Compressive Sensing:

A new approach to data acquisition

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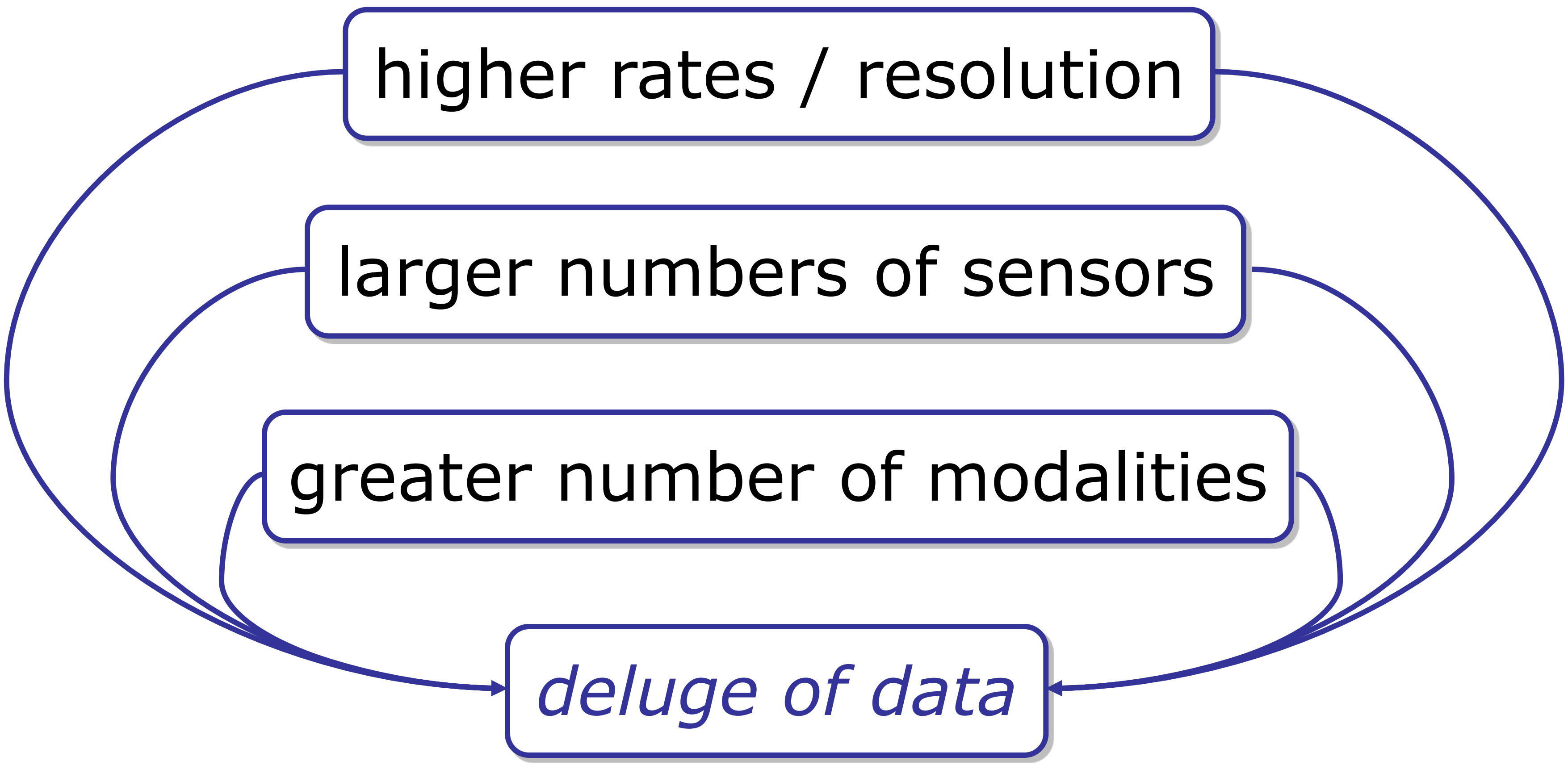
Pressure is on Signal Processing

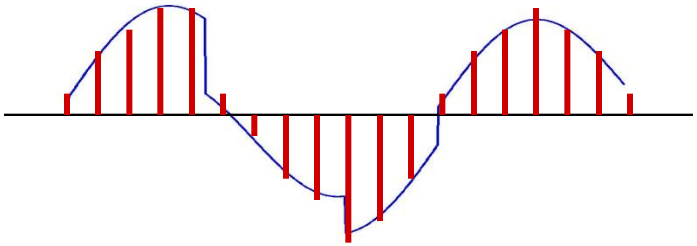
- Increasing pressure on signal/image processing hardware and algs to support

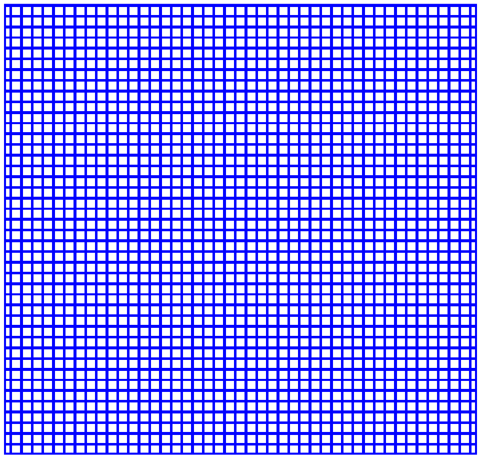


Sensing by Sampling

Data Acquisition and Representation

- Time: A/D converters, receivers, …
- Space: cameras, imaging systems, …





- Foundation: *Shannon sampling theorem*

Must sample at 2x highest frequency of the signal (Nyquist rate)

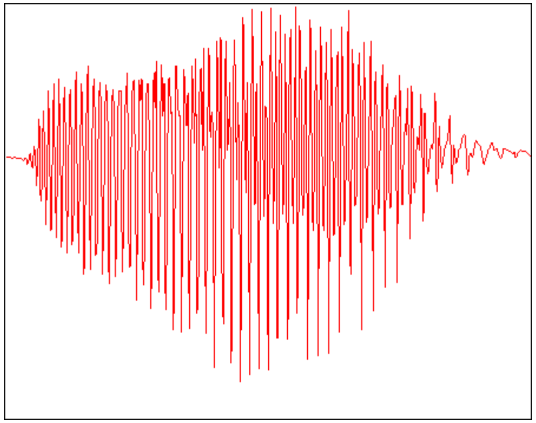
Sparsity

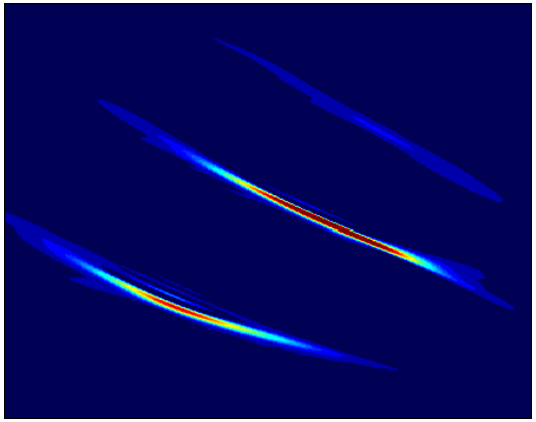
- Many signals can be ***compressed*** in some representation/basis (Fourier, wavelets, ...)

N
pixels

$K \ll N$
large
wavelet
coefficients

N
wideband
signal
samples

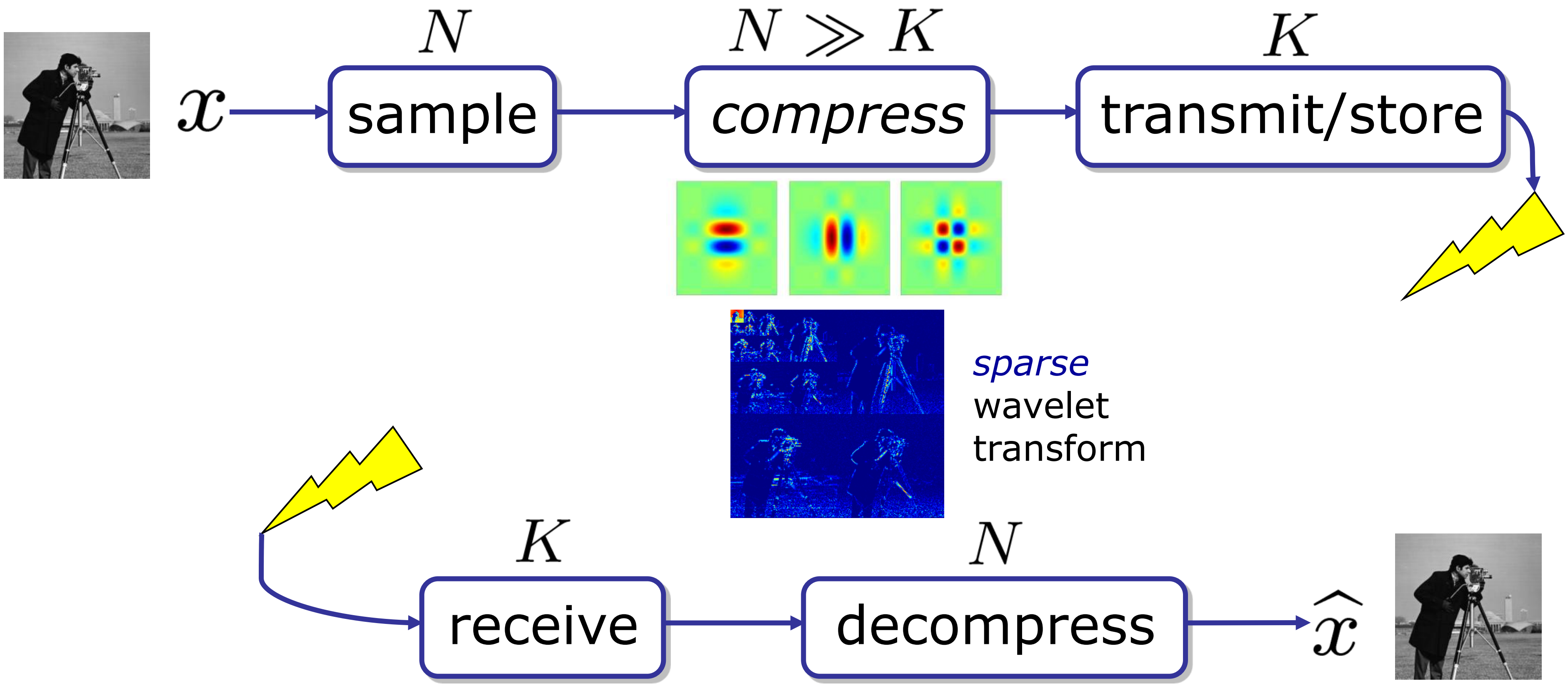




$K \ll N$
large
Gabor
coefficients

Sensing by *Sampling*

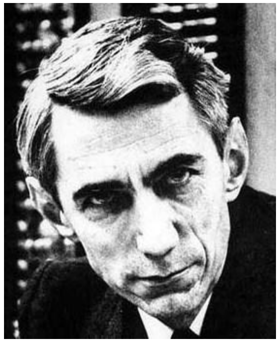
- Standard paradigm for digital data acquisition
  - ***sample*** data (ADC, digital camera, …)
  - ***compress*** data (signal-dependent, nonlinear)



