Diagnosis and Prognosis of Scrubber Faults for Underwater Rebreathers based on Stochastic Event Models

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***Abstract*—Imperfect CO_2 removal mechanisms of CO_2 scrubbers often lead to the existence of CO_2 in gas inhaled by a diver from underwater rebreathers. This may cause CO_2 related rebreather faults and subsequently would increase the risk of human injuries. We introduce a stochastic model for three CO_2 related rebreather faults: CO_2 bypass, scrubber exhaustion, and scrubber breakthrough. We establish the concept of CO_2 channeling that describes the cause of the faults and present a CO_2 channeling model based on a stochastic process driven by a Poisson counter. This helps us to investigate how CO_2 flow inside the rebreather is affected by CO_2 related faults. Fault diagnosis/prognosis algorithms are developed based on the stochastic model and are tested in simulation.**

I. Introduction

Underwater breathing apparatus (UBA) are used for dive assistance. In consideration of military use or saturation diving, one of the most widely used UBA is the closed circuit rebreather (CCR) [1]. A rebreather reuses exhaled gas from divers by employing a scrubber to remove CO_2 from the exhaled gas. However, CO_2 removal mechanisms are usually not perfect. The phenomena of excessive CO_2 in inhaled gas and the loss of CO_2 absorption capability of the scrubber are considered as severe faults of the rebreather, which may lead to life threatening injuries to divers due to the intake of the excessive CO_2 [2], [3], [4].

Efforts to analyze or detect possible faults of rebreathers have been made in [4], [5], [6]. Deep Life, Ltd. published research data on rebreathers, and their work including a MATLAB/SIMULINK rebreather model is accessible [4]. A computer simulation model for the chemical kinetics of CO_2 absorption by a scrubber was developed in [5] to investigate the characteristics of the scrubber. This model is utilized to monitor the status of the scrubber by observing temperature changes induced by chemical reactions between CO_2 and the scrubing chemicals. In addition, a fault tree was designed in [6] to identify risks of rebreather faults.

Aside from rebreather fault modeling, the physical characteristics of UBA such as gas dynamics under pressure at various locations are modeled in [7], [8], [9]. A computer simulation tool [7] analyzes the overall gas flow in the UBA. The breathing dynamics of closed-circuit UBA or CCRs are discussed in [8]. The main interest there is how to maximize performance in terms of breathing characteristics including work of breathing and peak to peak pressure. The authors of [9] extended [7], [8] by applying network theory to describe components in the breathing loop that affect gas flow.

In this paper, we focus on CO_2 related faults of a rebreather and model how CO_2 flow inside the rebreather is affected by CO_2 related faults. The faults we are interested in include CO_2 bypass, scrubber exhaustion, and scrubber breakthrough. We define a random event that a small amount of CO_2 passes through the scrubber without being absorbed as CO_2 channeling, which can be described by a Poisson process. Accumulated CO_2 channeling will lead to the faults. The CO_2 flow will be affected by the faults and can be modeled using stochastic differential equations.

Our approach on the stochastic representation of CO_2 related rebreather faults is novel. The model captures the stochastic nature of faults that are difficult to detect and predict. This leads to the development of stochastic model based fault diagnosis/prognosis algorithms for CO_2 related rebreahter faults without modeling the process of chemical reactions inside a scrubber which might require more computing power. Our contribution may lead to detection and prediction of scrubber faults in real time.

In the next section, we briefly explain terminologies and mechanisms of rebreathers, and in Section III, we model respiration of divers that provides the input and the output of a rebreather. In Section IV, we simplify gas dynamics of an oxygen rebreather to focus on CO_2 related scrubber faults and investigate the influence of the faults on CO_2 flow in the simplified rebreather by introducing a CO_2 channeling model driven by a stochastic process. The description of the particle filter for the CO_2 flow model is presented in Section V followed by the introduction to fault diagnosis and prognosis for CO_2 related rebreather faults in Section VI. Simulation results are shown in Section VII and conclusions are provided in Section VIII.

II. Mechanisms of a Rebreather

As shown in Fig. 1, a typical oxygen rebreather consists of a mouth-piece, a scrubber canister, an oxygen cylinder and a pressure gauge. Air flow inside a rebreather is driven by a diver's exhalation and inhalation forming a closed breathing loop [10]. From the viewpoint of the scrubber, the portion of the breathing loop where gas flows from the diver through the mouth-piece to the scrubber is named the *incoming path*, and the portion of the breathing loop where gas flows out of the scrubber back to the diver is named the *outgoing path*. The direction of gas flow is regulated by one-way valves in the mouth-piece. When a diver inhales, the valves open the outgoing path and block the incoming path so that the gas in the incoming path cannot be breathed back into the diver. In contrast, when a diver exhales, the valves close the outgoing path and direct the exhaled gas to the incoming path to prevent it from being mixed with the gas in the outgoing path.

During the use of the rebreather, pure oxygen is injected into the breathing loop from the oxygen cylinder. This can be performed either actively at a constant rate, which must satisfy the individual diver's personal needs based on the diver's metabolism, or by manually adding oxygen using a hand-operated valve. It can also be controlled based on the difference between ambient pressure and the breathing loop pressure. The pressure of the oxygen inside the cylinder is checked with the pressure gauge.

