Problem 1:

$$Y(x,t) = 1.5\cos\left(2\pi \cdot 10^8 t - \frac{2\pi}{3}x + \frac{\pi}{3}\right)e^{-.0005x} \quad (Volts)$$

(a) The Amplitude is 1.5 Volts at spatial position x = 0m. It has an attenuation of $e^{-.0005x}$.
The Period, $T = \frac{2\pi}{2\pi \cdot 10^8} s = 10^{-8} s$
The frequency is 1/T = 10^8 Hz = 100 MHz
The wavelength, λ=c/f=3m
The phase, $\phi = \pi/3$ rad
The attenuation factor, α = .0005 Np/m (assuming the negative sign is intrinsic to +x prop).

(b) $\beta=2\pi/\lambda=2\pi/3$ (rad/m)
$\omega=2\pi f=2\pi\cdot 10^8$ (rad/s)

(c) The wave propagates in the +x direction due to negative sign in front of β. This occurs due to a phase front needing to remain constant – increasing time requires a positive increase in x to maintain neutrality with time for the phase front.

(d) The amplitude at t = 10^{-5} s and x = 3000m is given by $1.5\cos(2000\pi-2000\pi+\pi/3)e^{-1.5}$
This value is .167 Volts which satisfies the .1V requirement for reception.

Problem 2:

(a) L = .1m, f = 1 GHz $\Rightarrow \lambda$ = .3m, so L/λ = 1/3 >> .01 $\Rightarrow$do not ignore tline effects

(b) L = 1m, f = 1.8 GHz $\Rightarrow \lambda$ = .1667m, so L/λ = 6 >> .01 $\Rightarrow$ do not ignore tline effects

(c) L = .01m, f = .9 GHz $\Rightarrow \lambda$ = .333m, so L/λ = .03 > .01 $\Rightarrow$ do not ignore tline effects (close)

(d) L = .05m, f = 60 GHz $\Rightarrow \lambda$ = .005m, so L/λ = 10 >> .01 $\Rightarrow$ do not ignore tline effects

Problem 3:

(a) $\Gamma = \frac{30-j60-50}{30-j60+50} = .2 - .6j$

(b) SWR = $\frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+.6325}{1-.6325} = 4.4415$

(c) $Z_{in}(.35\lambda) = \frac{(30-j60)+j50\tan(.7\pi)}{50+j(30-j60)\tan(.7\pi)}(50) = 78.4644 + j98.2431$

(d) Under a perfect match, 10W can be delivered. This can be interpreted as the input power to the system is 10W. For the match at hand, $|\Gamma|$ = .6325. Thus, the power received is given by $(1-|\Gamma|^2)P_{in}$= 6W

Problem 4:

(a) Z_o = 50Ω, L = λ/4, Z_L = 60 Ω. Find the input impedance. For a quarter-wave length section, the input impedance is given by $Z_{in} = Z_o^2/Z_L = 2500/60 = 41.67\ \Omega$

(b) For an open circuit, Γ = 1. For an open circuit $Z_{in} = -jZ_o\cot(\beta l)$. So for L = λ/8, Z_{in} = -j50 Ω This circuit behaves as a capacitor due to the negative complex impedance.

(c) For a short circuit, Γ = -1. For a short circuit, $Z_{in} = jZ_o\tan(\beta l)$. For for L = λ/6, Z_{in} = j86.6 Ω This circuit behaves as an inductor due to the positive complex impedance.

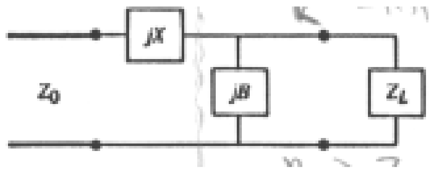
Problem 5:

(a) Because the load is a short circuit, the reflection coefficient is -1.
(b) Because the input impedance is dependent on a tangent function, it is periodic with pi. Thus, every .5λ returns to the load impedance. Thus, 2.3λ is equivalent to .3λ. So, Z_{in} = j50*tan(.6π) = -j153.88 Ω.
(c) The input admittance is given by $1/Z_{in}$ = .0065j S (Siemens or Ω^{-1}).

