1. a) $y[n] + .5\,y[n-1] = 2x[n]$

$$Y(z)(1 + .5z^{-1}) = 2X(z)$$

$$H(z) = \frac{2}{1+.5z^{-1}} = \frac{2z}{z+.5}$$

b) $y[n] + 2y[n-1] - y[n-2] = 2x[n] - x[n-1] + 2x[n-2]$

$$Y(z)(1 + 2z^{-1} - z^{-2}) = X(z)(2 - z^{-1} + 2z^{-2})$$

$$H(z) = \frac{2 - z^{-1} + 2z^{-2}}{1 + 2z^{-1} - z^{-2}} = \frac{2z^2 - z + 2}{z^2 + 2z - 1}$$

c) $y[n] + y[n-2] = 2x[n] - x[n-1]$

$$Y(z)(1 + z^{-2}) = X(z)(2 - z^{-1})$$

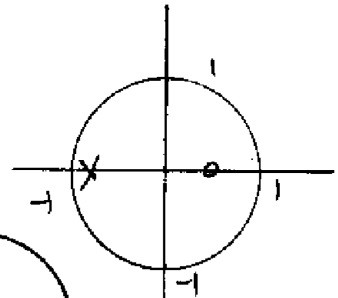
$$H(z) = \frac{2 - z^{-1}}{1 + z^{-2}} = \frac{2z^2 - z}{z^2 - 1}$$

d) $y[n] = x[n] - 2x[n-1] + x[n-2]$

$$Y(z) = X(z)(1 - 2z^{-1} + z^{-2})$$

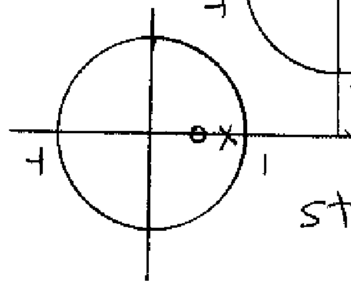
$$H(z) = 1 - 2z^{-1} + z^{-2} = \frac{z^2 - 2z + 1}{z^2}$$

2 a) $H(z)=\dfrac{z-.5}{z+.75}$



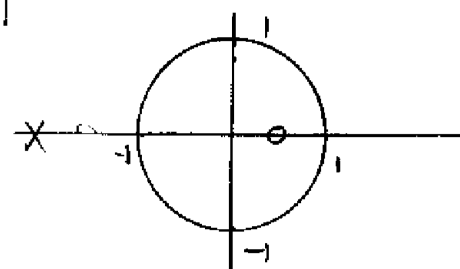
stable

b) $H(z)=\dfrac{z-.5}{z-.75}$



stable

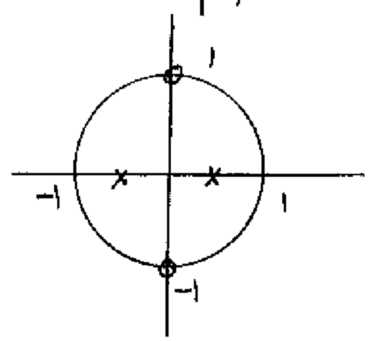
c) $H(z)=\dfrac{z-.5}{z+2}$



unstable

d) $H(z)=\dfrac{z^2+1}{z^2-0.25}$

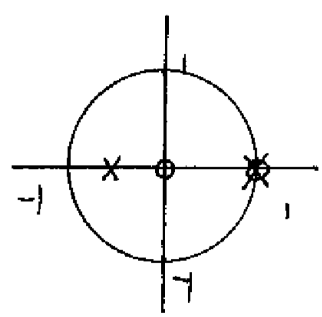
$=\dfrac{(z-j)(z+j)}{(z-\frac{1}{2})(z+\frac{1}{2})}$