Compressive Sensing

From Samples to *Measurements*

- Shannon was a *pessimist*
  - worst case bound for *any* bandlimited signal



- ***Compressive sensing*** (CS) principle

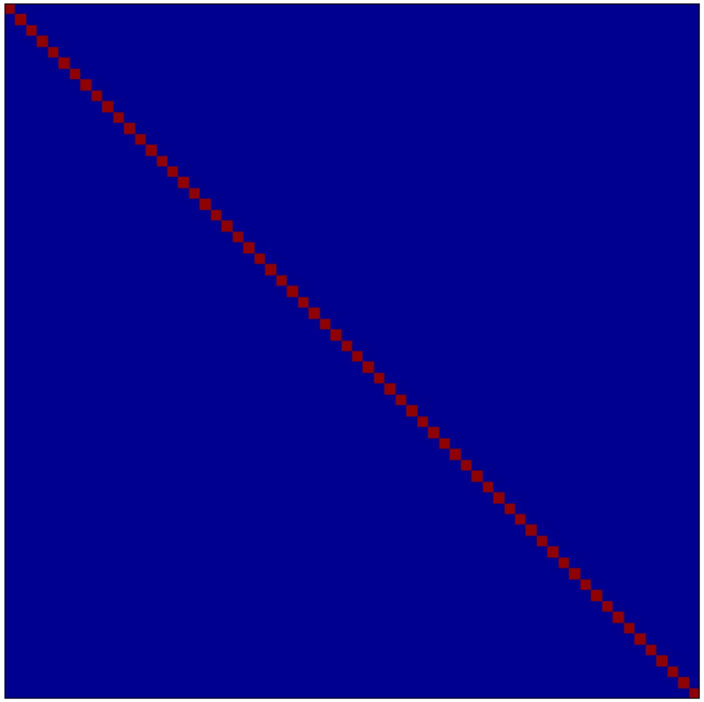
> "***sparse*** signals can be recovered from a small number of ***nonadaptive linear measurements***"

  - integrates sensing, compression, processing
  - based on new *uncertainty principles* and the concept of *incoherency* between two bases

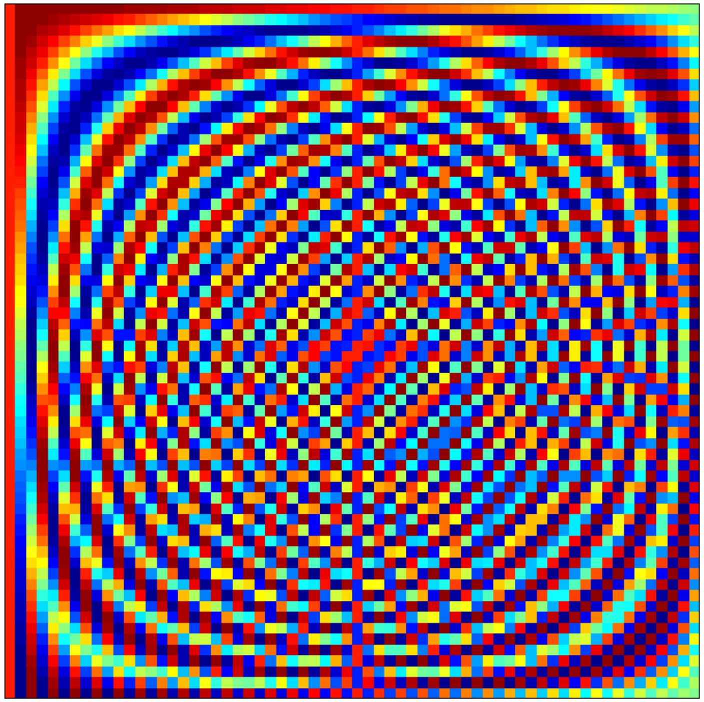
Incoherent Bases

- Spikes and sines (Fourier)
  (Heisenberg)

$$\Psi = I$$



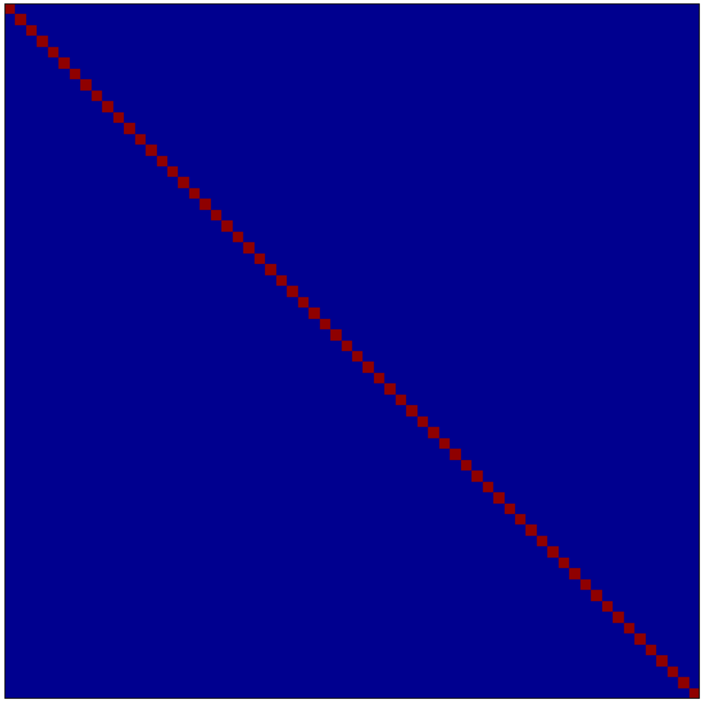
$$\Phi = \mathrm{idct}(I)$$



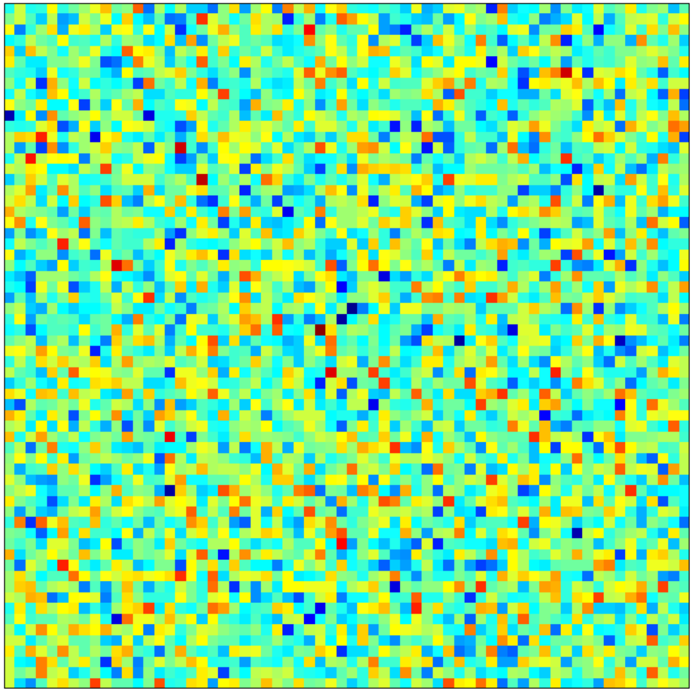
Incoherent Bases

- Spikes and "random basis"

$$\Psi = I$$



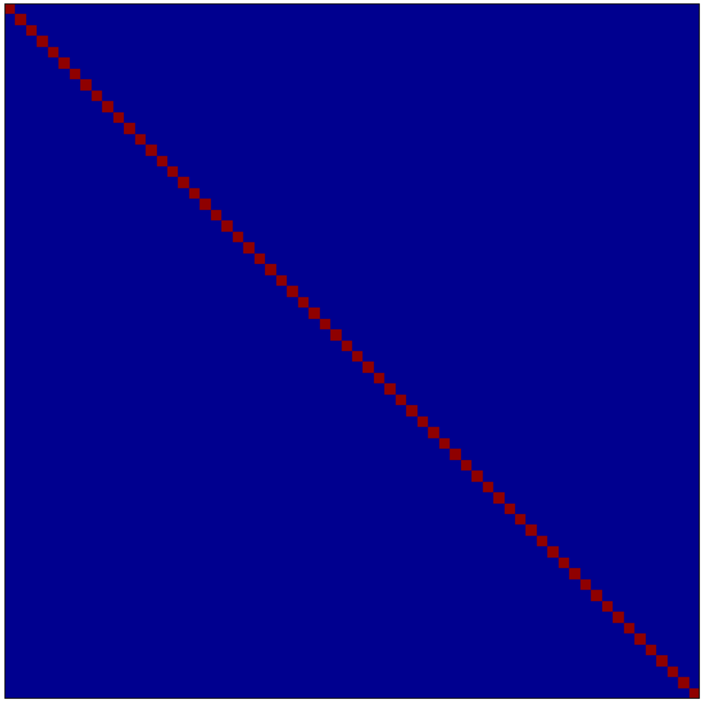
$$\Phi = \text{randn}(N, N)$$



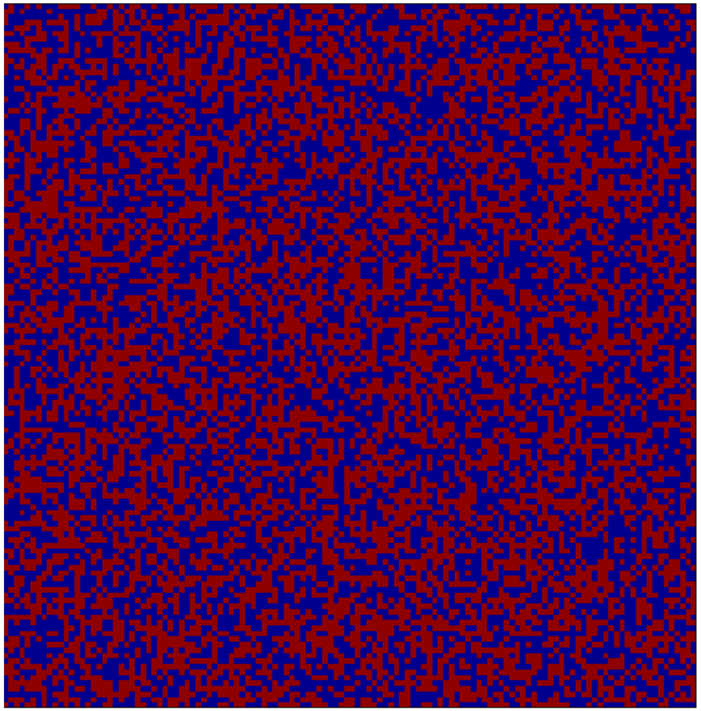
Incoherent Bases

- Spikes and "random sequences" (codes)

