1. a) $x[n] = u[n] - u[n-4]$

$$X(z) = \sum_{n=0}^{3} z^{-n} = 1 + z^{-1} + z^{-2} + z^{-3}$$

b) $x[n] = 0.5^n u[n]$

$$X(z) = \sum_{n=-\infty}^{\infty} 0.5^n u[n] z^{-n}$$

$$= \sum_{n=0}^{\infty} (0.5 z^{-1})^n$$

use geometric series convergence:

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad \text{if} \quad |a|<1$$

$$X(z) = \frac{1}{1 - 0.5z^{-1}}$$

c) $x[n] = [1 \quad 4 \quad 8 \quad 2]$

$$X(z) = 1 + 4z^{-1} + 8z^{-2} + 2z^{-3}$$

d) $x[n] = [0 \quad 1 \quad 2 \quad 3 \; 4]$

$$X(z) = z^{-1} + 2z^{-2} + 3z^{-3} + 4z^{-4}$$

e) $x[n] = 2(.8)^n u[n]$

$$X(z) = \frac{2}{1 - 0.8z^{-1}}$$

2. a) $X(z) = \dfrac{(z-1)(z+0.8)}{(z-0.5)(z+0.2)}$

$$\frac{X(z)}{z} = \frac{c_1}{z} + \frac{c_2}{z-.5} + \frac{c_3}{z+0.2}$$

$$c_1 = \frac{X(z)}{z} z \Big|_{z=0} = 8 \qquad c_2 = \frac{X(z)}{z}(z-.5)\Big|_{z=.5} =$$

$$= \frac{(z-1)(z+.8)}{z(z+.2)}\Big|_{z=.5} = -1.857$$

$$c_3 = \frac{X(z)}{z}(z+.2)\Big|_{z=-.2} = -5.143$$

$$X(z) = 8 - \frac{1.857\, z}{z-.5} - \frac{5.143 z}{z+0.2}$$

$$x[n] = 8\delta[n] - 1.857\,(0.5)^n u[n] - 5.143\,(-0.2)^n\; u[n]$$

b) $X(z) = \dfrac{z+0.8}{(z-0.5)(z+.2)}$

$$X(z) = \frac{1.857}{z-.5} - \frac{.8571}{z+.2}, \text{ let } X_1(z) = \frac{1.857 z}{z-.5} - \frac{.8571 z}{z+.2}$$

then $x_1[n] = \left(1.857\,(.5)^n - .8571\,(-0.2)^n\right) u[n]$

$$X(z) = z^{-1} X_1(z) \Rightarrow x[n] = \left(1.857\,(0.5)^{n-1} - .8571\,(-0.2)^{n-1}\right) u[n-1]$$

c) $X(z) = \dfrac{z^3+z+1}{(z^2-.5z+.25)(z-1)}$

$$\frac{X(z)}{z} = \frac{-4}{z} + \frac{4}{(z-1)} + \frac{c_1 z + c_2}{z^2-.5z+.25}$$

$$z^3+z+1 = (-4z+4)(z^2-.5z+.25) + 4z(z^2-.5z+.25)$$

$$+(c_1 z + c_2)(z^2-z)$$

z^3: $1 = -4+4+c_1 \Rightarrow c_1 = 1$

z^1: $1 = -1-2+1-c_2 \Rightarrow c_2 = -3$

$$X(z) = -4 + \frac{4z}{z-1} + \frac{z^2-3z}{z^2-0.5z+.25}$$

let $a = 0.5$, $-2a\cos(\Omega) = -0.5$

$\Rightarrow \Omega = 1.047$ rad

$$\frac{z^2-3z}{z^2-0.5z+0.25} = \frac{z^2 - a\cos(\Omega)z + 2.75}{z^2-2a\cos(\Omega)z + a^2}$$ $= 6.35 a \sin \Omega$

$$x[n] = -4\delta[n] + 4u[n] + (0.5)^n \cos(1.047n)u[n]$$

$$+(0.5)^n\, 6.35 \sin(1.047n)u[n]$$

d) $X(z) = \dfrac{(z^2-1)(z+0.8)}{(z-0.5)^2(z+0.2)}$

$$\frac{X(z)}{z} = \frac{C_1}{z} + \frac{C_2}{z+.2} + \frac{C_3}{z-.5} + \frac{C_4}{(z-.5)^2}$$

$C_1 = -16$, $C_2 = 5.88$, $C_4 = -2.79$

$$C_3 = \frac{d}{dz}\left(\frac{(z^2-1)(z+.8)}{z(z+.2)}\right)\Bigg|_{z=.5} = 11.12$$

$$x[n] = -16\,\delta[n] + 5.88(-0.2)^n u[n] + 11.12\,(0.5)^n u[n]$$

$$-2.79n\,(0.5)^n u[n]$$

3 a) $X(z) = \frac{(z-1)(z+0.8)}{(z-0.5)(z+0.2)}$

$$\lim_{n\to\infty} x[n] = \left[(z-1)X(z)\right]_{z=1}$$

$$= \frac{(z-1)^2(z+.8)}{(z-0.5)(z+0.2)}\bigg|_{z=1} = 0$$

matches 2a)

b) $X(z) = \frac{z+.8}{(z-.5)(z+.2)}$

$$\lim_{n\to\infty} x[n] = \frac{(z-1)(z+.8)}{(z-.5)(z+.2)}\bigg|_{z=1} = 0$$

matches 2b)

c) $X(z) = \frac{z^3+z+1}{(z^2-.5z+.25)(z-1)}$

$$\lim_{n\to\infty} x[n] = \frac{(z^3+z+1)\cancel{(z-1)}}{(z^2-.5z+.25)\cancel{(z-1)}}\bigg|_{z=1} = 4 \qquad \text{matches 2c)}$$

d) $X(z) = \frac{(z^2-1)(z+.8)}{(z-.5)^2(z+.2)}$, $\lim_{n\to\infty} x[n] = 0$

matches 2d)

4 a)

y[n] +3y[n-1] + 2y[n-2] = 2x[n] - x[n-1]; y[-1]= 0; y[-2] = 1, x[n] = u[n]

$$Y(z) + 3(z^{-1}Y(z) + y[-1]) + 2(z^{-2}Y(z) + y[-2] + z^{-1}y[-1])$$

$$= 2X(z) - z^{-1}X(z) - x[-1]$$

$$Y(z)(1+3z^{-1}+2z^{-2}) + 2 = \frac{2z}{z-1} - \frac{1}{z-1}$$

$$Y(z) = \frac{z^2}{z^2+3z+2} \quad \frac{2z-1-2z+2}{z-1}$$

$$= \frac{z^2}{(z+1)(z+2)(z-1)}$$

$$= \frac{C_1}{z+1} + \frac{C_2}{z+2} + \frac{C_3}{z-1}$$

$$= \frac{-1/2}{z+1} + \frac{4/3}{z+2} + \frac{1/6}{z-1}$$

$$zY(z) = \frac{-1/2\,z}{z+1} + \frac{4/3\,z}{z+2} + \frac{1/6\,z}{z-1}$$

$$\longrightarrow \left(-\frac{1}{2}(-1)^n + \frac{4}{3}(-2)^n + \frac{1}{6}\right)u[n]$$

$$y[n] = \left(-\frac{1}{2}(-1)^{n-1} + \frac{4}{3}(-2)^{n-1} + \frac{1}{6}\right)u[n-1]$$