Sparse Spectral Unmixing

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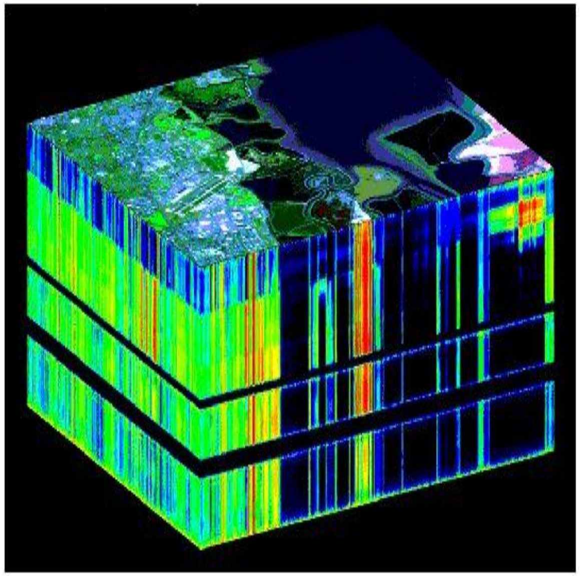
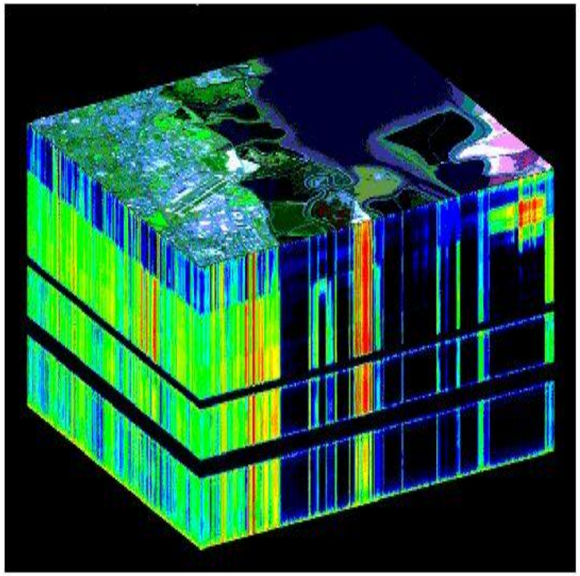
Hyperspectral Imaging



- Images simultaneously acquired in many narrow, adjacent frequency bands
- A hyperspectral image is not just an image
  - provides detailed information about chemical compositions of materials present
  - great potential for classification/anomaly detection applications

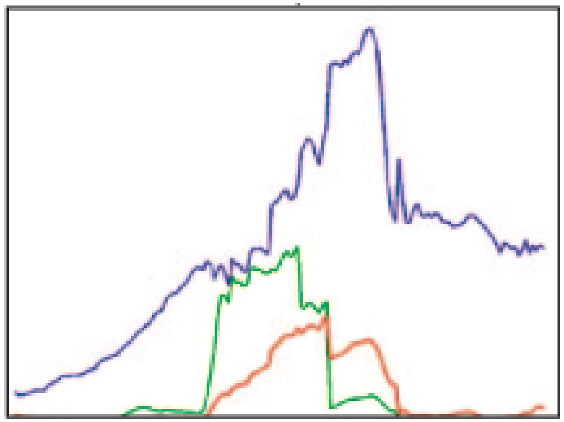
Spectral Unmixing

- Challenges in hyperspectral imaging:
  - often limited by large pixel size
  - spectrum observed at a single pixel may actually be a mixture of multiple spectra
- We would like to identify:
  - the separate materials present at a given pixel
  - the quantities of each material



Supervised Spectral Unmixing

- Begin by assuming we have a dictionary of spectral signatures (*endmembers)*
  - water
  - soil
  - metal
  - man-made materials



- Traditional approaches
  - least squares without noise: ULS, NNLS, POCS
  - least squares with noise: MVUE, Gaussian MVUE
  - max entropy, fuzzy membership, log-odds…

Sparsity

- Many natural images can be *compressed* in some representation/basis (Fourier, wavelets)