Fig. 1. A pure oxygen rebreather (Cobra).

CO_2 is absorbed by the scrubber using chemicals such as sodalime stored in the canister [11], [12], [13]. The scrubber has a limited life time since the reaction of the scrubbing chemicals with CO_2 is irreversible. In fact, even when sodalime is not fully consumed, CO_2 may pass through the scrubber by a large amount, causing CO_2 related faults. Thus, the status of the scrubber consumption should be monitored in order for divers to avoid severe injuries due to the intake of excessive CO_2, i.e., hypercapnia. However, due to their stochastic nature, these faults are difficult to predict, and when they happen, divers often have little time to react. One of our goals is to understand the stochastic nature of the CO_2 related faults towards better predictions.

III. Modeling Inhalation and Exhalation Rates

Divers interact with rebreathers by respiration, i.e., inhalation and exhalation. A rebreather can be viewed as a system that takes exhalation as an input and produces an output for inhalation. An advanced computer simulation model of the human respiration is dealt with in [14] and its application to semi-closed circuit underwater breathing equipment is discussed in [15]. For rebreather fault simulations, a simplistic respiration model is constructed in this section in accordance with European Standard EN14143:2003 [16].

European standard EN14143:2003 [16] requires a sinusoidal waveform of breathing simulator and recommends CO_2 absorption endurance test at ventilation rate 40 L/min and at CO_2 generation rate 1.6 L/min. Moreover, the breathing simulator should handle breathing frequency of 20/min when ventilation rate is 40 L/min. According to these recommendations, we define the ventilation rate r and the frequency f as $r = 40\ [L/min]$ and $f = \frac{1}{3}\ [Hz]$. Note that the period T is just $\frac{1}{f}$ such that $T = 3\ [s]$.

In reality, respiration rates are different from person to person even among people who have similar physical conditions. We can easily see that the respiration rate is expected to vary along with workload changes which have influence on the heart rate. Let us define $s = \frac{\mathsf{HR}}{\mathsf{HR}_{\mathsf{ref}}}$ as the heart rate ratio where $\mathsf{HR}_{\mathsf{ref}}$ is the reference heart rate and HR is the present heart rate. Let $\xi(s)$ be a function which affects the magnitude of the respiration rate and $\chi(s)$ be a function which affects the frequency of the respiration rate. Based on the above discussion, the normalizing factor L, the inhalation rate v and the exhalation rate u with heart rate variations are

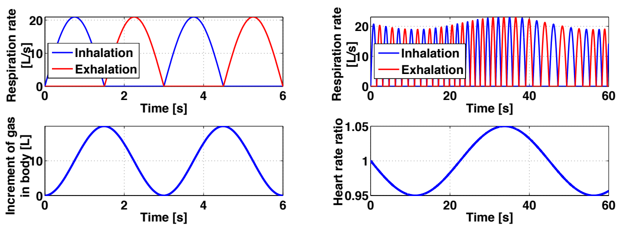
$$L(s,t) = \begin{cases} \int_{\frac{nT}{\chi(s)}}^{\frac{(n+\frac{1}{2})T}{\chi(s)}} \sin(2\pi f \chi(s) t) dt & , t \in I_1 \\ \int_{\frac{(n+\frac{1}{2})T}{\chi(s)}}^{\frac{(n+1)T}{\chi(s)}} \sin(2\pi f \chi(s) t + \pi) dt & , t \in I_2 \end{cases}, \tag{1}$$

$$v(s,t) = \begin{cases} \frac{r\xi(s)}{60 f L(s,t)} \sin(2\pi f \chi(s) t) & , t \in I_1 \\ 0 & , t \in I_2 \end{cases}, \tag{2}$$

$$u(s,t) = \begin{cases} 0 & , t \in I_1 \\ \frac{r\xi(s)}{60 f L(s,t)} \sin(2\pi f \chi(s) t + \pi) & , t \in I_2 \end{cases} \tag{3}$$

where $\xi(1) = 1$, $\chi(1) = 1$, $I_1 = [\frac{nT}{\chi(s)}, \frac{(n+\frac{1}{2})T}{\chi(s)})$ and $I_2 = [\frac{(n+\frac{1}{2})T}{\chi(s)}, \frac{(n+1)T}{\chi(s)})$ for $n = \{0, 1, 2, \cdots\}$.

The inhalation rate and the exhalation rate are listed separately as in (2) and (3) since inhalation and exhalation do not happen at the same time. A combination of one cycle of inhalation and one cycle of exhalation produces a single cycle of respiration. In other words, in the case that the heart rate ratio s is one and the respiration rate is 1/3 Hz, then each of inhalation and exhalation takes place in every 1.5 seconds, leading to a cycle of respiration in every 3 seconds.



(a) Respiration rate over time with unit heart rate ratio

(b) Respiration rate along with heart rate change

Fig. 2. In (a), the top plot shows rate of inhalation and exhalation for each cylce, and the bottom plot shows the amount of gas ventilated through a diver. In (b), the top plot shows the variation of the respiration rate related to the heart rate ratio shown in the bottom plot.