stable

e) $H(z)=\dfrac{z(z-1)}{z^2-0.5z-0.5}$

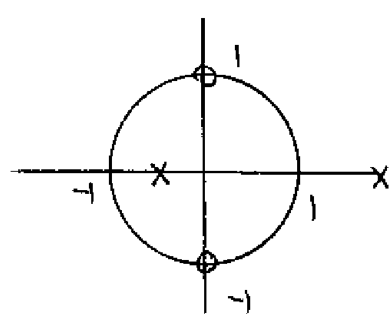
$=\dfrac{z(z-1)}{(z-1)(z+.5)}$



marginally stable

f) $H(z)=\dfrac{z^2+1}{z^2-1.5z-1}$

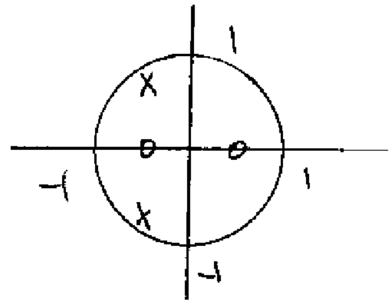
$=\dfrac{(z+j)(z-j)}{(z-2)(z+0.5)}$



unstable

g) $H(z) = \frac{(z-.5)(z+.5)}{z^2+z+.74}$

$$= \frac{(z-.5)(z+.5)}{(z+.5+0.7j)(z+0.5-0.7j)}$$

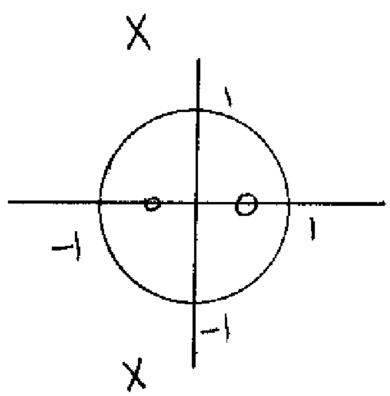


poles at $0.86\angle 125^\circ \Rightarrow$ stable

h) $H(z) = \frac{(z-0.5)(z+0.5)}{z^2+z+4.25}$

$$= \frac{(z-0.5)(z+0.5)}{(z+0.5+2j)(z+0.5-2j)}$$

poles at $2.06\angle 104^\circ \Rightarrow$ unstable



3. a) $H(z) = \dfrac{z - .5}{z + 0.75}$, pole at -0.75

$$y[n] = c_1(-0.75)^n u[n] + \delta[n] c_2$$

b) $H(z) = \dfrac{z - 0.5}{z - 0.75}$, pole at 0.75

$$y[n] = c_1 (0.75)^n u[n] + c_2 \delta[n]$$

c) $H(z) = \dfrac{z - 0.5}{z + 2}$, pole at -2

$$y[n] = c_1 (-2)^n u[n] + c_2 \delta[n]$$

d) $H(z) = \dfrac{z^2 + 1}{z^2 - 0.25}$, poles at $0.5, -0.5$

$$y[n] = c_1 (0.5)^n u[n] + c_2 (-0.5)^n u[n] + c_3 \delta[n]$$

e) $H(z) = \dfrac{z(z-1)}{z^2 - 0.5z - 0.5}$, poles at $1, -0.5$

$$y[n] = c_1 u[n] + c_2 (-0.5)^n u[n] + c_3 \delta[n]$$

f) $H(z) = \frac{z^2+1}{z^2-1.5z-1}$, poles at $2, -0.5$

$y[n] = c_1\, 2^n u[n] + c_2 (-0.5)^n u[n] + c_3\, \delta[n]$

g) $H(z) = \frac{(z-0.5)(z+0.5)}{z^2+z+0.74}$, poles at $0.86 \angle 125^\circ$, 2.19 rad

$y[n] = c\,(0.86)^n \cos(2.19\, n + \theta)\, u[n] + c_2\, \delta[n]$

h) $H(z) = \frac{(z-0.5)(z+0.5)}{z^2+z+4.25}$, poles at $2.06 \angle 104^\circ$, 1.81 rad