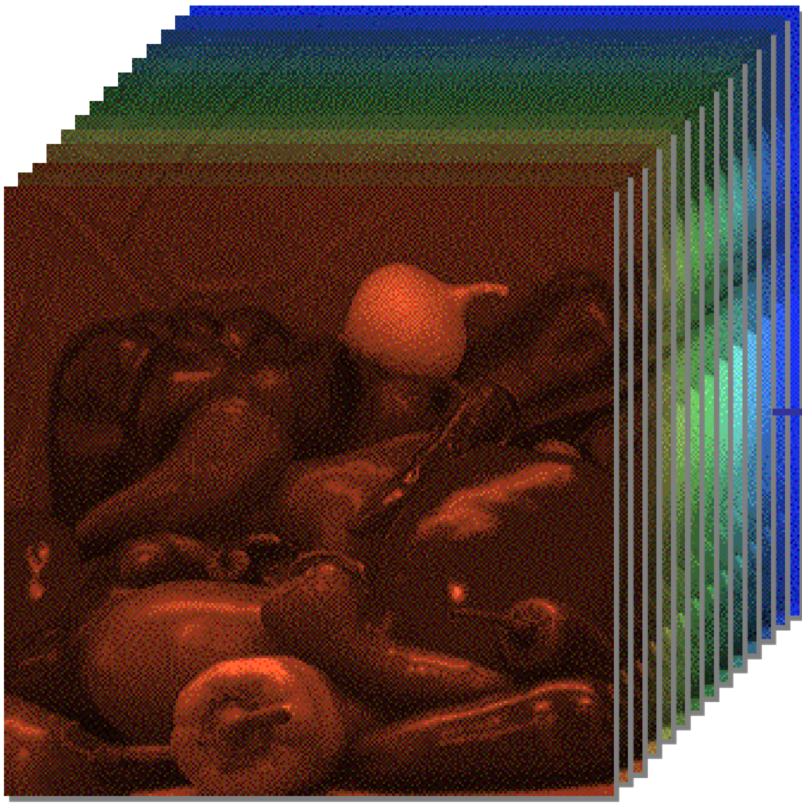
N
pixels

$K \ll N$
large
wavelet
coefficients

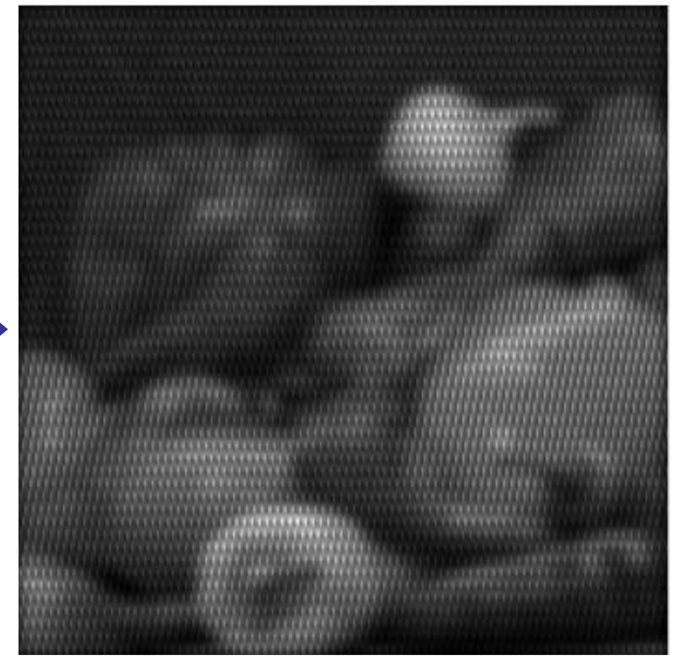
Sparse Spectral Unmixing

- We can exploit sparsity to solve the unmixing problem
  - the amount of a particular substance present will tend to vary smoothly from pixel to pixel (spatial-regularity)
  - each pixel will only have contributions from a small number of spectral signatures (spectral mixture sparsity)