Fig. 2 demonstrates our respiration model. In the top row, the blue and the red curves indicate inhalation and exhalation rates, respectively. The respiration rate and the amount of gas flow with no heart rate changes, i.e., $s = 1$, are shown in Fig. 2(a). The inhalation and exhalation rates when the heart rate varies are demonstrated in Fig. 2(b).

IV. Modeling CO_2 Flow

In this section, we analyze how CO_2 related faults affect CO_2 flow based on the following assumptions which simplify gas dynamics in a rebreather: (*i*) when a diver breathes out, exhaled gas flows to a scrubber canister instantly, i.e., we ignore gas dynamics in the incoming path; (*ii*) elements of gas are uniformly distributed throughout the outgoing path and the ratio of the gas elements breathed in by a diver is identical to the ratio of the gas elements in the outgoing path; (*iii*) the amount of CO_2 breathed in by a diver is breathed out later without loss.

We first model CO_2 flow in each case of inhalation and exhalation assuming that CO_2 is absorbed by an ideal CO_2 scrubber. The two separate parts based on inhalation and exhalation can later be combined. We describe CO_2 related faults following a stochastic approach and analyze how CO_2 flow is affected by CO_2 related faults using a stochastic representation of the faults.

A. CO_2 Flow with no Fault

The volume of CO_2 in the outgoing path will be measured using proper CO_2 sensors. Let x be the volume of CO_2 in the outgoing path. We define the ratio of CO_2 in the outgoing path c_1 as

$$c_1(t) = \frac{x(t)}{V_L} \tag{4}$$

where V_L is the volume of the total gas in the outgoing path.

During the inhalation cycle, the volume of inhaled CO_2 is determined by the inhalation rate v and the ratio of CO_2 in the outgoing path c_1. Let us define y as the volume of CO_2 removed by a scrubber. By our assumptions, gas in the incoming path does not flow to the outgoing path during inhalation, so CO_2 absoprtion y is zero. On the other hand, the amount of CO_2 in the outgoing path x will be reduced due to the inhalation by a diver. Therefore, CO_2 flow during inhalation ($t \in I_1$) is described as

$$\begin{aligned} \dot{x} &= -c_1(t) \cdot v(t) \\ &= -\frac{v(t)}{V_L} \cdot x(t) \qquad (5) \\ \dot{y} &= 0 \qquad (6) \end{aligned}$$

where $x(0) = 0$, $y(0) = 0$ and $I_1 = [\frac{nT}{\chi(s)}, \frac{(n+\frac{1}{2})T}{\chi(s)})$ for $n = \{0, 1, 2, \cdots\}$.

We now consider the exhalation cycle. According to [17], the volume of CO_2 generated by a diver changes along with the heart rate variations. However, the variations of the CO_2 generation rate, which is related to O_2 metabolic consumption rate, based on workload is not easy to predict [10]. We use a simplified model based on the heart rate ratio here. We assume that the change in CO_2 generation rate is proportional to the heart rate ratio s. Let us define the CO_2 generation ratio c_2 as

$$c_2(s) = a \cdot s \tag{7}$$

where a is a scaling factor with a typical value $a = 0.04$ and s is the heart rate ratio.

In the ideal cases, the scrubber absorbs all the CO_2 coming into a scrubber during exhalation. In other words, no CO_2 will flow to the outgoing path of a rebreather. The volume of CO_2 coming into a scrubber is determined by the exhalation rate u and the CO_2 generation ratio c_2 Thus, x and y during exhalation ($t \in I_2$) can be described as follows.

$$\dot{x} = 0 \tag{8}$$

$$\dot{y} = c_2(s) \cdot u(t) \tag{9}$$

where $x(0) = 0$, $y(0) = 0$, $I_2 = [\frac{(n+\frac{1}{2})T}{\chi(s)}, \frac{(n+1)T}{\chi(s)})$ for $n = \{0, 1, 2, \cdots\}$ and s is the heart rate ratio. When CO_2 related faults happen, CO_2 flow will change as will be discussed in the following subsections.

B. Influence of CO_2 related Faults on Respiration

Since CO_2 related faults cause CO_2 leaks to the outgoing path from the incoming path without being absorbed by a scrubber, the volume of CO_2 in the outgoing path x during exhalation is not zero anymore. Moreover, divers inhale CO_2 and this affects the volume of CO_2 contained in exhaled breath. Assuming that human-beings do not consume CO_2, then CO_2 breathed in will remain intact in exhaled gas. Let us define δx describing the total amount of CO_2 inhaled by a diver over one inhalation period as

$$\delta x(n) = x(nT) - x((n+\frac{1}{2})T) \tag{10}$$

where $n = \{0, 1, 2, \cdots\}$. Assuming that all the CO_2 inhaled by a diver is exhaled during the next exhalation cycle without loss, we define an additional exhalation rate u_1 for this portion of returned CO_2 as follows.

$$u_1(t,n) = \begin{cases} 0 & , t \in I_1 \\ \frac{\delta x(n)}{L}\sin(2\pi f t + \pi) & , t \in I_2 \end{cases} \tag{11}$$

where $I_1 = [\frac{nT}{\chi(s)}, \frac{(n+\frac{1}{2})T}{\chi(s)})$ and $I_2 = [\frac{(n+\frac{1}{2})T}{\chi(s)}, \frac{(n+1)T}{\chi(s)})$ for $n = \{0, 1, 2, \cdots\}$.