$y[n] = c\,(2.06)^n \cos(1.81\, n + \theta)\, u[n] + c_2\, \delta[n]$

4. a) $H(z) = \frac{(z-0.5)(z-0.1)}{(z+0.75)^2}$

$y[n] = c_1(-0.75)^n u[n] + c_2 n(-0.75)^n u[n] + c_3 \delta[n]$

b) $H(z) = \frac{(z-0.5)(z+0.5)(z+1)}{(z^2+z+0.74)(z-0.75)}$

poles at 0.75, $0.86 \angle 2.19$ rad

$y[n] = c_1(0.75)^n u[n] + c_2 (0.86)^n \cos(2.19n + \theta) u[n] + c_3 \delta[n]$

c) $H(z) = \frac{(z-0.5)^2(z+0.5)(z+1)^2}{(z^2+z+0.74)^2(z-0.75)}$

poles at $0.75, 0.86 \angle 2.19..., 0.86 \angle 2.19..$

$y[n] = c_1(0.75)^n u[n] + c_2(0.86)^n \cos(2.19n + \theta_1) u[n]$

$+ c_3(0.86)^n n \cos(2.19n + \theta_2) u[n] + c_4 \delta[n]$

5, a) $H(z) = \dfrac{z - 0.5}{z + 0.75}$, $X(z) = \dfrac{z}{z-1}$

$$Y(z) = H(z)X(z) = \frac{(z-0.5)z}{(z+0.75)(z-1)}$$

$$\frac{Y(z)}{z} = \frac{0.71}{z+0.75} + \frac{0.285}{z-1}$$

$$Y(z) = \frac{0.71z}{z+0.75} + \frac{0.285z}{z-1}$$

$$y[n] = \left(0.71\,(-0.75)^n + 0.285\right)u[n]$$

b) $H(z) = \dfrac{z - 0.5}{z - 0.75}$

$$\frac{Y(z)}{z} = \frac{z-0.5}{(z-0.75)(z-1)}$$

$$Y(z) = \frac{-z}{(z-0.75)} + \frac{2z}{z-1}$$

$$y[n] = \left(-(0.75)^n + 2\right)u[n]$$

c) $H(z) = \frac{z - 0.5}{z + 2}$

$$\frac{Y(z)}{z} = \frac{z - 0.5}{(z+2)(z-1)}$$

$$Y(z) = \frac{0.833z}{z+2} + \frac{0.167z}{z-1}$$

$$y[n] = \left(0.833(-2)^n + 0.167\right)u[n]$$

d) $H(z) = \frac{z^2 + 1}{z^2 - 0.25}$

$$\frac{Y(z)}{z} = \frac{z^2 + 1}{(z-0.5)(z+0.5)(z-1)}$$

$$Y(z) = \frac{-2.5z}{z-0.5} + \frac{0.833}{z+0.5} + \frac{2.67}{z-1}$$

$$y[n] = \left(-2.5(0.5)^n + 0.833(-0.5)^n + 2.67\right)u[n]$$

e) $H(z) = \dfrac{z(z-1)}{z^2 - 0.5z - 0.5}$

$$\frac{Y(z)}{z} = \frac{z(z-1)}{(z-1)(z+0.5)(z-1)} = \frac{z}{(z+0.5)(z-1)}$$

$$Y(z) = \frac{0.33z}{z+0.5} + \frac{0.67z}{z-1}$$

$$y[n] = \left(0.33(-0.5)^n + 0.67\right)u[n]$$

f) $H(z) = \dfrac{z^2+1}{z^2 - 1.5z - 1} = \dfrac{z^2+1}{(z-2)(z+0.5)}$

$$\frac{Y(z)}{z} = \frac{z^2+1}{(z-2)(z-1)(z+0.5)}$$

$$Y(z) = \frac{2z}{z-2} + \frac{-1.33z}{z-1} + \frac{0.33z}{z+0.5}$$

$$y[n] = \left((2)^{n+1} - 1.33 + 0.33(-0.5)^n\right)u[n]$$

g) $H(z) = \dfrac{(z-.5)(z+.5)}{z^2+z+0.74}$

$$\frac{Y(z)}{z} = \frac{(z-.5)(z+.5)}{(z^2+z+0.74)(z-1)}$$

$$= \frac{C_1}{z-1} + \frac{C_2 z + C_3}{z^2+z+0.74}$$

$C_1 = .273$

$$z^2 - 0.25 = .273(z^2+z+0.74) + (z-1)(C_2 z + C_3)$$
$$C_2 = 0.727 \quad , C_3 = +0.452$$