Spatial Sparsity



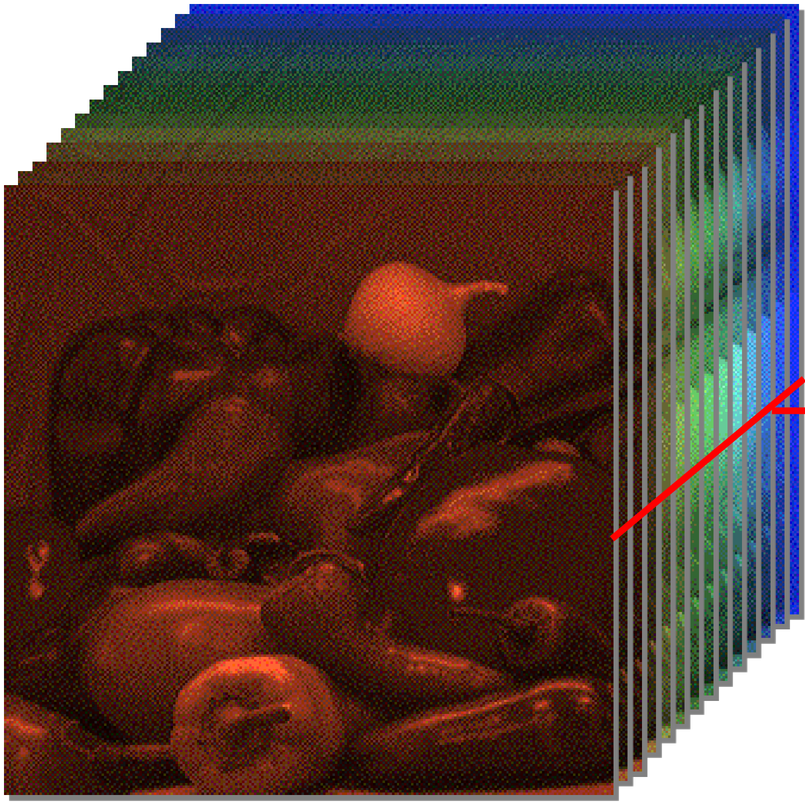
x



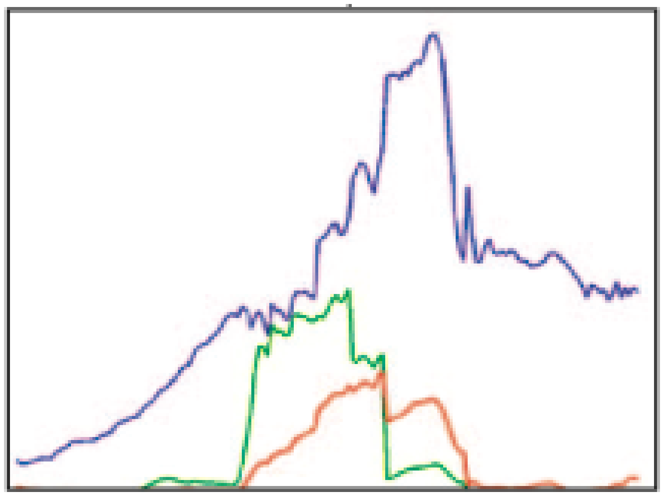
spatial sparsity
(wavelets)