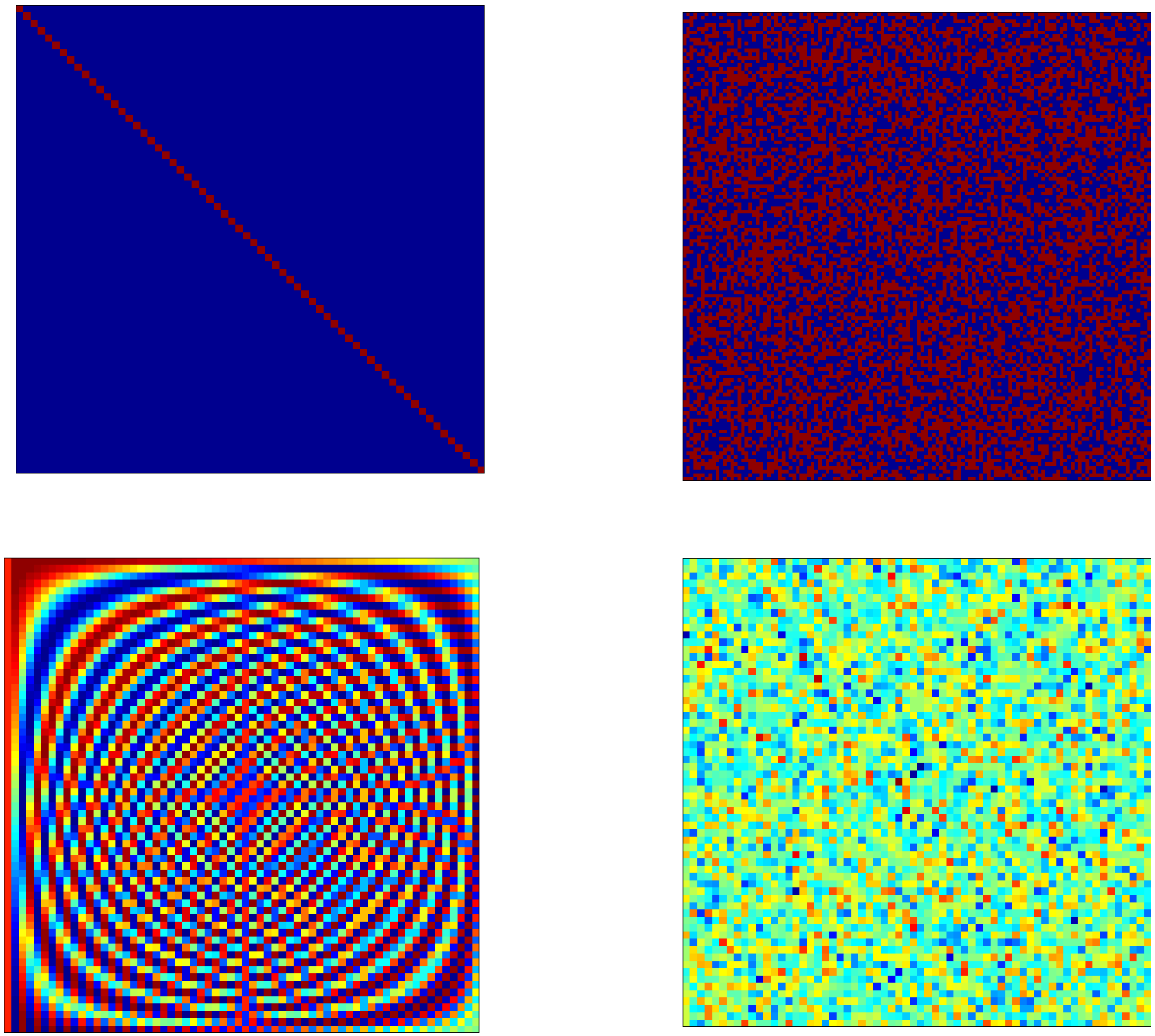
$$\Psi = I$$



$$\Phi$$



Incoherent Bases



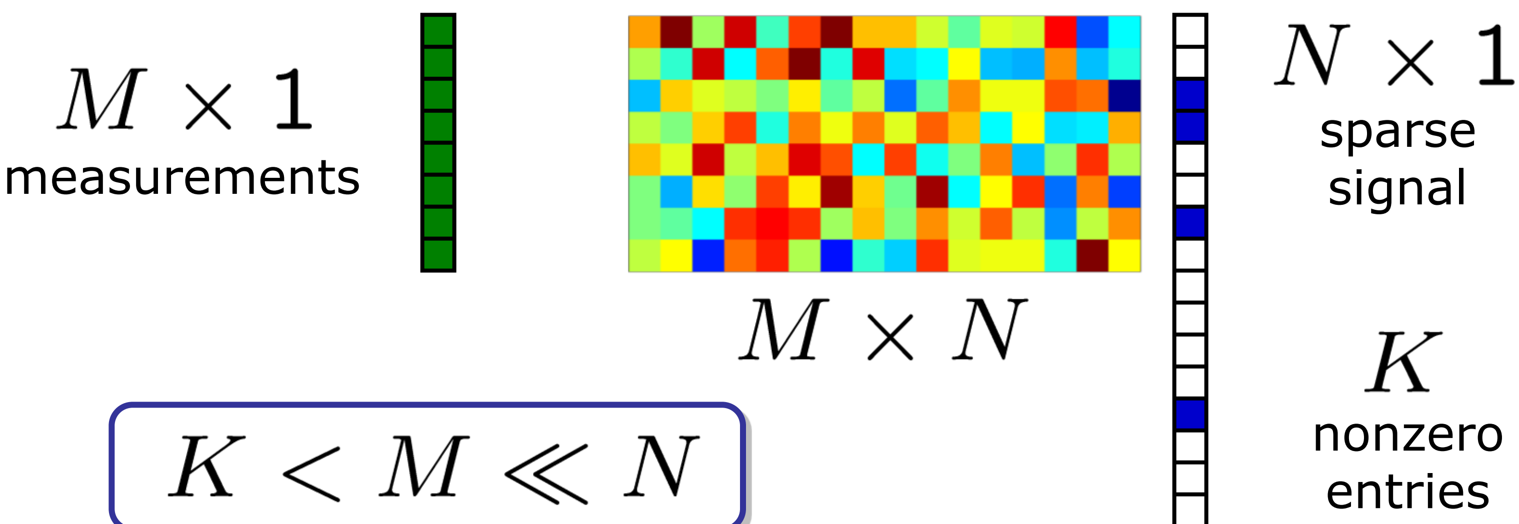
Compressive Sensing

[Candes, Romberg, Tao; Donoho]

- Signal x is K-*sparse* in basis/dictionary Ψ
  - WLOG assume sparse in space domain $\Psi = I$

- Replace samples with *linear projections*

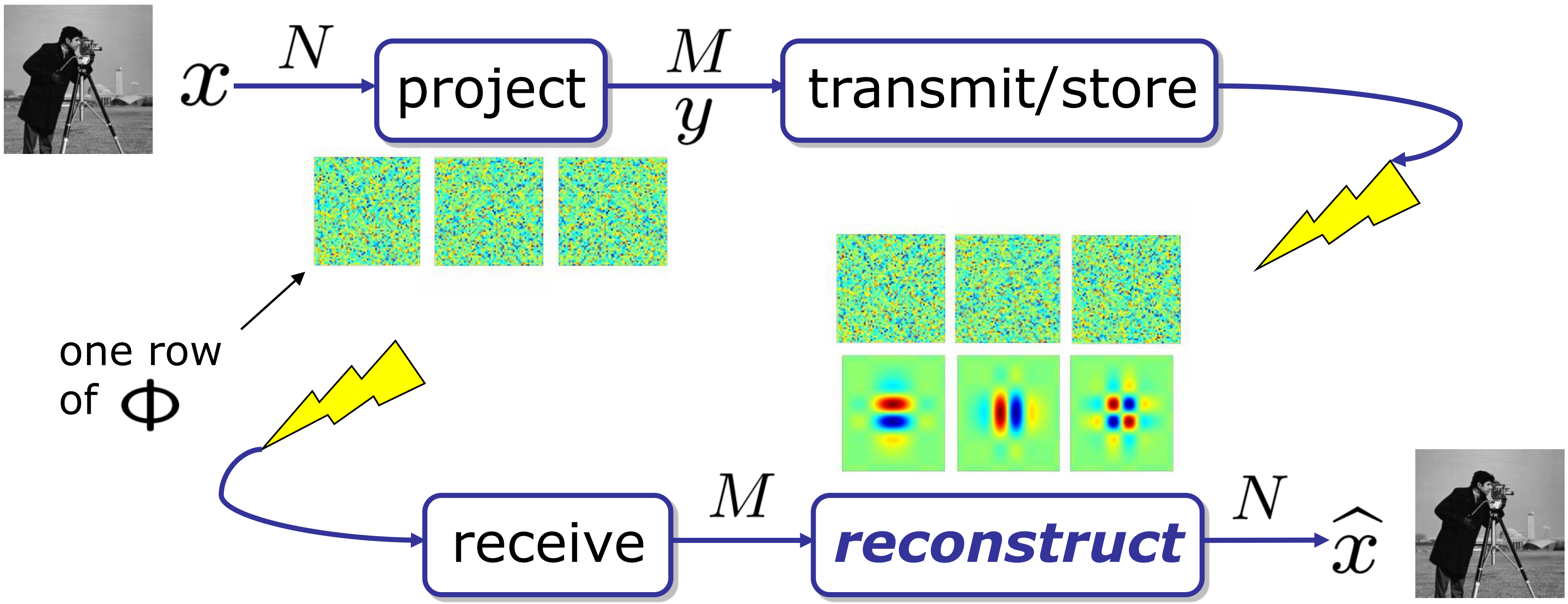
$$y = \Phi x$$



$$K < M \ll N$$

Compressive Sensing

- Measure linear projections onto *incoherent* basis where data is *not sparse/compressible*



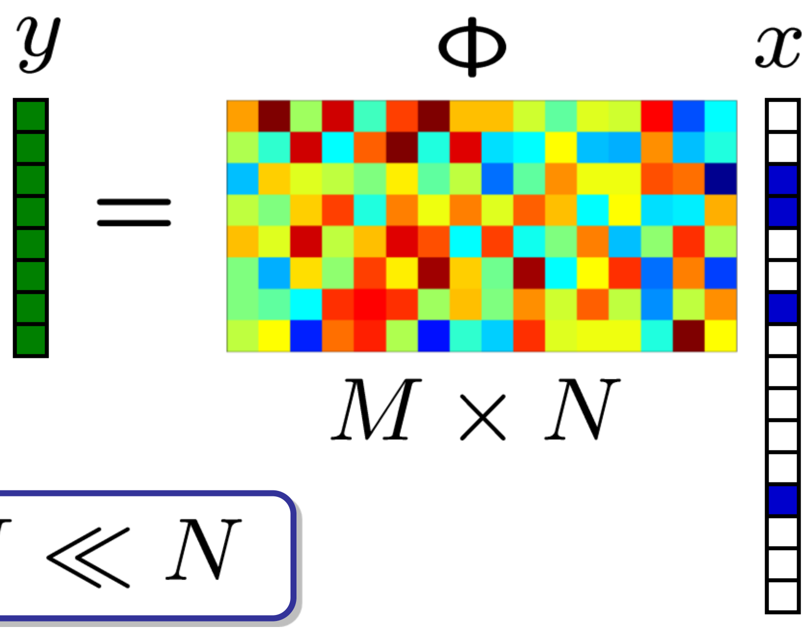
- Reconstruct via *nonlinear processing* (optimization)

CS Signal Recovery

- Reconstruction/decoding:

given $y = \Phi x$
find x

ill-posed inverse problem



$$K < M \ll N$$

CS Signal Recovery

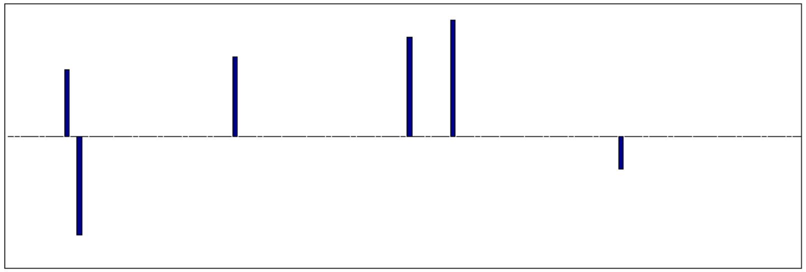
- Reconstruction/decoding:
  (ill-posed inverse problem)

given $y = \Phi x$
find x

- $\boldsymbol{L_2}$: $\widehat{x} = \arg\min_{y=\Phi x} \|x\|_2 \longrightarrow \widehat{x} = (\Phi^T\Phi)^{-1}\Phi^T y$

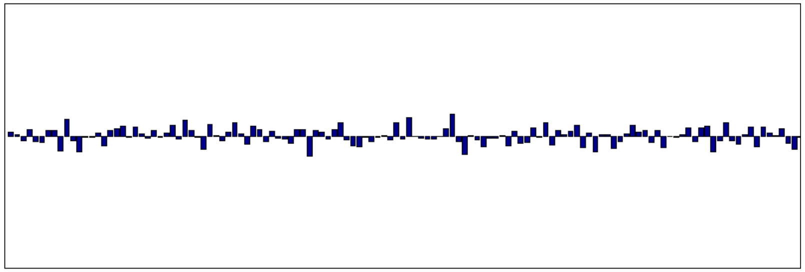
- Fast, but *wrong*

x



- Solution is *almost never* sparse

$\widehat{x}$



CS Signal Recovery

- Reconstruction/decoding:
  (ill-posed inverse problem)

given $y = \Phi x$
find x

- $\boldsymbol{L_2}$: $\widehat{x} = \arg\min_{y=\Phi x} \|x\|_2$

- $\boldsymbol{L_0}$: $\widehat{x} = \arg\min_{y=\Phi x} \|x\|_0$ ← *number of nonzero entries*

