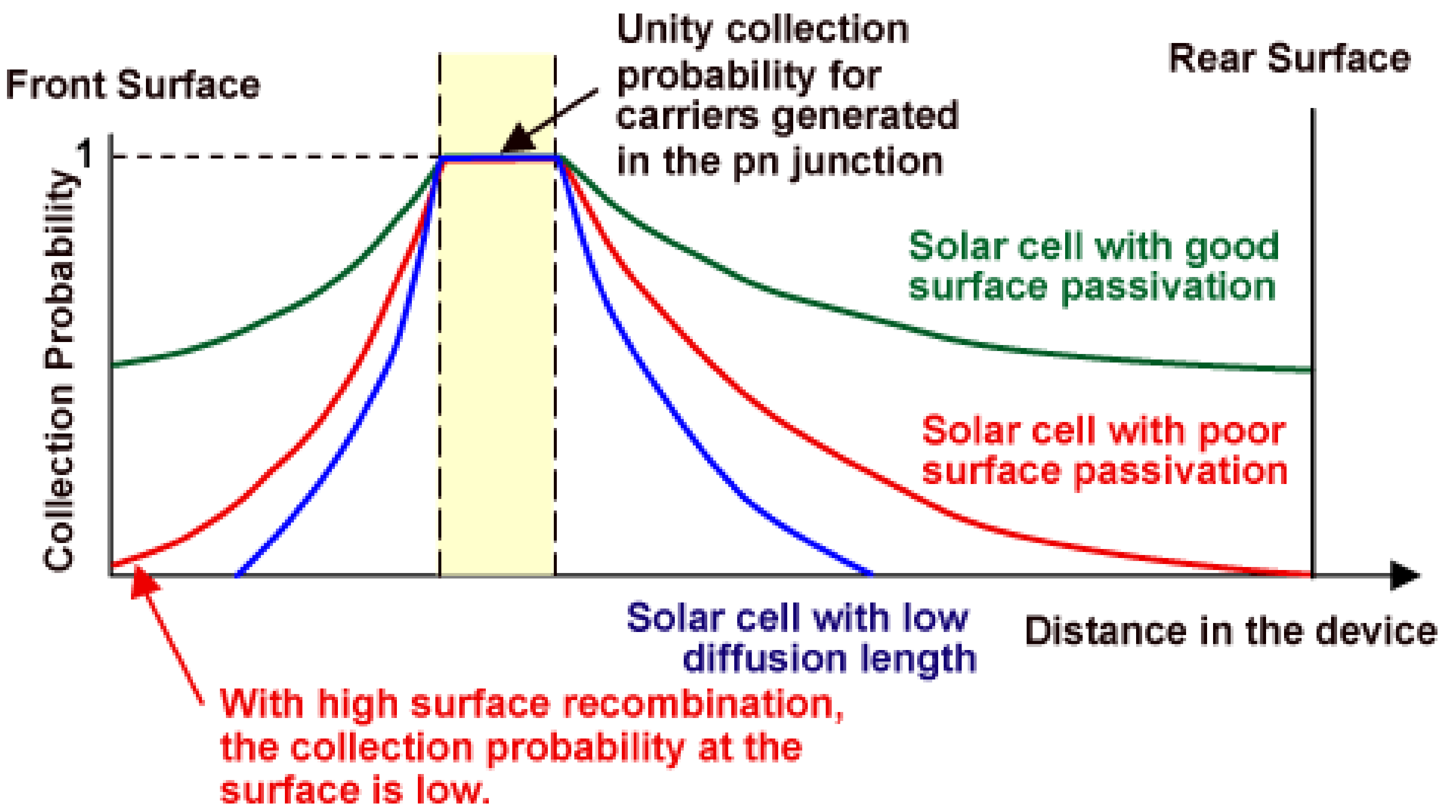
- A possibility of higher efficiency
- A possibility of lower cost.
- I_{sc} depends linearly on light intensity
- Efficiency improves due to the logarithmic dependence of the open-circuit voltage on short circuit:

$$V'_{OC} = \frac{nkT}{q}\ln\left(\frac{XI_{SC}}{I_0}\right) = \frac{nkT}{q}\left[\ln\left(\frac{I_{SC}}{I_0}\right) + \ln X\right] = V_{OC} + \frac{nkT}{q}\ln X$$

where X is the concentration of sunlight.