Ψ_S

Spectral Sparsity



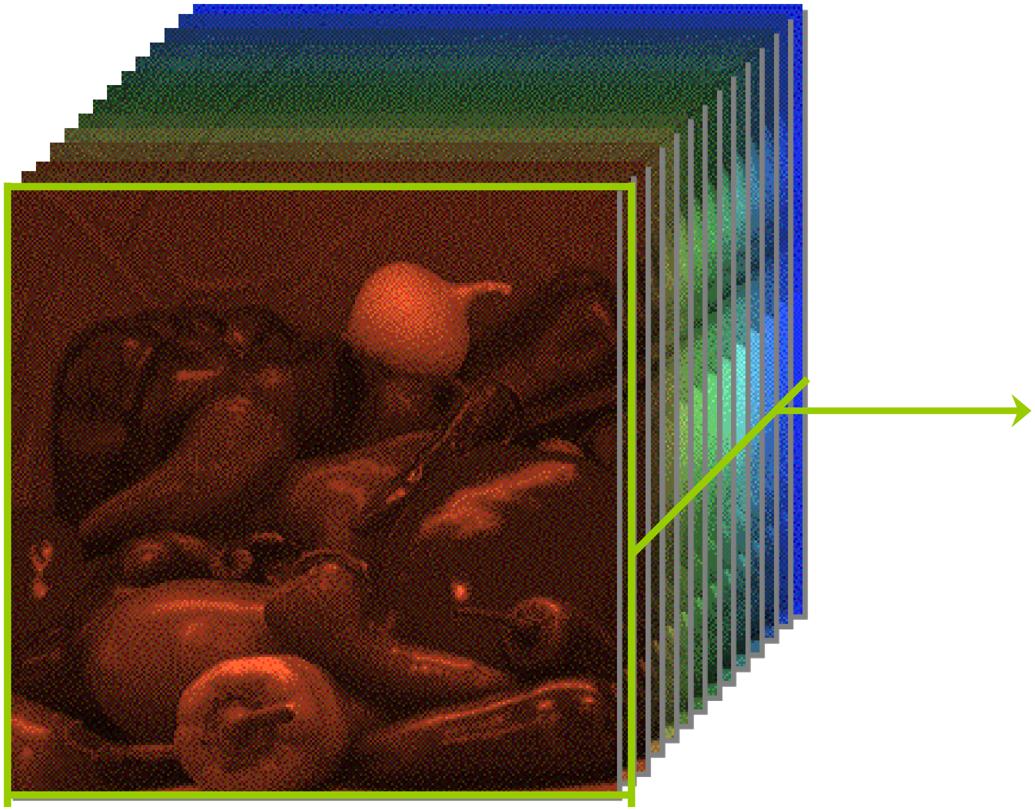
x



spectral sparsity
dictionary

Ψ_F

Tensor Product Dictionary



x

datacube sparsity
(tensor product
of spatial and spectral
dictionaries)

$$\Psi = \Psi_S \otimes \Psi_F$$

Sparse Spectral Unmixing

$$\boxed{\begin{array}{c}\text{Given } x = \Psi\alpha \\ \text{find } \alpha\end{array}}$$

assume α
is sparse

- Recovery algorithms:
  - linear programming/basis pursuit

$$\widehat{\alpha} = \arg\min_{x=\Psi\alpha} \|\alpha\|_1$$

  - greedy algorithms (OMP, ROMP)
  - many variations… dsp.rice.edu/cs

Theoretical Guarantees

- When can we guarantee that one of these algorithms can unmix the data?

- The spectral dictionary must satisfy

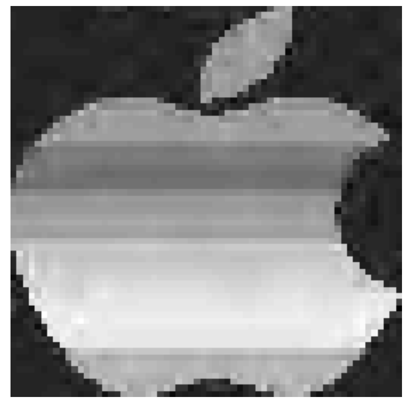
$$\boxed{||\Psi_F \alpha||_2 \approx ||\alpha||_2}$$

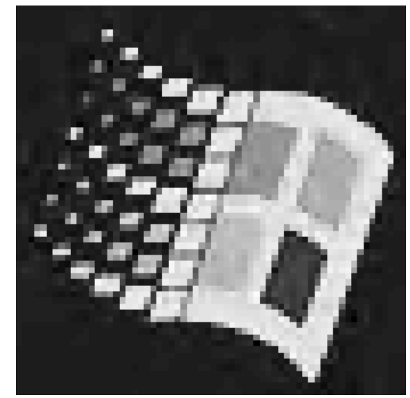
for all sparse α

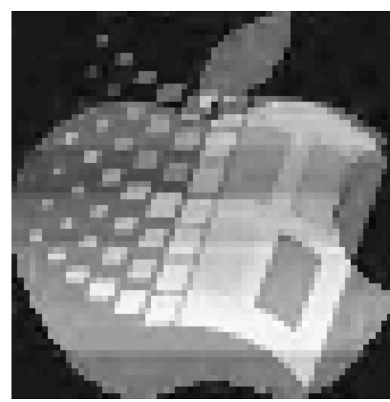
- Note: we cannot provide good guarantees for arbitrary spectral dictionaries

Synthetic Experiment

- Each logo is assigned 5 random elements (and weights) from spectral dictionary
- Weights are proportional to image intensity
- Additive i.i.d. Gaussian noise
- Perform unmixing using GPSR
  [Figueiredo, Nowak, Wright]