- Correct, but *slow* (NP-Hard)

- $M = K + 1$ measurements suffice
  [Bresler; Wakin]

CS Signal Recovery

- Reconstruction/decoding:
  (ill-posed inverse problem)

given $y = \Phi x$
find x

- ***L*$_2$**: $\widehat{x} = \arg\min_{y=\Phi x} \|x\|_2$

- ***L*$_0$**: $\widehat{x} = \arg\min_{y=\Phi x} \|x\|_0$

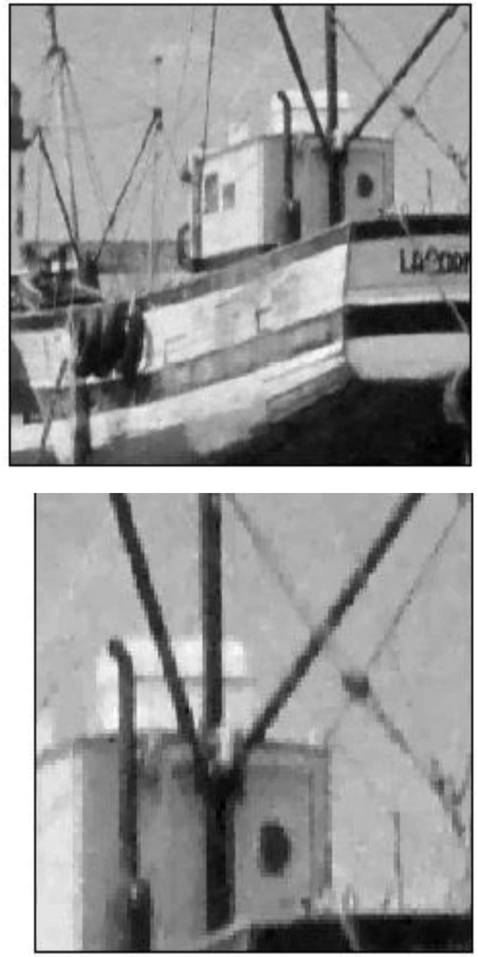
- ***L*$_1$**: $\widehat{x} = \arg\min_{y=\Phi x} \|x\|_1$ ⟵ *linear program*

- Gives same answer as ***L*$_0$**, mild increase in M
  [Candes et al, Donoho]

$$M = O(K \log(N/K) \ll N)$$

CS Signal Recovery

original (65k pixels)



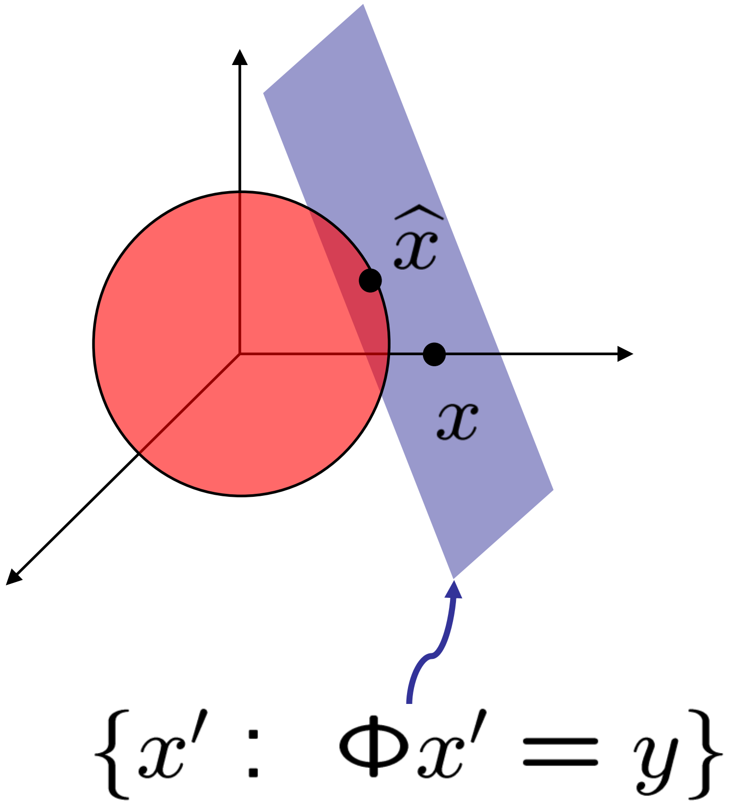
20k random projections

7k–term wavelet approximation

E. J. Candès and J. Romberg, "Practical Signal Recovery from Random Projections," 2004.

Why $\boldsymbol{L_2}$ Doesn't Work

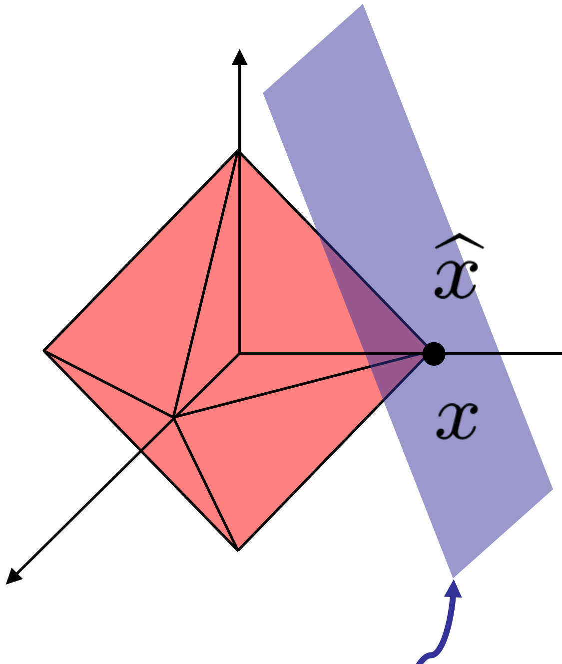
$$\widehat{x} = \arg\min_{y=\Phi x'} \|x'\|_2$$



least squares,
minimum $\boldsymbol{L}_2$ solution
is almost **never sparse**

Why L_1 Works

$$\widehat{x} = \arg \min_{y=\Phi x'} \|x'\|_1$$

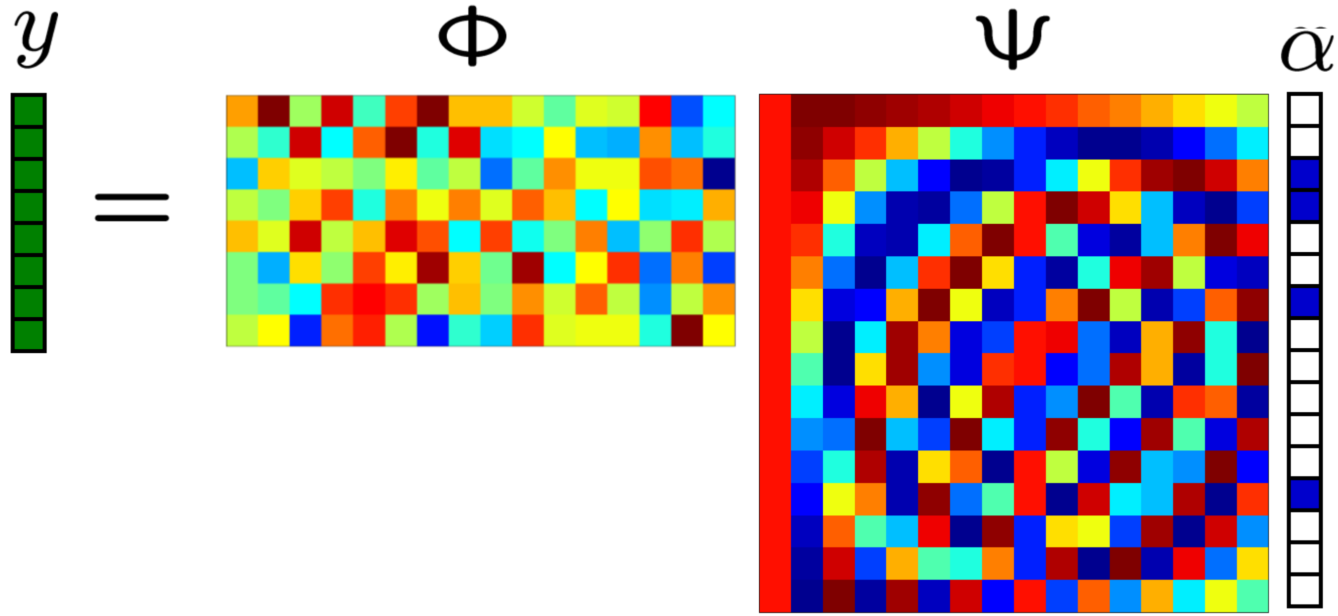


minimum L_1 solution
= sparsest solution if

$$M = O(K \log(N/K)) \ll N$$

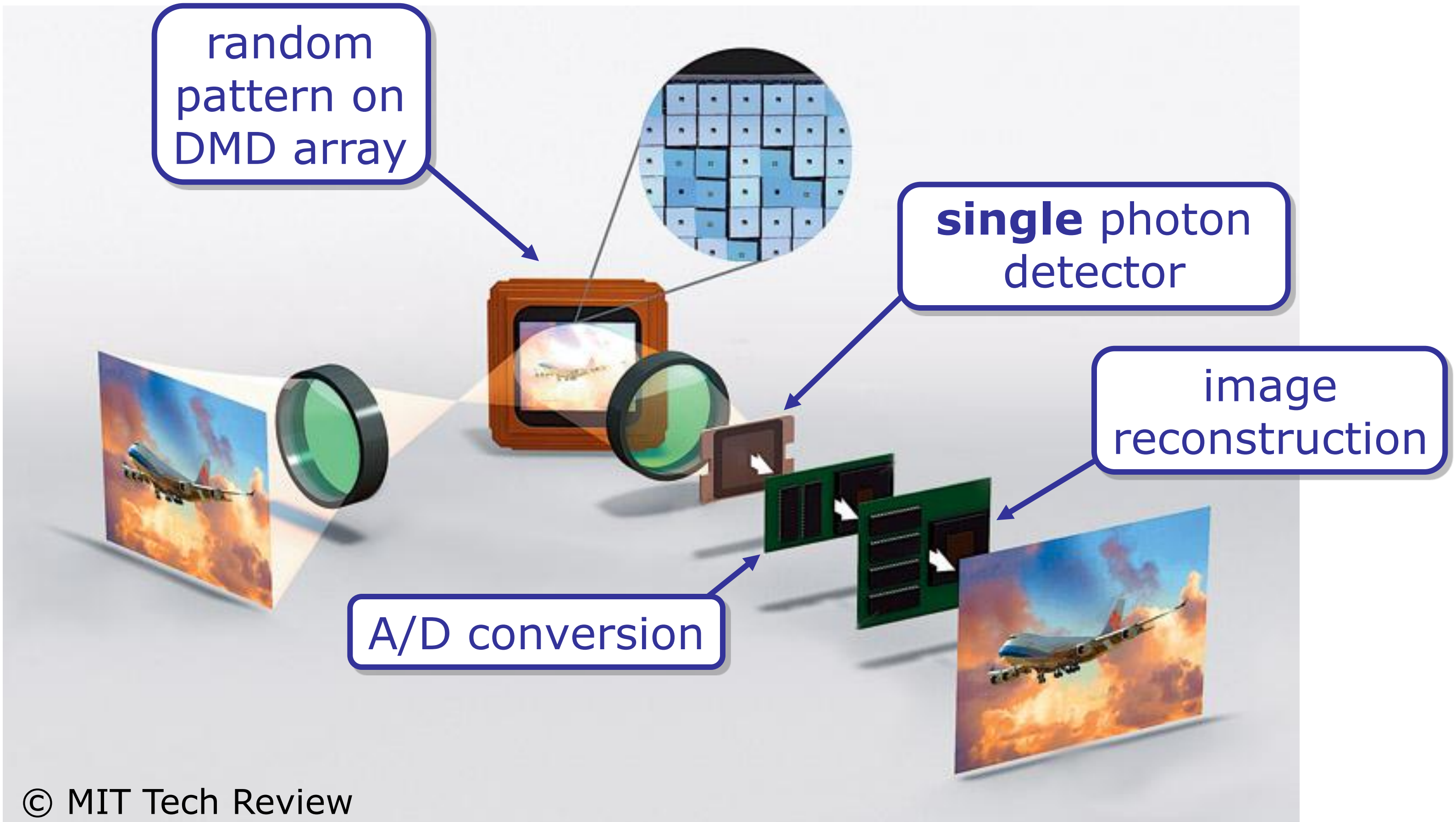
Universality

- Random matrix is incoherent with ***any*** fixed orthonormal basis (with high probability)