From the equation above, a doubling of the light intensity (X=2) causes a 18 mV rise in silicon's V_{OC}.

Other Measurements: How to Quantify Collection Probability



Run PVCDROM Applet

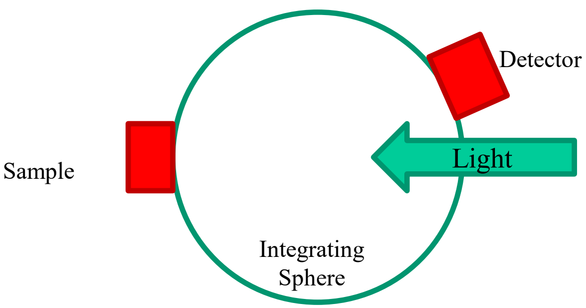
Other Measurements: How to Quantify Collection Probability – Quantum Efficiency

Lock-in Amplifier and Mechanical (grating) motor controller

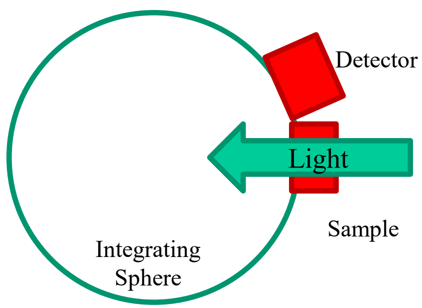
Lamp

Beam Chopper

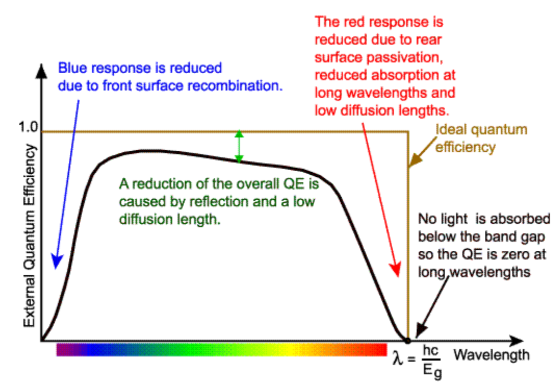
Diffraction grating selects wavelengths



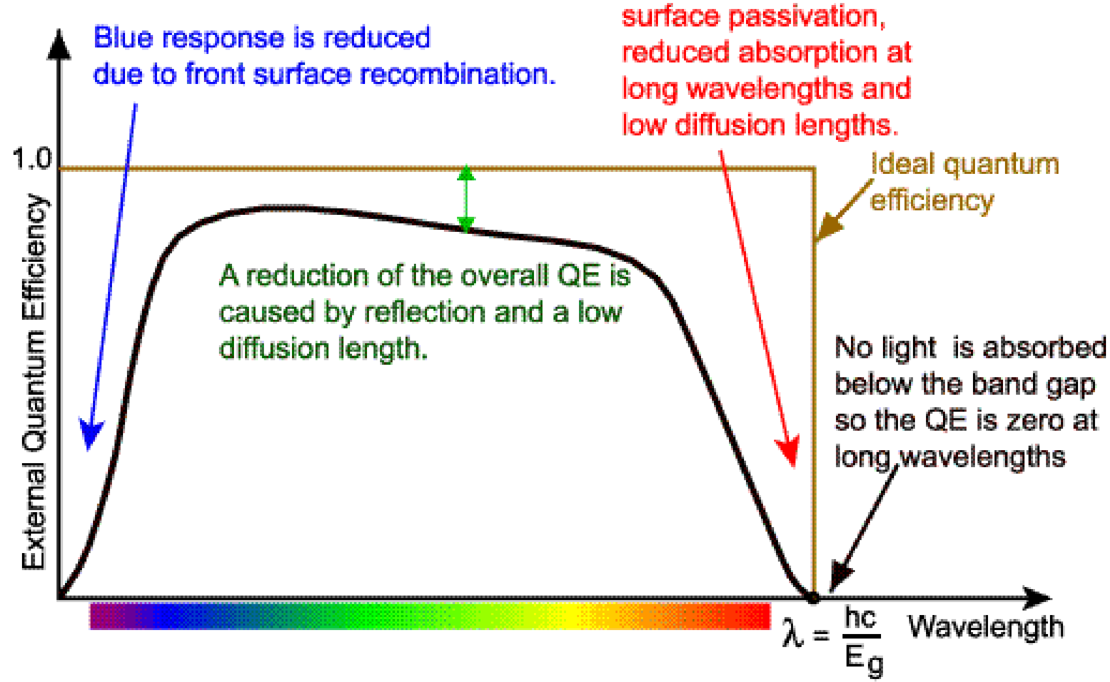
Reflection and Spectral Response Measurement