Considering that the returned CO_2 is not involved in CO_2 generation, we define the volume of CO_2 in exhaled breath using u and u_1 as

$$\mathscr{U}(t,n) = \begin{cases} 0 & , t \in I_1 \\ c_2(s)(u(t) - u_1(t,n)) + u_1(t,n) & , t \in I_2 \end{cases} \tag{12}$$

where s is the heart rate ratio.

C. CO_2 Channeling and Stochastic Modeling

We focus on the following CO_2 related rebreather faults: CO_2 bypass, scrubber exhaustion, and scrubber breakthrough. The descriptions of the faults are as follows. (*i*) *CO_2 bypass*: Existence of CO_2 beyond a safety level in the outgoing path of the rebreather. (*ii*) *Scrubber exhaustion*: Complete consumption of the CO_2 scrubber. (*iii*) *Scrubber breakthrough*: Loss of the CO_2 absorption capability of the CO_2 scrubber before the depletion of the scrubber. We will see later that these faults can be rigorously defined using a stochastic model.

Typically, even when a scrubber functions properly, a small amount of CO_2 will pass through the scrubber from the incoming path to the outgoing path. We define this phenomenon as *CO_2 channeling*. Each CO_2 channeling event happens randomly and is not considered as a fault. However, CO_2 channeling will lead to the three CO_2 related faults, and our stochastic model will reveal the mechanism.

Considering the randomness of CO_2 channeling events, an arrival of CO_2 at the outlet of a scrubber canister can be modeled as a Poisson process. We define a Poisson counter $N(t)$ such that $N(t)$ satisfies the following Poisson distribution:

$$P[(N(t+\tau) - N(t)) = d] = \frac{e^{-\lambda\tau}(\lambda\tau)^d}{d!} \tag{13}$$

where $d = \{0, 1, \cdots\}$ is the index of events and λ is the expected number of events over unit length of time. Since each CO_2 channeling event happens independently, we define $dN(t)$ as one single CO_2 channeling event over dt.

The amount of the CO_2 channeling during one event can be modeled by a scaling factor, and it is affected by two factors: the total volume of CO_2 absorbed by the scrubber y and the volume of CO_2 coming into a scrubber canister $\mathscr{U}$. Then, the scaling factor G can be described as a function of y and $\mathscr{U}$ such that

$$G = G(y(t), \mathscr{U}(t,n)). \tag{14}$$

Since we define $dN(t)$ as an event of CO_2 channeling within dt, the change in CO_2 volume in the outgoing path due to CO_2 channelings can be expressed as

$$dx = G(y(t), \mathscr{U}(t,n))dN(t). \tag{15}$$

The two parameters for the Poisson process, G and λ, need to be carefully selected to reflect the physical behavior of the rebreather. At the beginning of the rebreather use, CO_2 channelings are very unlikely, but as the scrubber being consumed, the probability of CO_2 channeling increases. After the scrubber is depleted, CO_2 channeling will happen with probability 1. According to this, λ of the Poisson process can be assumed to be zero for a new scrubber but will become ∞ as the scrubber is getting fully consumed. Suppose the amount of CO_2 absorbed by the scrubber y is zero initially and its maximum capacity is y_{max}, then the rate parameter should satisfy

$$\lambda(0) = 0, \qquad \lambda(y_{max}) = \infty \tag{16}$$
$$\dot{\lambda}(0) = 0, \qquad \dot{\lambda}(y_{max}) = 0. \tag{17}$$

One possible function that satisfies the above conditions is

$$\lambda(y) = \frac{y(t)}{y_{max} - y(t)} \tag{18}$$

where y_{max} is the maximum capacity of the CO_2 scrubber and $y(t)$ is the remaining CO_2 absorption capacity of the CO_2 scrubber.

The function $G(y, \mathscr{U})$ will describe how much of CO_2 could pass through a scrubber when a CO_2 channeling event happens. Initial possible amount of CO_2 channeling can be assumed to be zero for a new scrubber, but when a CO_2 scrubber reaches its maximum capacity, all CO_2 coming into the scrubber will pass through. Let us assume that G is a polynomial function increasing from zero to $\mathscr{U}$ as $y(t)$ goes from zero to y_{max}, then one possible choice of function G is

$$G(y, \mathscr{U}) = \left(\frac{y(t)}{y_{max}}\right)^{\alpha} \mathscr{U}(t,n) \tag{19}$$

where $\alpha > 0$ is a stretching factor. We see that when $y = 0$, no CO_2 channeling can happen, but when $y = y_{max}$, all CO_2 in the incoming path will pass through the scrubber in a single channeling event.

D. CO_2 Flow under CO_2 related Faults

In this subsection, we will specify CO_2 flow dynamics in the rebreather induced by CO_2 related faults using the stochastic CO_2 channeling model with respect to x and y. Parameters and variables of the model are summarized in table I.