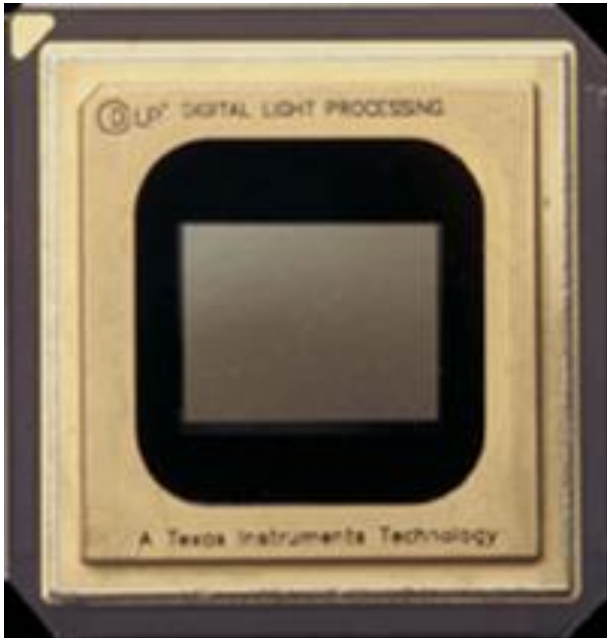
Compressive Sensing in Action

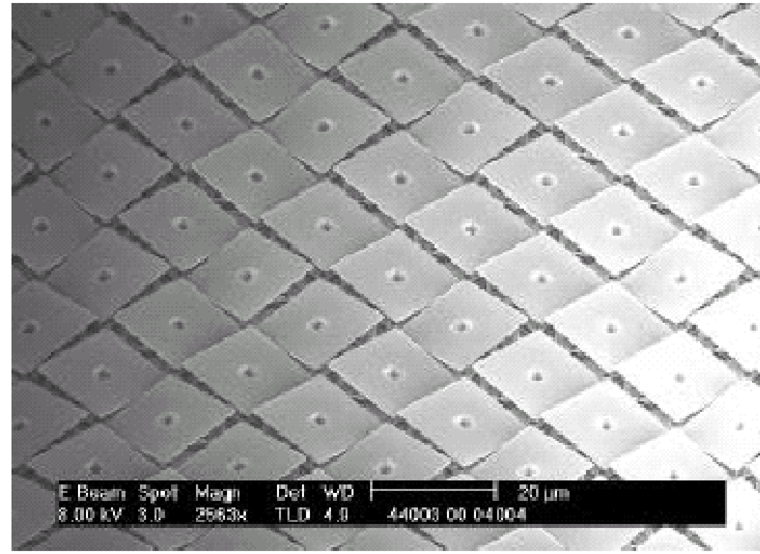
Single-Pixel CS Camera



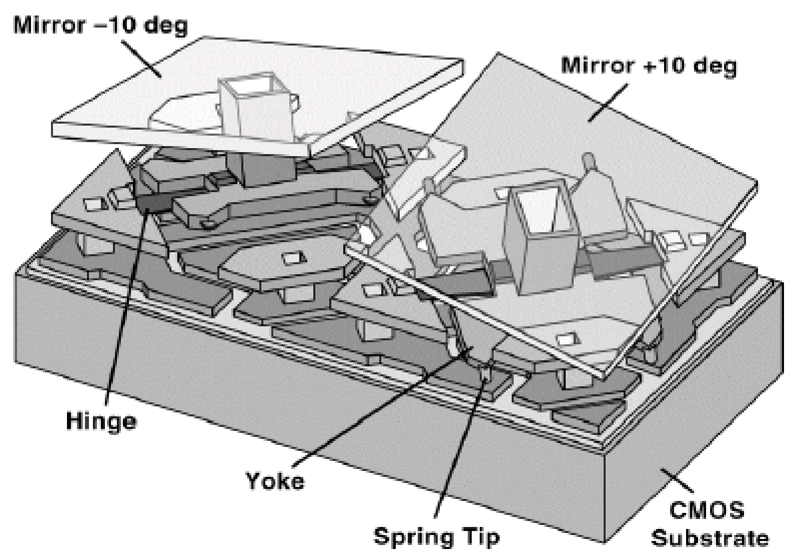
- New modalities
- Low cost
- Low power

TI Digital Micromirror Device (DMD)

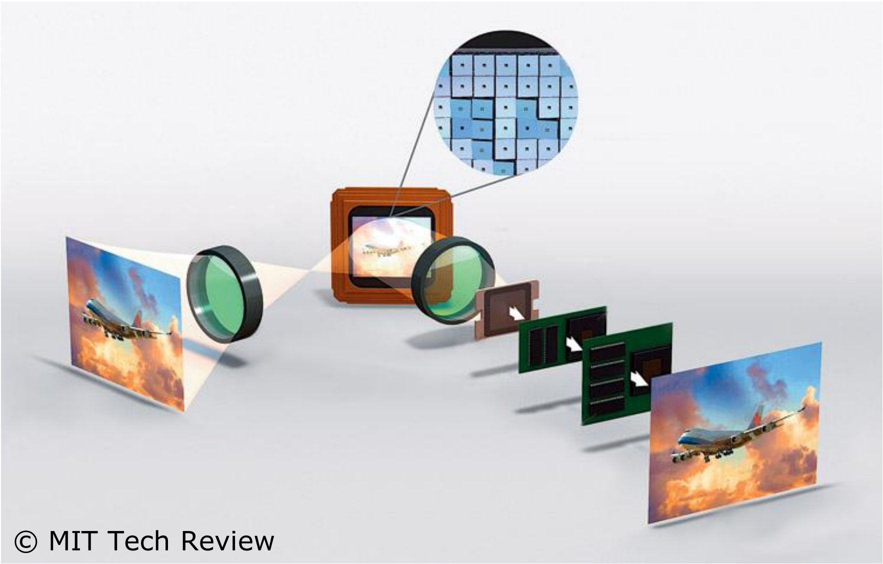




PICTURE BY
DLP™
A TEXAS INSTRUMENTS TECHNOLOGY



First Image Acquisition



ideal
256x256 pixels

20x
sub-Nyquist

50x
sub-Nyquist

Second Image Acquisition

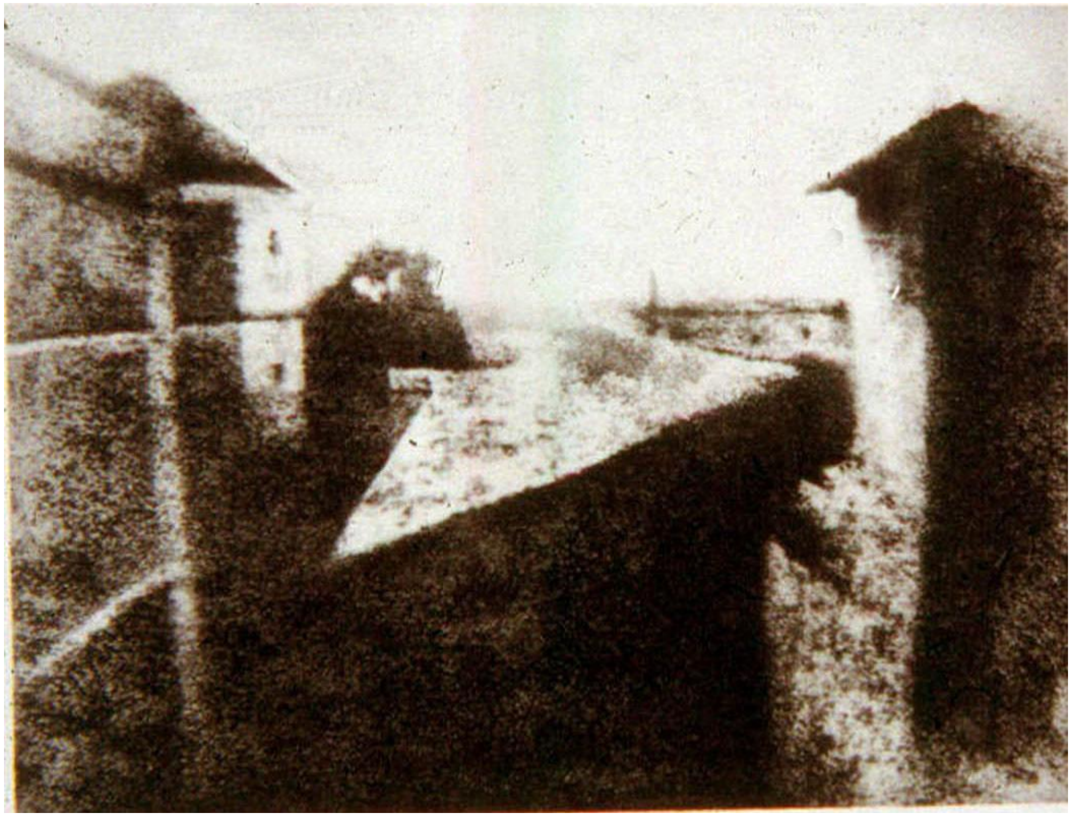
- Low-light scenario (photomultiplier tube)
- Used three color filters, separately reconstruct each color range

ideal
256x256 pixels

10x sub-Nyquist

World's First Photograph

- 1826, Joseph Niepce
- Farm buildings and sky
- 8 hour exposure



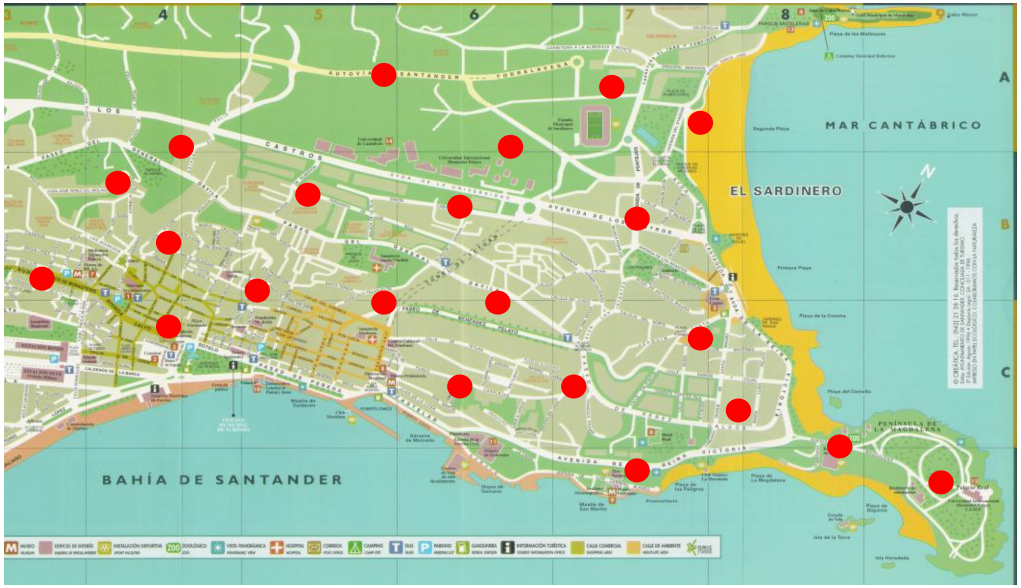
CS Hallmarks

- CS changes the rules of data acquisition
  - exploits a priori signal *sparsity* information
- **Universal**
  - same random projections / hardware can be used for *any* compressible signal class (*generic*)
- **Democratic**
  - each measurement carries the same amount of information
  - simple encoding
  - robust to measurement loss and quantization
- **Asymmetrical**
  - most processing at decoder