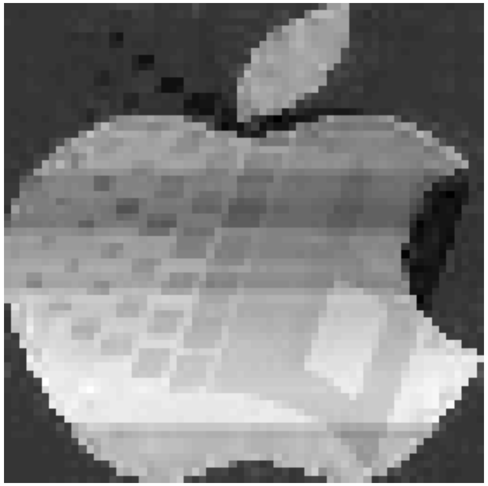


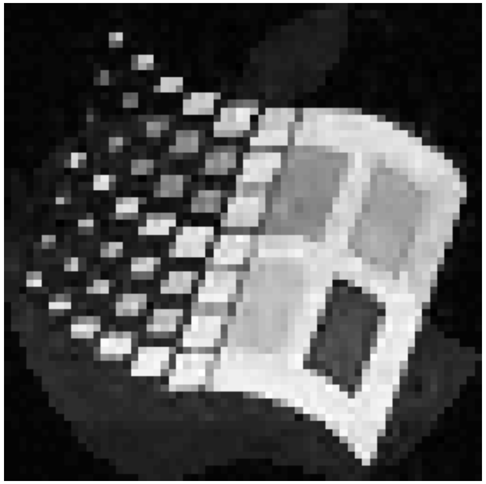


single
frequency
slice

Results

- Using GPSR followed by a simple thresholding of the wavelet coefficients
  - correctly identify all significant spectral signatures
  - no false alarms
  - relatively reliable estimates of the original mixing coefficients





Shortcomings

- Acquiring and storing the entire dataset can be expensive
  - current systems often overcome the storage issue through *dimensionality reduction* (PCA)
  - can something similar work for sparse spectral unmixing?
- Our approach requires that we know the dictionary – *supervised* spectral unmixing
  - can sparse spectral unmixing be generalized to the unsupervised case?