By the assumption that gas in the incoming path does not flow to the outgoing path during inhalation, CO_2 related

TABLE I
VARIABLES AND PARAMETERS OF THE CO_2 FLOW MODEL

Parameter	Value/unit	Meaning
x	$[L]$	Volume of CO_2 existing in outgoing path
y	$[L]$	Volume of CO_2 absorbed by CO_2 scrubber
p	$[L]$	Overall CO_2 flow in rebreather
$\mathscr{U}$	$[L/min]$	CO_2 exhalation rate
v	$[L/min]$	Inhalation rate
c_1		Ratio of CO_2 in the outgoing path
c_2		CO_2 generation ratio
s		Heart rate ratio ($^{\mathsf{HR}}/_{\mathsf{HR_{ref}}}$)

scrubber faults do not have influence on CO_2 flow during inhalation. Thus, (5) and (6) still hold.

However, the faults affect CO_2 flow during exhalation. The volume of CO_2 in the outgoing path x is as defined in (15). The volume of CO_2 absorbed by the scrubber y is determined by the difference between the volume of CO_2 arriving at the inlet of a scrubber canister due to exhalation and the volume of a CO_2 channeling event.

During exhalation ($t \in I_2$):

$$dx = G(y(t), \mathscr{U}(t,n))dN(t) \tag{15}$$

$$dy = \mathscr{U}(t,n)dt - dx \tag{20}$$

where $x(0) = 0$, $y(0) = 0$ and $I_2 = [\frac{(n+\frac{1}{2})T}{\chi(s)}, \frac{(n+1)T}{\chi(s)})$ for $n = \{0, 1, 2, \cdots\}$.

We define p such that $p(t) = x(t) + y(t)$ which represents the total CO_2 flow in the system, i.e., how much CO_2 enters or leaves a rebreather. Then, the rebreather system can be modeled as a stochatic system with $p(t)$ and $x(t)$ as state variables.

During inhalation ($t \in I_1$):

$$dx = -\frac{v(t)}{V_L} \cdot x(t)dt \tag{21}$$

$$dp = dx \tag{22}$$

During exhalation ($t \in I_2$):

$$dx = G(p(t) - x(t), \mathscr{U}(t,n))dN(t) \tag{23}$$

$$dp = \mathscr{U}(t,n)dt \tag{24}$$

where $x(0) = 0$, $p(0) = 0$, $I_1 = [\frac{nT}{\chi(s)}, \frac{(n+\frac{1}{2})T}{\chi(s)})$ and $I_2 = [\frac{(n+\frac{1}{2})T}{\chi(s)}, \frac{(n+1)T}{\chi(s)})$ for $n = \{0, 1, 2, \cdots\}$.

The volume of CO_2 in the outgoing path x will be measured by a CO_2 sensor. However, the total CO_2 flow p is not able to be measured. Thus, the noisy measurement $m(t)$ for $x(t)$ can be modeled as an output of the stochastic systems in (21)-(24).

$$m(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ p(t) \end{bmatrix} + \nu(t) \tag{25}$$

where $\nu(t)$ represents measurement noise.

V. PARTICLE FILTER

Since the model is driven by a Poisson process, there are limited choices of filters for state estimation. In particular, Kalman filter may not be very efficient. In this paper, the particle filter is applied to estimate the states $x(t)$ and $p(t)$. We first briefly introduce the particle filtering algorithm [18], and then apply the filter to estimate the states. The particle filter is performed by the iteration of the following four steps.

(i) Particle Creation

To initialize the filter, N samples for each x and p are randomly generated from the initial probability distribution functions $\rho(x_0)$ and $\rho(p_0)$. These samples are denoted as $x_i^+(0)$ and $p_i^+(0) (i = 1, \cdots, N)$, respectively. The values of the mean and the variance of the initial particles are chosen according to the fact that the volume of CO_2 is always positive and the amount of CO_2 is very small when an event of CO_2 channeling occurs. The values we empirically found are on the order of 10^{-8}. Since these values are very small, A scaling factor γ is used to increase the value of particles to reduce numerical errors. The value used for simulation is $\gamma = 10^6$

(ii) Prediction (Diffusion)

At each time step k, the particles are propagated to the next time step based on the system equations:

During inhalation ($kh \in I_1$):

$$x_i^-((k+1)h) = x_i^+(kh) - \gamma c_1(l)v(kh)h \tag{26}$$

$$p_i^-((k+1)h) = p_i^-(kh) + y_i^-((k+1)h) - y_i^-(kh) \tag{27}$$

During exhalation ($kh \in I_2$):

$$x_i^-((k+1)h) = x_i^+(kh) + G(kh)dN(t) \tag{28}$$

$$p_i^-((k+1)h) = p_i^+(kh) + \mathscr{U}(kh,n)h \tag{29}$$

where $G(kh) = G(p_i^+(kh) - y_i^+(kh), \mathscr{U}(kh,n))$, $i = \{1, \cdots, N\}$ and h is the step size.