Distributed Compressive Sensing

Sensor Networks

- Measurement, monitoring, tracking of *distributed physical phenomena* using wireless embedded sensors
  - environmental conditions
  - industrial monitoring
  - chemicals
  - weather
  - sounds
  - vibrations
  - seismic
  - wildfires
  - pollutants
  - …



Challenges

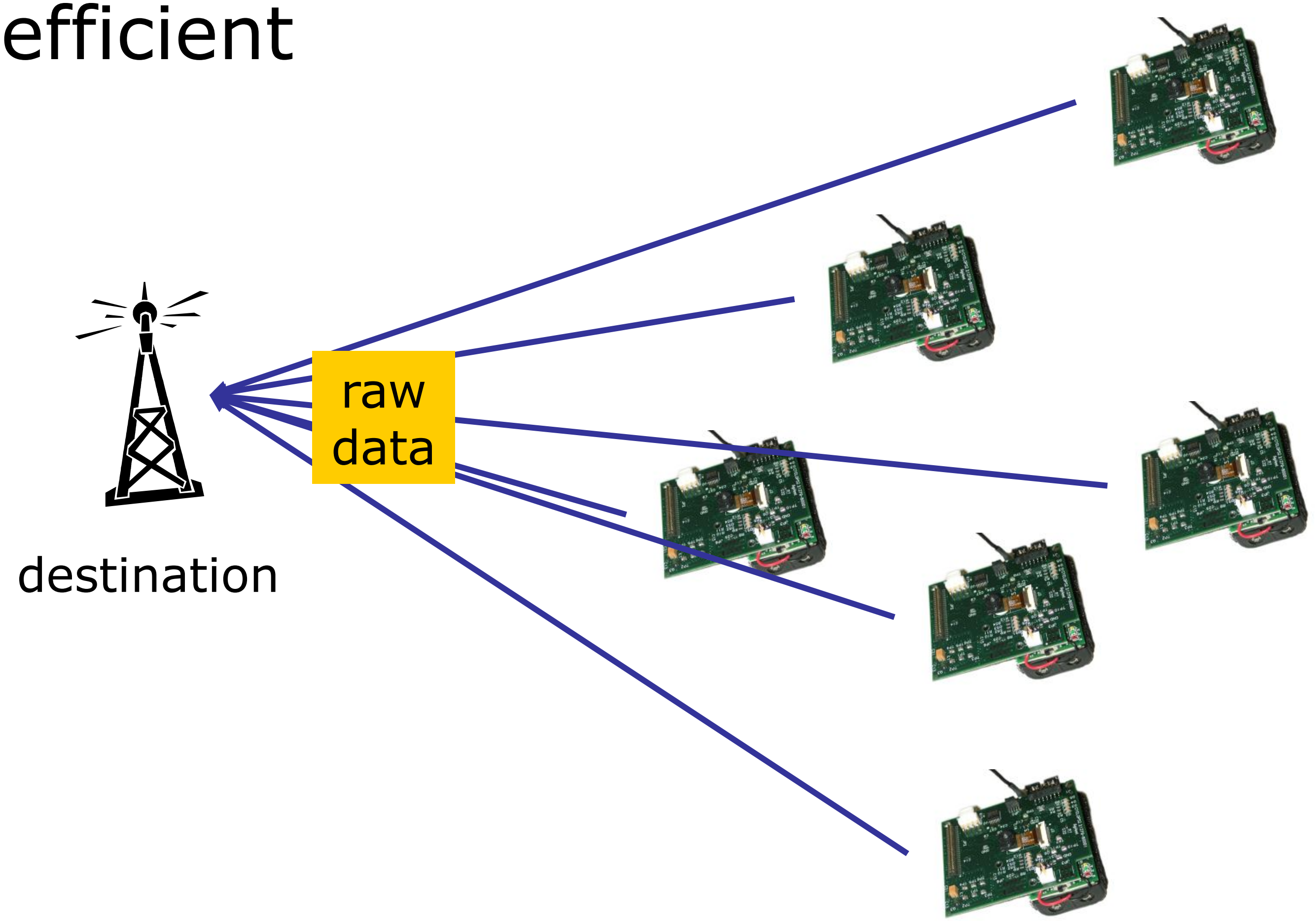
- Computational/power *asymmetry*
  - limited compute power on each sensor node
  - limited (battery) power on each sensor node

- Hostile *communication* environment
  - multi-hop
  - high loss rate

- Must be *energy efficient*
  - minimize communication

Distributed Sensing

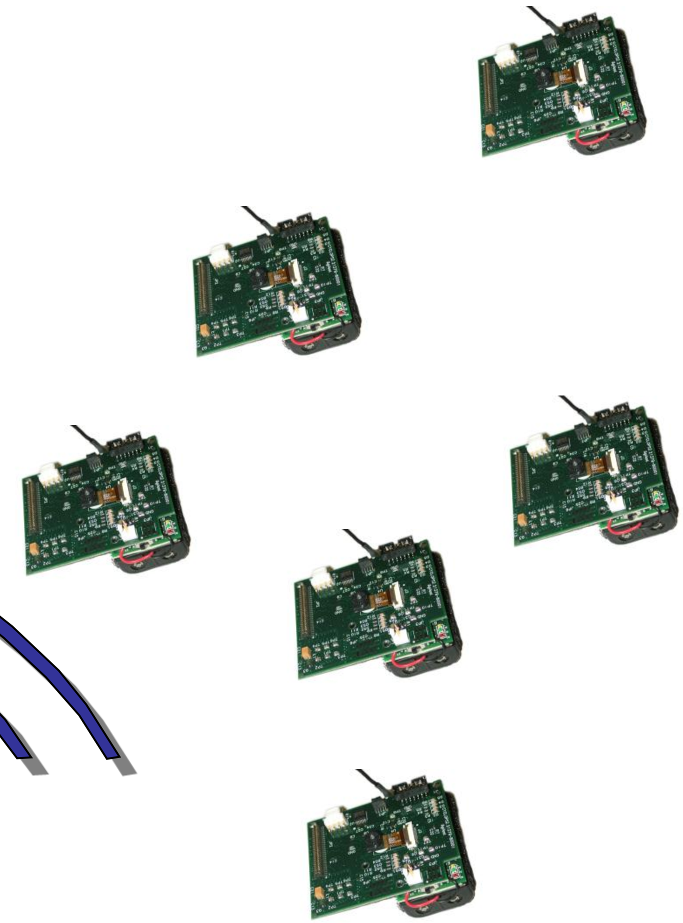
- Transmitting *raw data* can be inefficient



Distributed Sensing

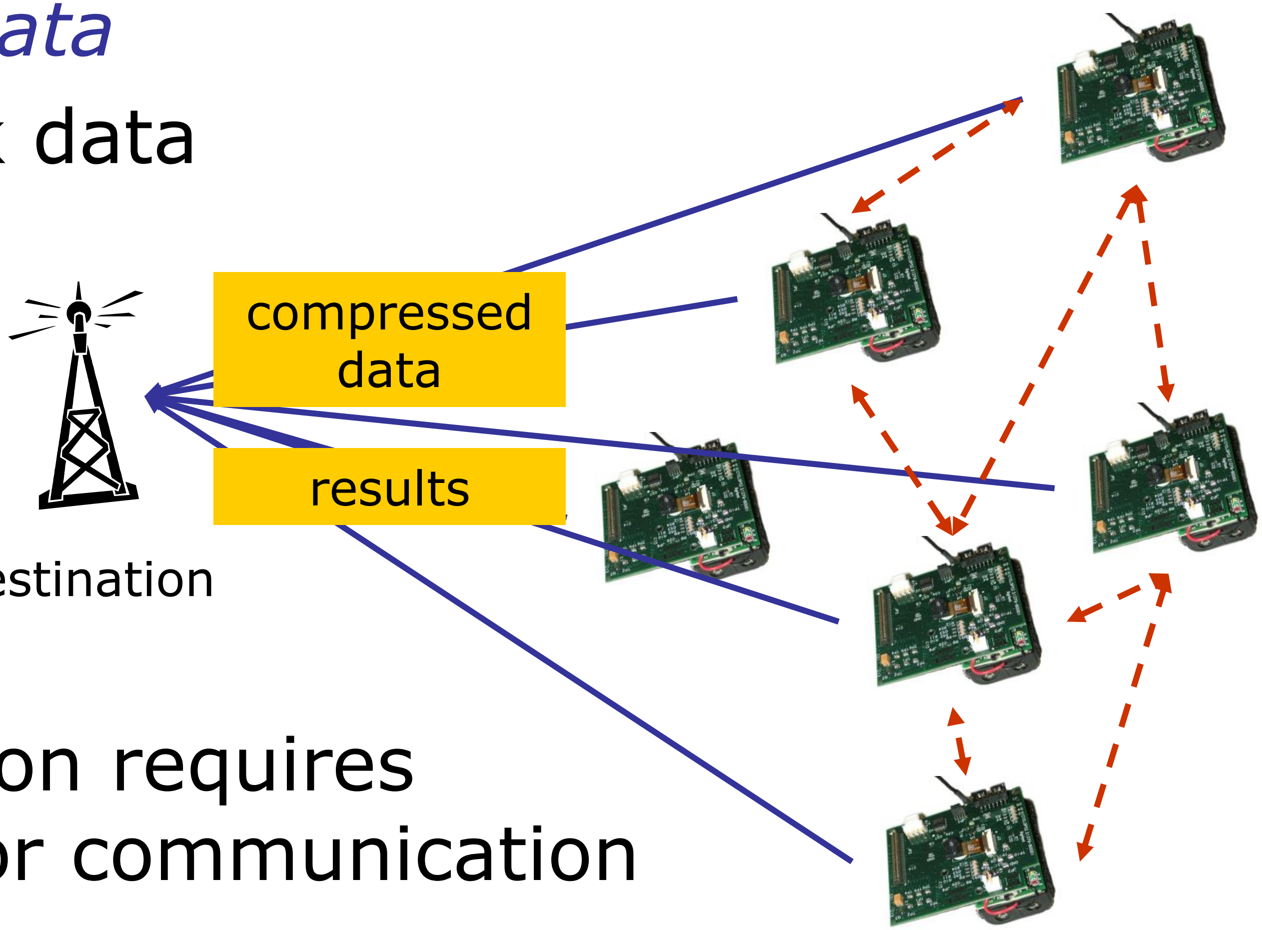
- Transmitting *raw data* can be inefficient

- Can we exploit
  - *intra-sensor* correlation?
  - *inter-sensor* correlation?