Dimensionality Reduction

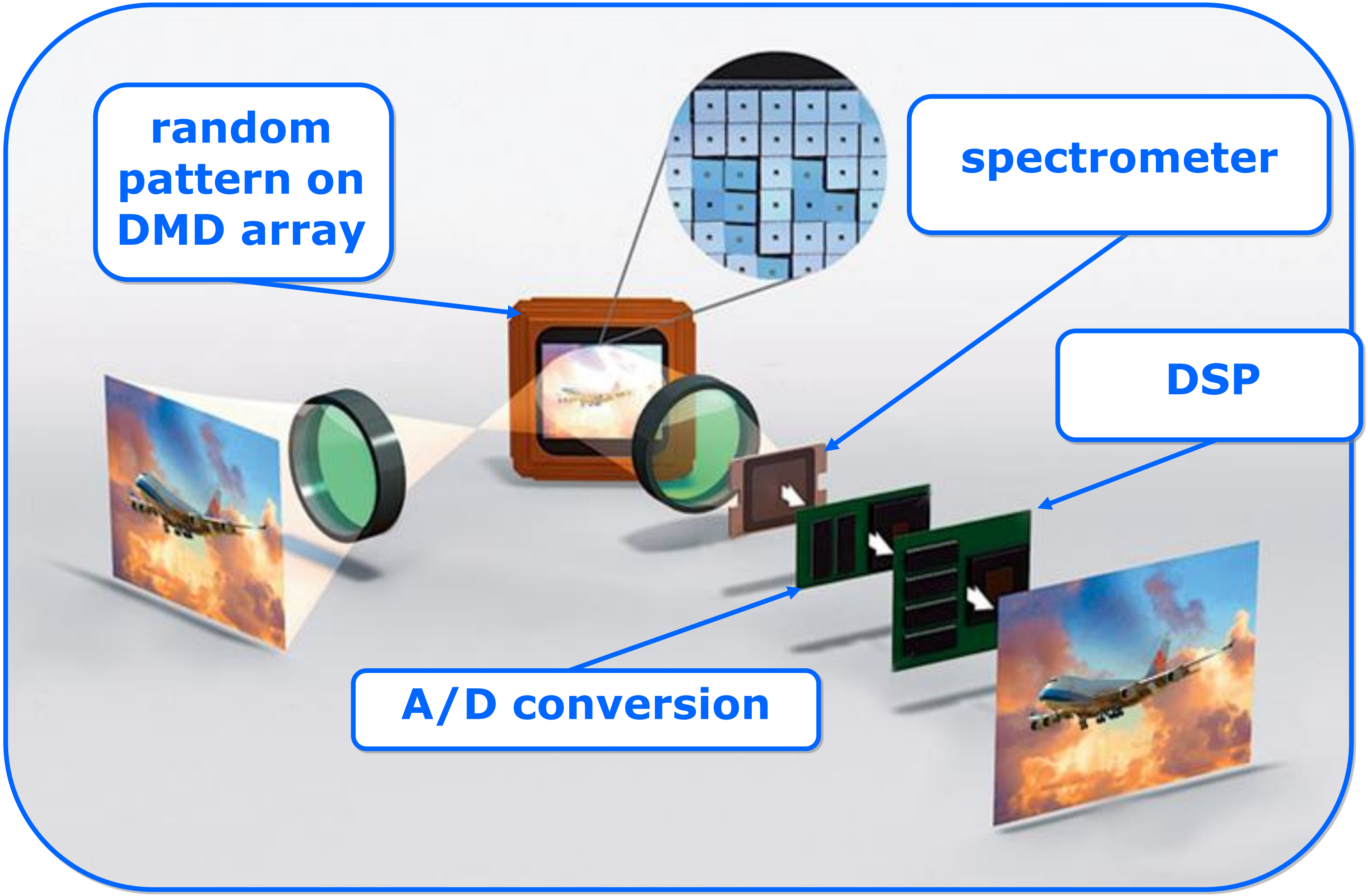
- For *sparse* data, PCA is doomed

- Compressive sensing: *random projections* preserve the information in sparse signals

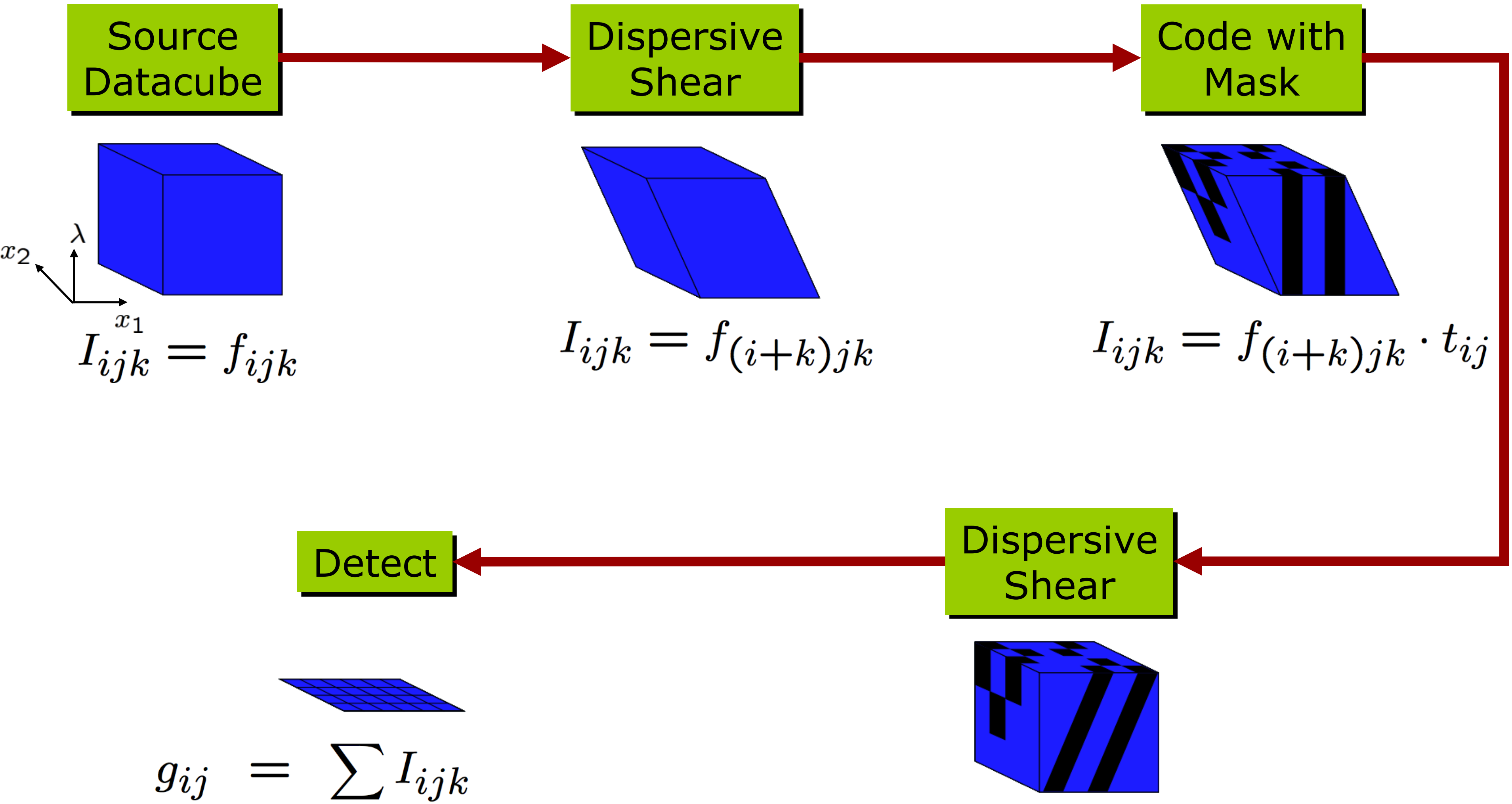
$$\boxed{\|\Phi\Psi\alpha\|_2 \approx \|\alpha\|_2}$$