A Poisson jump $dN(t)$ is a continuous-time process, but it can be simulated in discrete-time as $Poiss(kh)$. Computational implementation of a Poisson process is introduced in [19], [20], [21]. In this paper, a Poisson process is simulated in the way suggested by [19]. and $Poiss(kh)$ represents the implementation of the Poisson counter during the time interval $(kh, (k+1)h)$ where h is a small step size. If a randomly generated number which is uniformly distributed in the interval $[0, 1]$ is smaller than $1 - e^{-h\lambda}$, then $Poiss(kh) = 1$; otherwise, $Poiss(kh) = 1$. Then, we can express (28) as

$$x_i^-((k+1)h) = x_i^+(kh) + G(kh)Poiss(kh). \tag{30}$$

(iii) Likelihood Evaluation

We can obtain the output values using (25) as

$$m(kh) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_i^-(kh) \\ p_i^-(kh) \end{bmatrix} + \nu(kh) \tag{31}$$

where $\nu(kh) \sim \mathscr{N}(0, R)$.

After measuring $m(kh)$, the conditional relative likelihood is computed to evaluate $P(m(kh)|x_i^-(kh), p_i^-(kh))$. Given that noise ν is Gaussian, for a specific measurement m^*, a relative likelihood q_i can be computed as

$$q_i = P[(m_k = m^*)|(x_k = x_i^-(kh), p_k = p_i^-(kh))]$$
$$= P[\nu_k = m^* - h(x_i^-(kh), p_i^-(kh))]$$
$$\sim \frac{\exp\left(\frac{-r_i^T(kh)R^{-1}r_i(kh)}{2}\right)}{(2\pi)^{n/2}|R|^{1/2}} \quad (32)$$

where $r_i(kh) = [m^* - h(x_i^-(kh), p_i^-(kh))]$ and $n = 1$. Then, in order to ensure that the sum of all the likelihoods is equal to one, the relative likelihoods are normalized as $\bar{q}_i = q_i / \sum_{j=1}^N q_j$.

(iv) Resampling

Particles are resampled by using the following two steps for $i = 1, \cdots, N$.

1) Generate a uniformly distributed random number r on [0,1].
2) If $\sum_{m=1}^{j-1} \bar{q}_m < r$ but $\sum_{m=1}^{j} \bar{q}_m \geq r$, then $x_i^+(kh) = x_j^-(kh)$ and $p_i^+(kh) = p_j^-(kh)$.

Then, the estimates of x and p at time kh are determined by the statistical mean of the updated particles such that

$$\begin{bmatrix} \tilde{x}(kh) \\ \tilde{p}(kh) \end{bmatrix} = \begin{bmatrix} \frac{1}{N}\sum_{i=1}^N \{x_i^+(kh)\} \\ \frac{1}{N}\sum_{i=1}^N \{p_i^+(kh)\} \end{bmatrix}. \quad (33)$$

VI. Fault Diagnosis and Prognosis

Our CO_2 flow model helps the detection of the CO_2 related scrubber faults. Fault diagnosis and prognosis are achieved using the particle filtering results based on the CO_2 flow model.

A. Fault Detection and Isolation for Rebreathers

1) CO_2 bypass: The estimate of the volume of CO_2 channeling over one single period of exhalation $\Delta\tilde{x}$ is obtained using $\tilde{x}$ such that

$$\Delta\tilde{x}(n) = \tilde{x}((n+1)T) - \tilde{x}((n+\frac{1}{2})T), \quad (34)$$

and the occurrence of CO_2 bypass can be determined by summating the volume of CO_2 channelings over a finite time window such that the decision of CO_2 bypass is made based on the following criterion:

$$\sum_{i=n-M+1}^{n} \Delta\tilde{x}(i) > H_1 \quad (35)$$

where M is the width of the time window, and H_1 is the threshold for detection.

Equation (35) itself is enough to detect CO_2 bypass. However, a more detrimental factor induced by CO_2 bypass fault is the partial pressure of CO_2, $ppCO_2$. We assume that the pressure inside the rebreather is the same as the ambient pressure. Provided that the ambient pressure P_{amb} and the volume of gas in the outgoing path V_L are known, $ppCO_2$ can be computed similarly to ppO_2 in [10] as

$$ppCO_2 = 100 \cdot c_1 \cdot P_{amb} = \frac{\tilde{x} \cdot P_{amb}}{V_L} \quad (36)$$

Then, with a threshold value H_2, we can detect CO_2 bypass by

$$ppCO_2 > H_2. \quad (37)$$

The fault of CO_2 bypass will be detected if either of the two thresholds, H_1 or H_2, are exceeded.

2) CO_2 scrubber exhaustion and CO_2 scrubber breakthrough: CO_2 scrubber exhaustion and CO_2 scrubber breakthrough happen when CO_2 absorption capability is lost. To detect these faults we need to compare $\Delta\tilde{x}$ with the volume of CO_2 exhaled over one breathing period. Let us define

$$\Delta\mathscr{U}(n) = \int_{(n+\frac{1}{2})T}^{(n+1)T} \mathscr{U}(\tau, n)d\tau \quad (38)$$

where $n = \{0, 1, 2, \cdots\}$.