Transmission Measurement



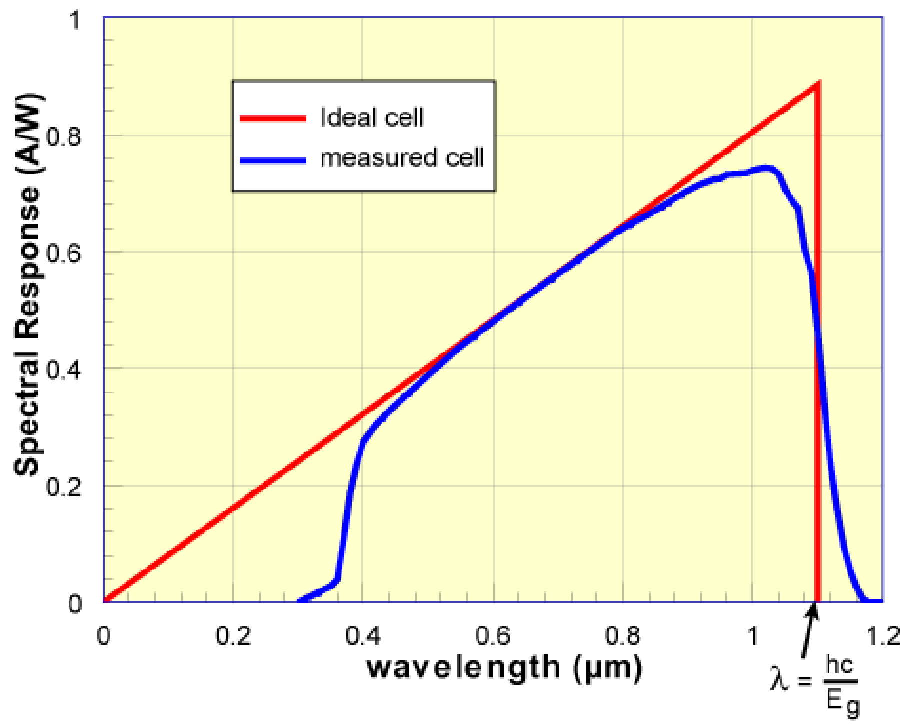
Other Measurements: How to Quantify Collection Probability – Quantum Efficiency



<u>The "quantum efficiency" (Q.E.) is the ratio of the number of carriers collected by the solar cell to the number of photons of a given energy incident on the solar cell</u>. The quantum efficiency may be given either as a function of wavelength or as energy. The "external" quantum efficiency of a silicon solar cell includes the effect of optical losses such as transmission and reflection. However, it is often useful to look at the quantum efficiency of the light left after the reflected and transmitted light has been lost. "Internal" quantum efficiency refers to the efficiency with which photons that are not reflected or transmitted out of the cell can generate collectable carriers. $IQE = EQE / (1 - R - T)$. By measuring the reflection and transmission of a device, the external quantum efficiency curve can be corrected to obtain the internal quantum efficiency curve.

Run PVCDROM Applet

Other Measurements: How to Quantify Collection Probability – Spectral Response



$$SR = \frac{q\lambda}{hc} QE$$

The spectral response is the ratio of the current generated by the solar cell to the **power** incident on the solar cell.
The ideal spectral response is limited at long wavelengths by the inability of the semiconductor to absorb photons with energies below the band gap. However, unlike the square shape of QE curves, the spectral response decreases at small photon wavelengths. At these wavelengths, each photon has a large energy, and hence the ratio of photons to power is reduced. Any energy above the band gap energy is not utilized by the solar cell and instead goes to heating the solar cell. The inability to fully utilize the incident energy at high energies, and the inability to absorb low energies of light represents a significant power loss in solar cells consisting of a single p-n junction.
Spectral response is important since it is the spectral response that is measured from a solar cell, and from this the quantum efficiency is calculated. The quantum efficiency can be determined from the spectral response by replacing the power of the light at a particular wavelength with the photon flux for that wavelength.