Collaborative Sensing

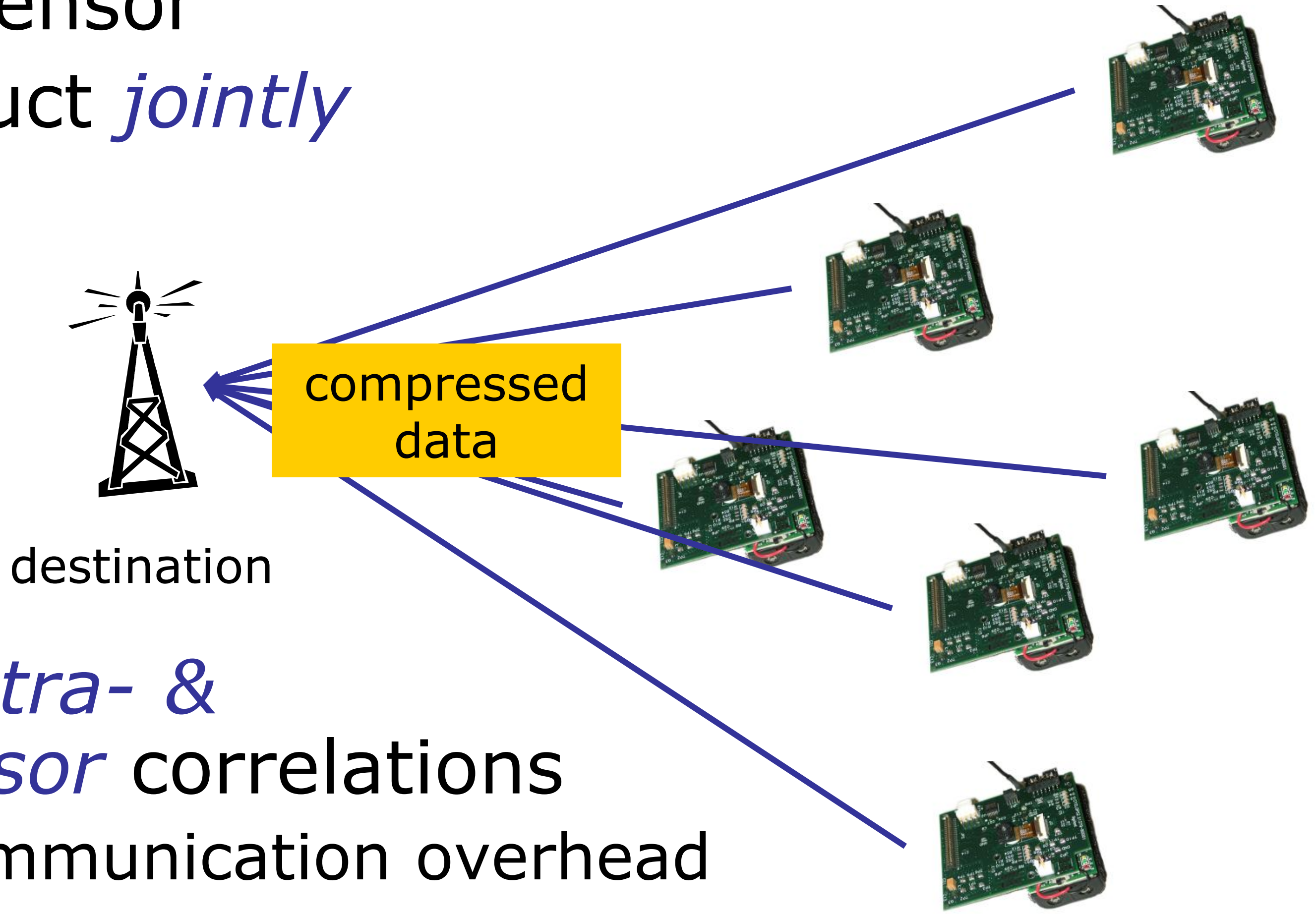
- Output *results* rather than *raw data*
- In-network data processing



- Collaboration requires inter-sensor communication

Distributed Compressed Sensing

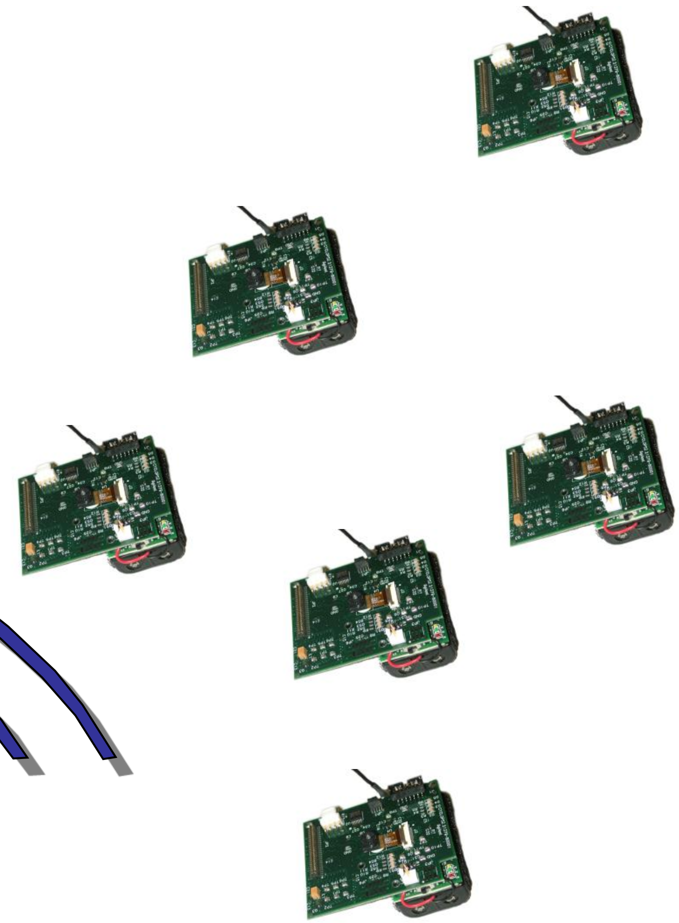
- Take random measurements at each sensor
- Reconstruct *jointly*



- Exploit *intra- & inter-sensor* correlations
  - Zero communication overhead
- Analogy w/ Slepian-Wolf coding

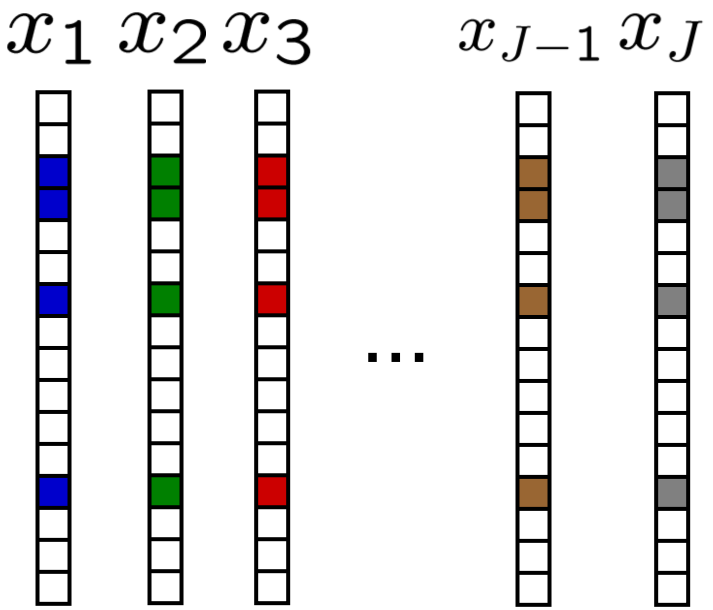
Common Sparse Supports Model

- **Example: audio signals**
  - sparse in Fourier Domain
  - same frequencies received by each node
  - different attenuations and delays (magnitudes and phases)



Common Sparse Supports Model

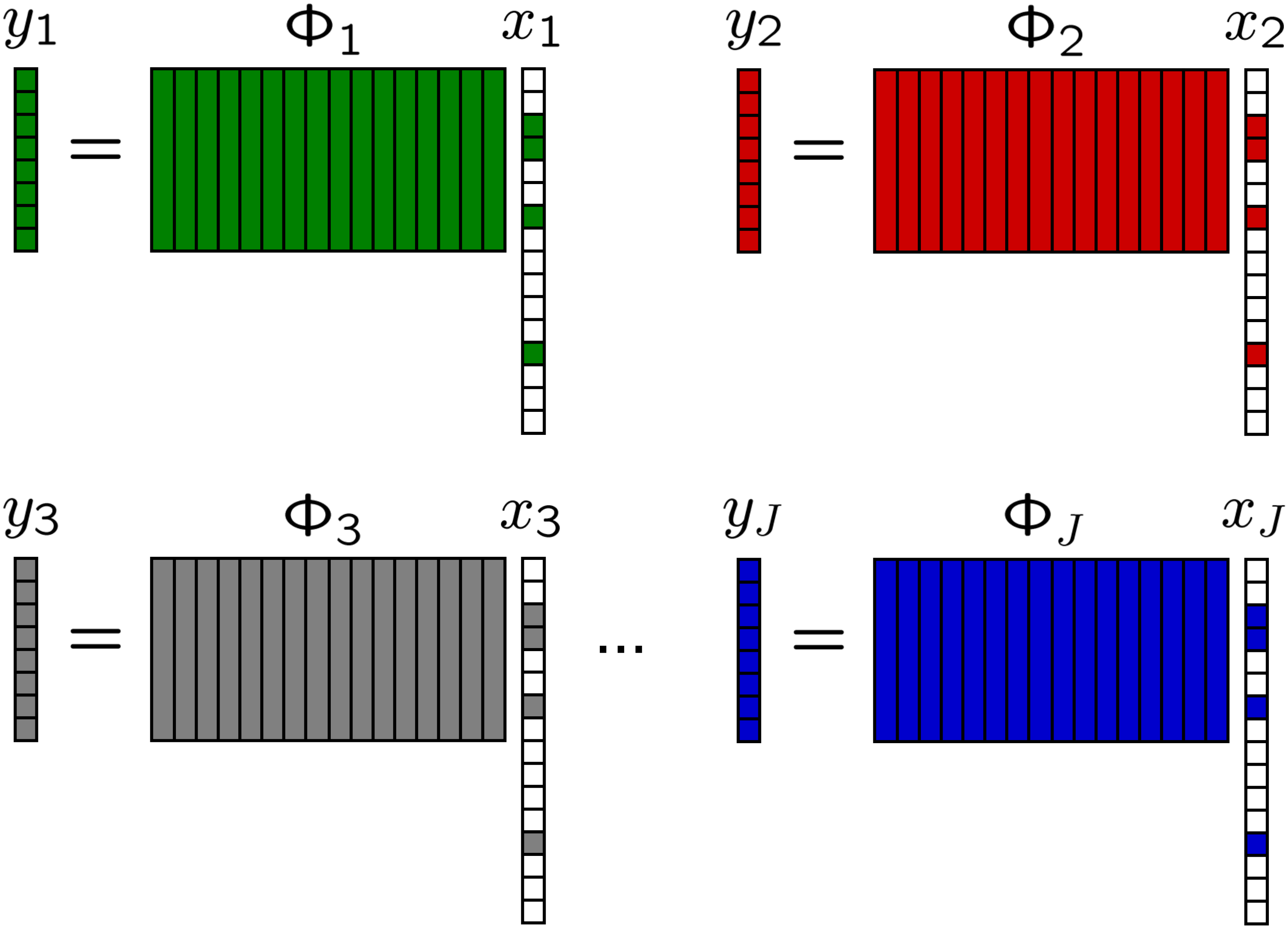
- Measure J signals, each K-sparse
- *Signals share sparse components but with different coefficients*