We can detect scrubber exhaustion or breakthrough by first checking whether

$$\Delta\mathscr{U}(n) - \Delta\tilde{x}(n) < H_3 \quad (39)$$

where H_3 is a threshold value. If the threshold is exceeded, then complete CO_2 channeling events is happening. Using $\tilde{x}$ and $\tilde{p}$ obtained by (33), we can determine whether the fault will lead to scrubber exhaustion or scrubber breakthrough. If the scrubber does not reach its maximum absorption capacity yet, complete CO_2 channelings indicate scrubber breakthrough. Let us define the remaining scrubber capacity ($\mathscr{C}$) as

$$\mathscr{C} = 1 - \frac{y(t)}{y_{max}} = 1 - \frac{p(t) - x(t)}{y_{max}} \quad (40)$$

where x is the volume of CO_2 in the outgoing path, y is the current CO_2 absorption capacity of the scrubber, p is the overall CO_2 flow and y_{max} is the maximum absorption capacity of the srubber. At the time of the fault occurrence, if

$$\tilde{\mathscr{C}} < H_4 \quad (41)$$

where H_4 is a threshold on the scrubber depletion, then the fault is identified as scrubber exhaustion, otherwise scrubber breakthrough.

B. Prognostics and Health Management

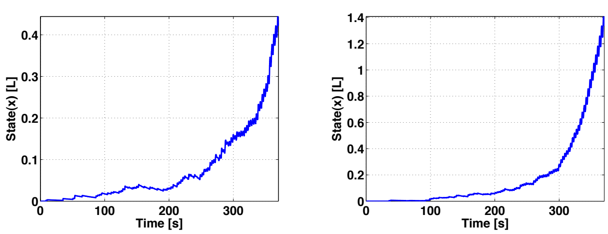
We can use the state estimates from the Particle Filter to predict future CO_2 channeling occurrences by propagating the state equations of the rebreather CO_2 flow model forward in time. Prognosis for the faults are implemented according to the fault criteria in (35), (37), (39) and (41) using the predicted values $\hat{x}$ and $\hat{p}$ obtained by Algorithm 1 instead of estimated values $\tilde{x}$ and $\tilde{p}$. The fault prediction algorithm at $t = kh$ for the future states at $t = (k+l)h$ is as follows. Note that the tilde represents the state estimate and the hat represents the state prediction.

Algorithm 1 Fault prediction at $t = kh$ for $t = (k+l)h$

$\hat{P}_{amb} = P_{amb}$, $\Delta\hat{\mathscr{U}} = \Delta\mathscr{U}$
$[\tilde{x}(kh),\ \tilde{p}(kh)]$ = ParticleFilter$(x(kh),\ p(kh))$
$\hat{x}(kh) = \tilde{x}(kh),\ \hat{p}(kh) = \tilde{p}(kh)$
for $i = k$ **to** $i = k+l-1$ **do**
  $\hat{x}((i+1)h) = \hat{x}(ih) + d\hat{x}(ih)$
  $\hat{p}((i+1)h) = \hat{p}(ih) + d\hat{p}(ih)$
end for
/* Check the following fault decision rules using $\hat{x}$ and $\hat{p}$ instead of $\tilde{x}$ and $\tilde{p}$ in the equations. */
if (35) or (37) is met **then**
  CO_2 bypass alarm
end if
if (39) is met **then**
  if (41) is met **then**
    CO_2 exhaustion alarm
  else
    CO_2 breakthrough alarm
  end if
end if

VII. Simulation Results

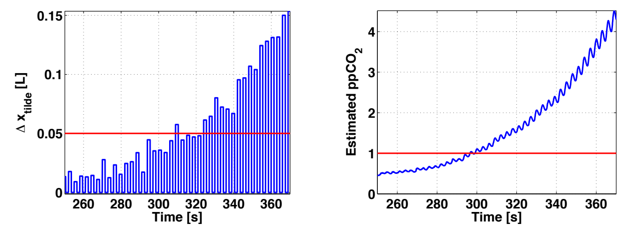
We perform simulations of the rebreather based on our stochastic CO_2 flow model at a constant depth with ambient pressure as $P_{amb} = 1$ to demonstrate the CO_2 related rebreather faults. The parameters we used in simulations are: the number of particles as $N = 800$; the capacity of the scrubber as $y_{max} = 10$; the size of the outgoing path in the breathing loop as $V_L = 30$; the heart rate ratio as $s = 1$; the stretching factor for the function G as $\alpha = 10^{-6}$; the window size for the detection of the CO_2 bypass fault as $M = 1$; and the time step as $h = 0.05$.



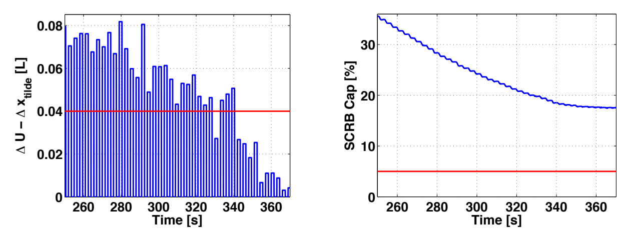
(a) State x under scrubber exhaustion. (b) State x under scrubber breakthrough.

