Notational Conventions

DC Quantity – Upper case-letter, upper-case subscript	V_{BE}, I_D
Small-Signal Quantity – Lower case-letter, lower-case subscript	v_{be}, i_d
Total Quantity – Lower case-letter, upper-case subscript	$v_{BE} = V_{BE} + v_{be}$, $i_D = I_D + i_d$
Phasor Quantity – Upper case-letter, lower-case subscript	V_{be}, I_d

Independent Sources

Figure 1(a) shows the diagram of an independent voltage source. The voltage v is independent of the current i that flows through the source. Fig. 1(b) shows the diagram of an independent current source. The current i is independent of the voltage v across the source.

Figure 1: (a) Independent voltage source. (b) Independent current source.

Dependent Sources

VCVS – Voltage Controlled Voltage Source

Figure 2(a) shows the diagram of a voltage controlled voltage source. The output voltage is given by a voltage gain A_v multiplied by an input voltage v_1. Such a source in SPICE is called an E source.

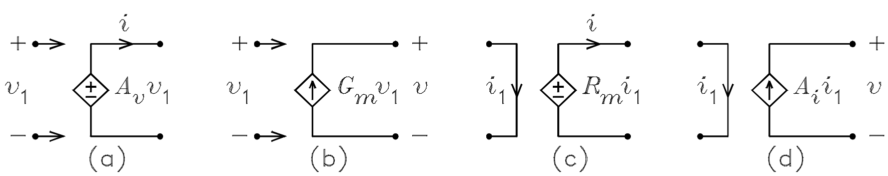

Figure 2: (a) Voltage controlled voltage source. (b) Voltage controlled current source. (c) Current controlled voltage source. (d) Current controlled current source.

VCCS – Voltage Controlled Current Source

Figure 2(b) shows the diagram of a voltage controlled current source. The output current is given by a transconductance G_m multiplied by an input voltage v_1. Such a source in SPICE is called a G source.

CCVS – Current Controlled Voltage Source

Figure 2(c) shows the diagram of a current controlled voltage source. The output voltage is given by a transresistance R_m multiplied by an input current i_1. Such a source in SPICE is called an F source.

CCCS – Current Controlled Current Source

Figure 2(d) shows the diagram of a current controlled current source. The output current is given by a current gain A_i multiplied by an input current i_1. Such a source in SPICE is called an H source.

Passive Elements

Resistor

Figure 3(a) shows the diagram of a resistor. The voltage across it is given by

$$v = iR$$

This relation is known as Ohm's law.

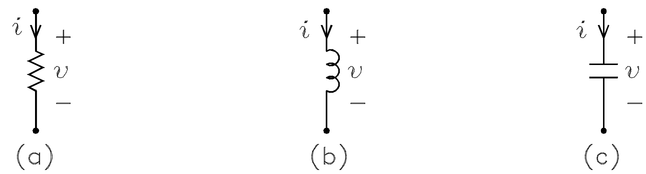

Figure 3: (a) Resistor. (b) Inductor. (c) Capacitor.

Inductor

Fig. 3(b) shows an inductor. The voltage across it is given by

$$v = L\frac{di}{dt}$$

In the analysis of circuits having sinusoidal excitations, phasor analysis is usually used. In this case, the voltage across the inductor is given by

$$V = LsI$$

where V and I are phasors, $s = j\omega$, and ω is the radian frequency of the excitation. In the phasor domain, a multiplication by s is equivalent to a time derivative in the time domain. This is because the time domain excitation is assumed to be of the form $\exp(st)$.

Capacitor

Fig. 3(c) shows a capacitor. The current through it is given by

$$i = C\frac{dv}{dt}$$

For phasor representation of the signals, the current through the capacitor is given by

$$I = CsV$$

Voltage Division and Current Division

Voltage Division

Figure 3(a) shows a two-resistor voltage divider. The voltages v_1 and v_2 are given by

$$v_1 = i_S R_1 = \left(\frac{v_S}{R_1 + R_2}\right) R_1 = v_S \frac{R_1}{R_1 + R_2}$$

$$v_2 = i_S R_2 = \left(\frac{v_S}{R_1 + R_2}\right) R_2 = v_S \frac{R_2}{R_1 + R_2}$$

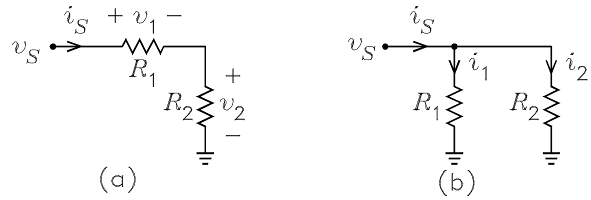

Figure 4: (a) Voltage divider. (b) Current divider.

Current Division

Figure 3(b) shows a two-resistor current divider. The currents i_1 and i_2 are given by

$$i_1 = \frac{v_S}{R_1} = \frac{i_S\left(R_1 \| R_2\right)}{R_1} = i_S \frac{R_2}{R_1 + R_2}$$

$$i_2 = \frac{v_S}{R_2} = \frac{i_S\left(R_1 \| R_2\right)}{R_2} = i_S \frac{R_1}{R_1 + R_2}$$

Use of Superposition in Solving Circuits

This section illustrates the use of superposition in solving circuits containing dependent sources. The correct use of superposition requires the dependent sources to be treated as independent sources in writing equations. When a source is zeroed, its controlling variable is not zeroed.