$$y(z) = \frac{.273z}{z-1} + \frac{.727(z^2+0.62z)}{z^2 - 2a\cos(\Omega)z + a^2}$$
$$a = 0.86 \ , \quad \Omega = 2.19 \text{ rad}$$

$$= \frac{.273z}{z-1} + \frac{.727(z^2 - a\cos(\Omega)z)}{z^2 - 2a\cos(\Omega) + a^2} + \frac{0.087 \quad z}{z^2 - 2a\cos(\Omega) + a^2}$$
where $0.087 = 0.125 \, a \sin \Omega$

$$y[n] = 0.273 + 0.727(0.86)^n \cos(2.19n)$$
$$+ 0.125(0.86)^n \sin(2.19n),$$

$n \geq 0$

h) $H(z) = \dfrac{(z-0.5)(z+0.5)}{z^2+z+4.25}$

$$\frac{Y(z)}{z} = \frac{z^2-0.25}{(z^2-2a\cos\Omega\, z+a^2)(z-1)} \quad , \; a = 2.06, \; \Omega = 1.816$$

$$\frac{Y(z)}{z} = \frac{0.12}{z-1} + \frac{C_1 z + C_2}{z^2-2a\cos\Omega\, z+a^2}$$

$$z^2-0.25 = 0.12(z^2+z+4.25) + (z-1)(C_1 z + C_2)$$

$$C_1 = 0.88 \; , \; C_2 = 0.76$$

$$Y(z) = \frac{0.12z}{z-1} + \underbrace{\frac{0.88z^2+0.76z}{z^2-2a\cos(\Omega)z+a^2}}$$

$$\frac{0.88(z^2-a\cos(\Omega)z) + .32z}{z^2-2a\cos(\Omega)z+a^2} \qquad .32 = 0.16a\sin(\Omega)$$

$$y[n] = \Big(0.12 + 0.88\,(2.06)^n\cos(1.816n)$$

$$+0.16\sin(1.816n)(2.06)^n\Big)\; u[n]$$

6.

a) $Y(z) = (1 - z^{-1} + z^{-2})X(z)$

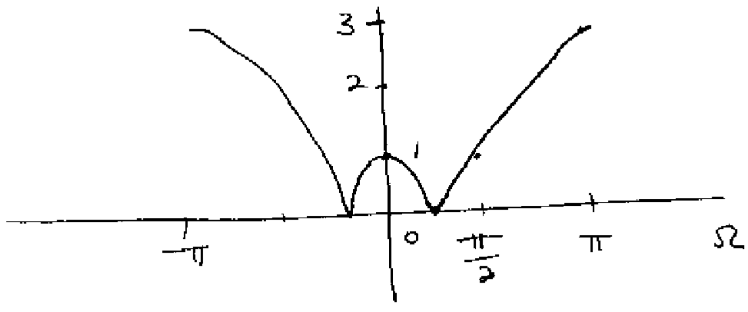
$$H(z) = 1 - z^{-1} + z^{-2} = \frac{z^2 - z^{1} + 1}{z^2}$$

b) $h[n] = \delta[n] - \delta[n-1] + \delta[n-2]$

c) this is an FIR filter $\Rightarrow$ $h[n] \to 0$ in finite # of steps $\Rightarrow$ it is stable (also note that the poles are inside the unit circle)

d)
$$H(\Omega) = H(z)\big|_{z=e^{j\Omega}} = 1 - e^{-j\Omega} + e^{-2j\Omega}$$
$$= e^{-j\Omega}\left(e^{j\Omega} - 1 + e^{-j\Omega}\right)$$
$$= e^{-j\Omega}\left(2\cos(\Omega) - 1\right)$$

$$|H(\Omega)| = |2\cos(\Omega) - 1|$$



band stop
(possibly highpass)