Fig. 3. The amount of CO_2 in the outgoing path under scrubber exhaustion and scrubber breakthrough up to $t = 370$ are shown in (a) and (b), respectively. Their patterns are almost identical except that they happen at different times. Note the difference in values of the state x at time t between the cases of scrubber exhaustion and scrubber breakthrough. CO_2 bypass happens due to the accumulated CO_2 in the outgoing path before the exhaustion or the breakthrough.

The CO_2 flow patterns under the faults of scrubber exhaustion and scrubber breakthrough are described in Fig. 3. Scrubber exhaustion happens when the scrubber is depleted and Fig. 3(a) shows that there is a dramatic increase in CO_2 in the outgoing path after $t = 330$ under the scrubber exhaustion fault. The scrubber breakthrough is intentionally made such that the maximum capacity of the scrubber y_{max} under scrubber breakthrough is reduced to $0.85 \cdot y_{max}$ from the beginning of the simulation. The CO_2 flow pattern under scrubber breakthrough is similar to the scrubber exhaustion case, but when scrubber breakthrough occurs, the increase in CO_2 in the outgoing path comes earlier as in Fig. 3(b), so there exists more CO_2 in the outgoing path in the same amount of dive time after CO_2 breakthrough occurs. CO_2 bypass happens at the early stages of scrubber exhaustion and scrubber breakthrough as CO_2 is getting accumulated in the outgoing path due to CO_2 channeling events.



(a) $\Delta\tilde{x}$ with the threshold value as $H_1 = 0.05$. (b) Estimated $ppCO_2$ with the threshold value as $H_2 = 1$.

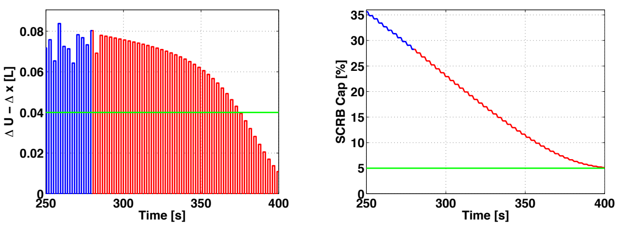


(c) $\Delta U - \Delta\tilde{x}$ with the threshold value as $H_3 = 0.04$. (d) The remaining capacity of the scrubber with the threshold value as $H_4 = 0.05$.

Fig. 4. Parameters for fault diagnosis are introduced with the threshold values (red lines) in the case of scrubber breakthrough. For CO_2 bypass fault detection, $\Delta\tilde{x}$ and $ppCO_2$ are used as in (a) and (b). Parameters for scrubber exhaustion/breakthrough detection are presented in (c) and (d).

To demonstrate the CO_2 related fault diagnosis and prognosis algorithms, we implement the particle filter on our stochastic CO_2 flow model. An example of fault diagnosis for scrubber breakthrough is presented in Fig. 4. The detection criteria on CO_2 bypass fault are shown with threshold values in Figs. 4(a) and 4(b). The occurrence of CO_2 bypass will be determined by checking $\Delta\tilde{x}$ and estimated $ppCO_2$. The criteria on the scrubber exhaustion/breakthrough are presented in Figs. 4(c) and 4(d). By setting a threshold value for $\Delta U - \Delta\tilde{x}$ as 0.04, scrubber exhaustion/breakthrough can be detected, and the remaining capacity of the scrubber can be checked to determine which fault between scrubber exhaustion and scrubber breakthrough happens.

The system propagation for fault prognosis is shown in Fig. 5. We use the same criteria and threshold values as used in previous figures. We see that the particle filter estimates are updated before $t = 280$, and system propagation is performed as the red line segments in Fig. 5. Then, a prediction for scrubber exhaustion/breakthrough can be achieved using $\Delta U - \Delta\tilde{x}$ in Fig. 5(a) and the remaining capacity of the scrubber as $100 \cdot \mathscr{C}$ in Fig. 5(b). Compared with the simulation results of fault diagnosis in Figs. 4, our model is optimistic, e.g., the predicted time of the fault occurrence is about 30 seconds later than the actual fault occurrence time. To improve the accuracy of the fault prediction under scrubber breakthrough, the change in the actual maximum capacity of a CO_2 scrubber needs to be studied as well.



(a) $\Delta U - \Delta\tilde{x}$ with the threshold value as $H_3 = 0.04$.

(b) The remaining capacity of the scrubber with the threshold value as $H_4 = 0.05$.

Fig. 5. State propagation after $t = 280$ for a scrubber breakthrough prediction. The blue and the red curves indicate the particle filter estimate and the system propagation, respectively. The threshold values are introduced as green lines.

VIII. Conclusions

We have presented fault diagnosis and prognosis algorithms for three rebreather faults: CO_2 bypass, scrubber exhaustion and scrubber breakthrough. As part of our achievements, we have investigated the process of CO_2 flow inside a rebreather and the influence of the faults on CO_2 flow. Consequently, we have developed a stochastic model of a CO_2 channeling event whose cumulative occurences result in the rebreather faults and a CO_2 flow model using the stochastic model. We have shown that the particle filter is applicable to obtain the state estimates of CO_2 flow, and the fault diagnosis/prognosis algorithms are tested in simulations based on the particle filter estimates.

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