- We can exploit this to build new hyperspectral imaging hardware

Rice Single-Pixel Hyperspectral Camera



Duke Hyperspectral Imager



$$I_{ijk} = f_{ijk}$$

$$I_{ijk} = f_{(i+k)jk}$$

$$I_{ijk} = f_{(i+k)jk} \cdot t_{ij}$$

$$I_{ijk} = f_{ijk} \cdot t_{(i-k)j}$$

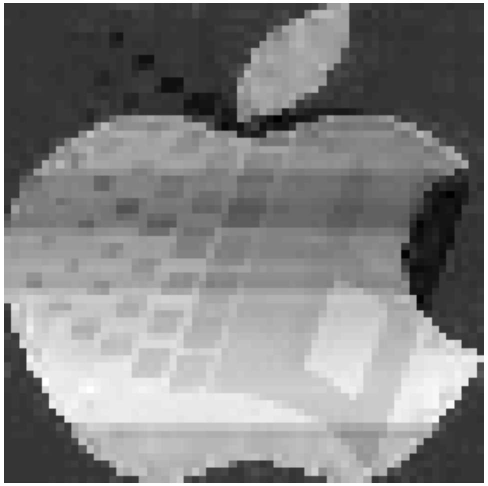
$$\begin{aligned} g_{ij} &= \sum_k I_{ijk} \\ &= \sum_k f_{ijk} \cdot t_{(i-k)j} \end{aligned}$$

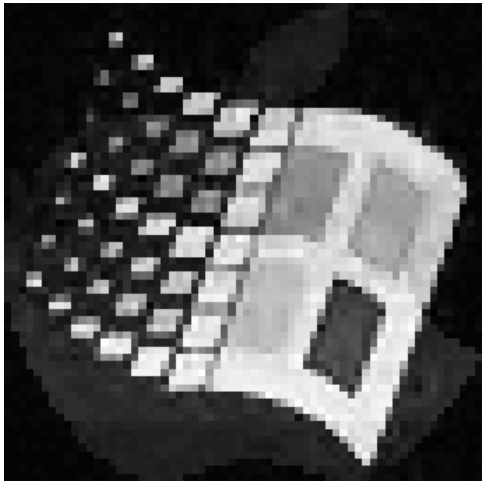
Compressive Spectral Unmixing

Given $x = \Phi\Psi\alpha$

find α

Using 10x fewer measurements





Unsupervised Spectral Unmixing

- We may not have enough prior information about the scene to build a dictionary of spectral signatures

- Traditional approach
  - ICA

- Learn the spectral signatures from the data by again exploiting sparsity
  - K-SVD [Aharon, Elad, Bruckstein]
  - *Sparse* ICA [Lennon, Mercier, Mouchot, Hubert-Moy]

Conclusions

- Sparse recovery provides a powerful framework for spectral unmixing
- Sparse spectral unmixing yields a recovery algorithm for compressive hyperspectral imaging systems
- Unsupervised sparse spectral unmixing should be possible, and are a necessary next step