$$x_j = \sum_{\omega \in \Omega} x_{j,\omega} \psi_\omega,$$

$$|\Omega| = K$$

Common Sparse Supports Model



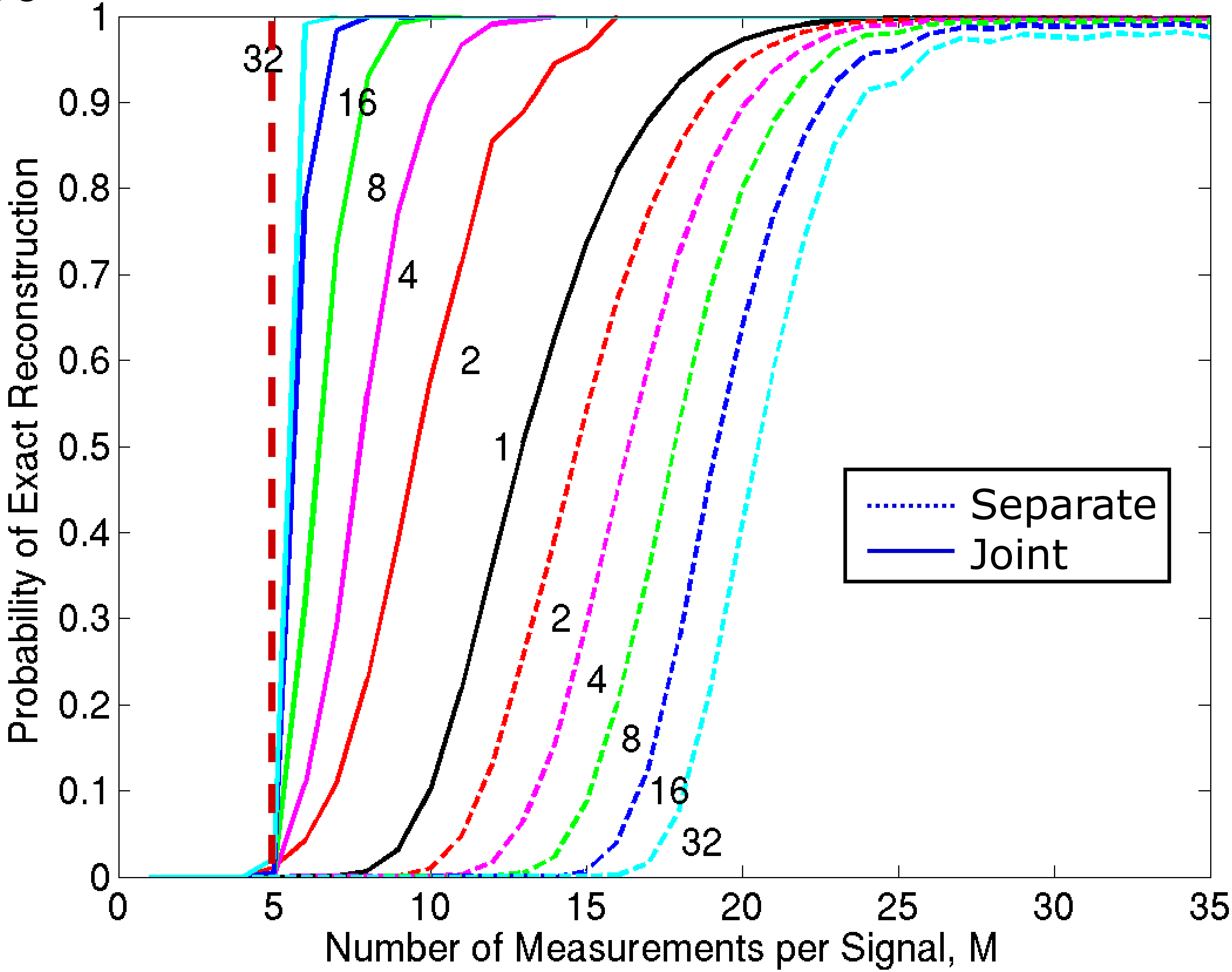
Ensemble Reconstruction Comparison

- Separate reconstruction using linear programming
  - measurements per sensor: $O(K \log(N/K))$
- Simultaneous Orthogonal Matching Pursuit
  - extends greedy algorithms to signal ensembles sharing a sparse support
    [Tropp, Gilbert, Strauss; Temlyakov]
  - measurements per sensor: cK

$$\lim_{J \to \infty} c = 1$$

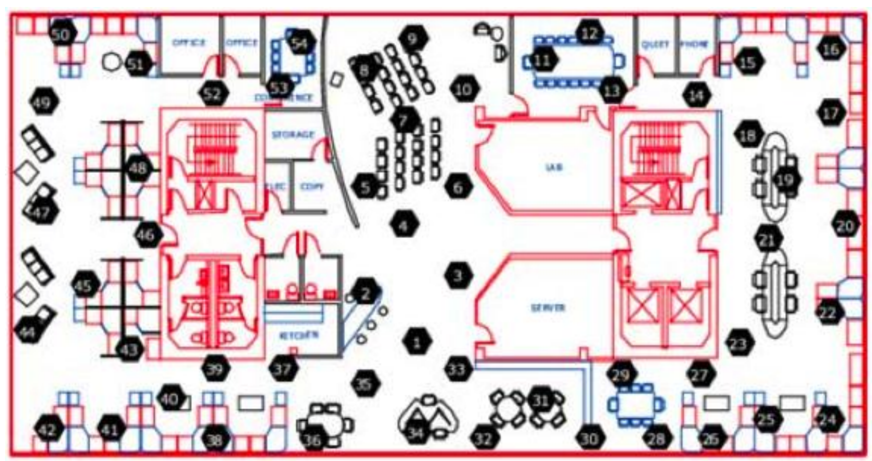
***K*=5**
N=50

Simulation

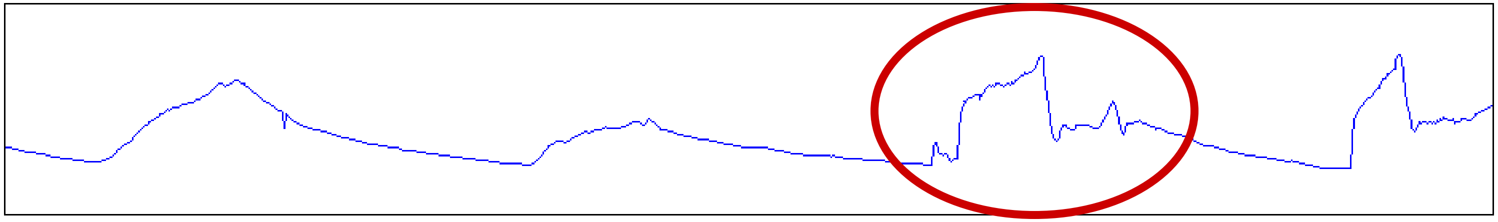


Real Data Example

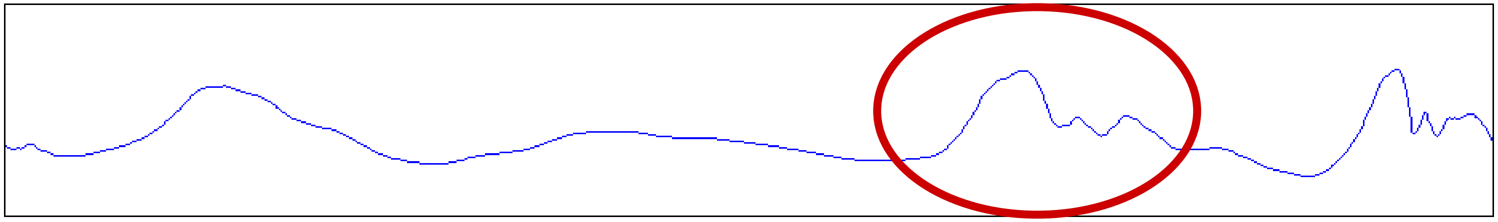
- Environmental Sensing in Intel Berkeley Lab
- J = 49 sensors, N = 1024 samples each
- Compare:
  - transform coding — K largest terms per sensor
  - independent CS — $4K$ measurements per sensor
  - DCS — $4K$ measurements per sensor



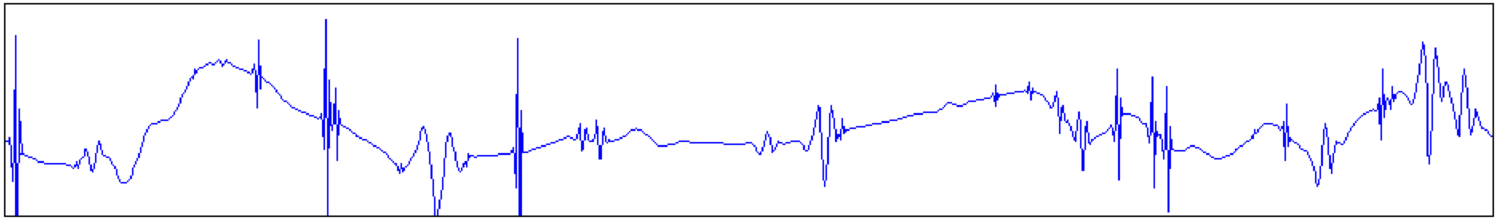
Temperature – Wavelets, $K = 20$



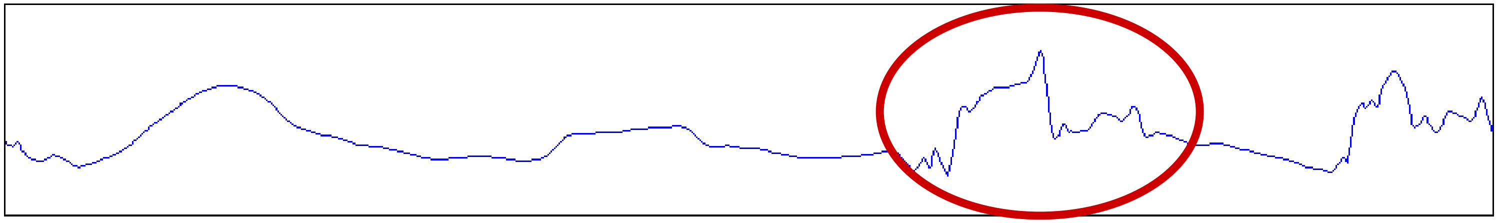
(a) Original



(b) Transform Coding, SNR = 25.9499 dB



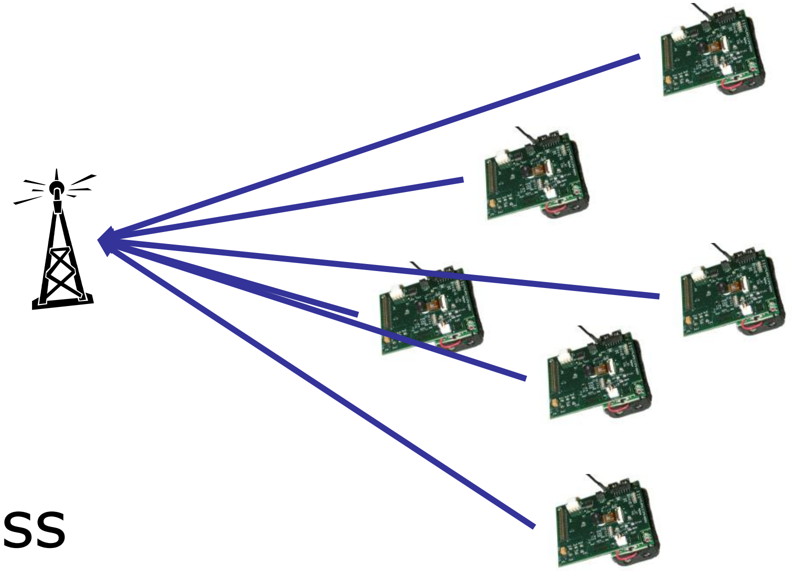
(c) Compressed Sensing, SNR = 16.8255 dB



(d) Distributed Compressed Sensing, SNR = 29.4149 dB

DCS Benefits

- Random projections for *sensing* and *encoding*
  - exploit both intra- and inter-sensor correlations
  - **joint source/channel coding**
- Universality
  - generic hardware
- Simple quantization
- Robust
  - to noise, quantization, loss
  - progressive
- Zero inter-sensor collaboration



Conclusions

- **Compressive sensing**
  - exploits signal sparsity/compressibility information
  - based on new uncertainty principles
  - integrates sensing, compression, processing
  - natural for sensor network applications

- Ongoing research
  - new algorithms for *analog-to-information* conversion
  - *fast algorithms* based on ECC matrices
  - manifold models for multiple signals/images
  - connections to machine learning

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