Problem 6:

$Z_o = 50\ \Omega$
$Z_L = 75 - j20\ \Omega$
$f_o = 6$ GHz

(a) Because the real component of the load is larger than Z_o, the correct circuit topology to be used is:



Using the design equations,

$$B = \frac{X_L \pm \sqrt{R_L/Z_o}\sqrt{R_L^2 + X_L^2 - Z_o R_L}}{R_L^2 + X_L^2} = .006373, -.013015$$

$$X = \frac{1}{B} + \frac{X_L Z_o}{R_L} - \frac{Z_o}{B R_L} = 38.9444, -38.9444$$

For (.006373, 38.9444)

$$L = \frac{X}{\omega} = \frac{38.9444}{2\pi \cdot 6 \cdot 10^9} = 1.033\ nH$$

$$C = \frac{B}{\omega} = \frac{.006373}{2\pi \cdot 6 \cdot 10^9} = .16913\ pF$$

For (-.013015, -38.9444)

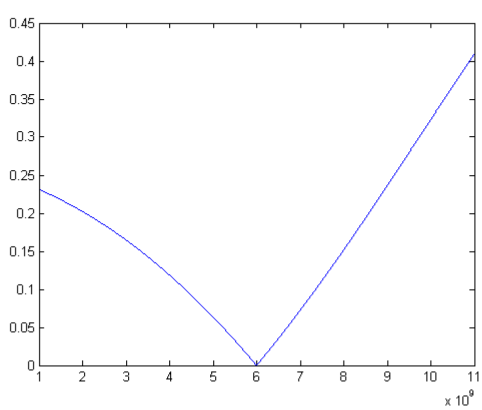
$$L = \frac{1}{\omega B} = 2.0381\ nH$$

$$C = \frac{1}{\omega X} = .68112\ nF$$

Zin for (1.033 nH, .16913 pF)

$$Z_{in} = j\omega L + j\omega C || Z_l$$

This results in the following gamma plot:

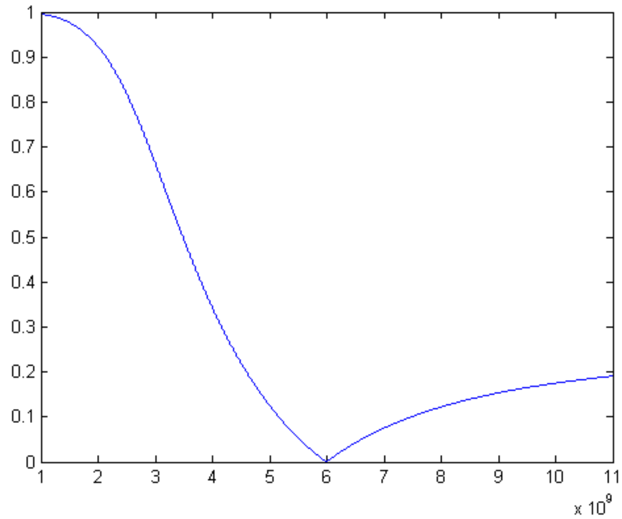


Using the cursor, Δf = 24.45% (.05 Γ_m)

For the second set (2.0381 nH, .68112 pF)

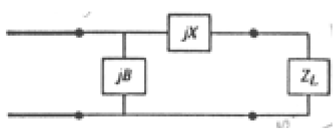
$$Z_{in} = -\frac{j}{\omega C} + \left(\frac{-j}{\omega L}\right) || Z_L$$

This results in the following gamma plot:



Using the cursors, Δf = 18.02%

(b) Z_L = 25-j20 Ω. This results in the second circuit topology for matching:



Using the design equations:

$$X = \pm\sqrt{R_L(Z_L - R_L)} - X_L = 45, -5$$

$$B = \pm\frac{\sqrt{(Z_o - R_L)/R_L}}{Z_o} = \pm.02$$

This results in the first LC pair as (B, X) ⇒

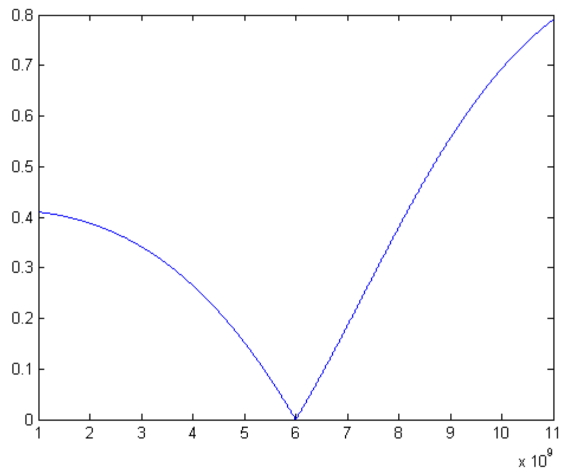
$$L = \frac{X}{\omega} = 1.1937\ nH$$

$$C = \frac{B}{\omega} = .53052\ pF$$

This produces a Zin of

$$Z_{in} = j\omega C || (j\omega L + Z_L)$$

This additionally yields the following gamma plot



This results in a Δf = 9.7%

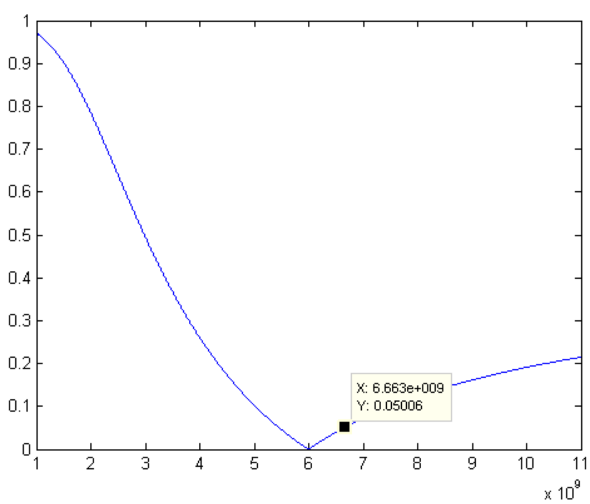
The second pair of LC (for the negative values)

$$L = \frac{1}{\omega B} = 1.3263\ nH$$
$$C = \frac{1}{\omega X} = 5.3052\ pF$$

This produces a Zin of:

$$Z_{in} = \left(\frac{-j}{\omega L}\right) || \left(\frac{-j}{\omega C} + Z_L\right)$$

This yields a gamma plot of:



This produces a Δf = 19.85%

Problem 7:

f = 100 MHz, Z_o = 300Ω, $Z_L = 73\ \Omega$

(a) Quarter-Wavelength Transformer (electrical length λ/4)

$$Z_{\lambda/4} = \sqrt{Z_o Z_L} = 147.99\ \Omega$$

(b) Determine the physical length:

$$\lambda = \frac{C_o}{\sqrt{\epsilon_r} f} = \frac{3 \cdot 10^8}{1.6 \cdot 10^8} = 1.875\ m \ \Rightarrow \frac{\lambda}{4} = .4688\ m$$

(c) Determine the bandwidth given Γ_m = .05

$$\frac{\Delta f}{f_o} = 2 - \frac{4}{\pi} \cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}} \frac{2\sqrt{Z_o Z_L}}{|Z_o - Z_L|}\right] = 2 - \frac{4}{\pi} \cos^{-1}\left[\frac{.05}{\sqrt{1-.05^2}} \frac{2\sqrt{73 \cdot 300}}{|300-73|}\right] = 8.32\%$$

(d) Use a 3 Stage Binomial Transformer to Design a more bandwidth efficient match:

The recursive formula for generating the section impedances is:

$$\ln\left(\frac{Z_{n+1}}{Z_n}\right) = 2^{-N} C_n^N \ln\left(\frac{Z_L}{Z_o}\right)$$
$$Z_o = 300\ \Omega$$
$$Z_1 = 251.42\ \Omega$$
$$Z_2 = 148\ \Omega$$
$$Z_3 = 87.11\ \Omega$$
$$Z_L = 73\ \Omega$$

(e) The bandwidth for this system is given by (N=3):

$$\frac{\Delta f}{f_o} = 2 - \frac{4}{\pi} \cos^{-1}\left(\frac{\frac{\Gamma_m}{2}}{\left|\frac{Z_L - Z_o}{2^N (Z_L + Z_o)}\right|}\right) = 57.3\%$$