Example 1 *For the circuit in Fig. 5(a), it is given that $R_1 = 20\,\mathrm{k\Omega}$, $R_2 = 1\,\mathrm{k\Omega}$, and $A_i = 50$. Solve for the Thévenin and Norton equivalent circuits seen looking into the output terminals.*

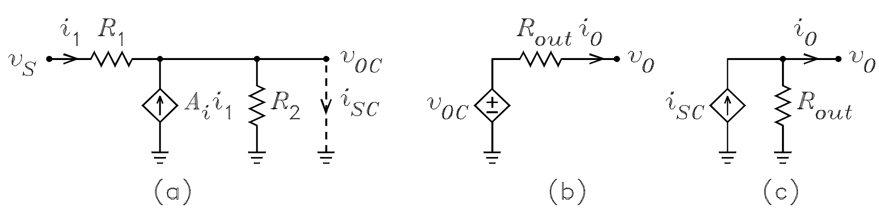

Figure 5: (a) Circuit for Example 1. (b) Thévenin equivalent. (b) Norton equivalent.

Solution. The Thévenin voltage is the open-circuit output voltage v_{OC}. The Norton current is the short-circuit output current i_{SC}. The output resistance is the ratio of the open-circuit output voltage to the short-circuit output current. First, we solve for the open-circuit output voltage. By superposition, we can write

$$v_{OC} = v_S \frac{R_2}{R_1 + R_2} + A_i i_1 \left(R_1 \| R_2\right)$$

Next, we use superposition to solve for i_1.

$$i_1 = \frac{v_S}{R_1 + R_2} - A_i i_1 \frac{R_2}{R_1 + R_2}$$

This can be solved for i_1 to obtain

$$i_1 = \frac{v_S}{R_1 + R_2} \frac{1}{1 + A_i \dfrac{R_2}{R_1 + R_2}} = \frac{v_S}{R_1 + \left(1 + A_i\right) R_2}$$

We substitute the solution for i_1 into the expression for v_{OC} to obtain

$$\begin{aligned}
v_{OC} &= v_S \frac{R_2}{R_1+R_2} + A_i \left(R_1 \| R_2\right) \frac{v_S}{R_1 + (1+A_i) R_2} \\
&= v_S \frac{R_2}{R_1+R_2} \left[R_2 + \frac{A_i R_1 R_2}{R_1 + (1+A_i) R_2}\right] \\
&= v_S \frac{20\,\mathrm{k}\Omega}{1\,\mathrm{k}\Omega + 20\,\mathrm{k}\Omega} \left[1\,\mathrm{k}\Omega + \frac{50 \times 20\,\mathrm{k}\Omega \times 1\,\mathrm{k}\Omega}{20\,\mathrm{k}\Omega + (1+50)\,1\,\mathrm{k}\Omega}\right] \\
&= \frac{v_S}{21} \left(1 + \frac{50 \times 20}{20+51}\right) \\
&= 0.718 v_S
\end{aligned}$$

The Thévenin equivalent circuit is shown in Fig. 5(b).

By superposition, the short-circuit output current is given by

$$i_{SC} = \frac{v_S}{R_1} + A_i i_1$$

where i_1 is given by

$$i_1 = \frac{v_S}{R_1}$$

We substitute the expression for i_1 into the expression for i_{SC} to obtain

$$i_{SC} = \frac{v_S}{R_1} + A_i \frac{v_S}{R_1} = v_S \frac{1+A_i}{R_1} = v_S \frac{1+50}{1\,\mathrm{k}\Omega} = 2.55 \times 10^{-3} v_S$$

The output resistance is given by

$$R_{out} = \frac{v_{OC}}{i_{SC}} = \frac{0.718 v_S}{2.55 \times 10^{-3} v_S} = \frac{0.718}{2.55 \times 10^{-3}} = 282\,\Omega$$

The Norton equivalent circuit is shown in Fig. 5(c).

Example 2 *For the circuit in Fig. 6(a), it is given that* $R_1 = 3\,\mathrm{k}\Omega$, $R_2 = 2\,\mathrm{k}\Omega$, *and* $G_m = 0.1\,\mathrm{S}$. *Solve for the input resistance to the circuit.*

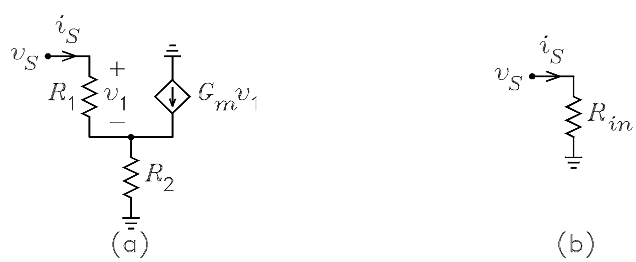

Figure 6: (a) Circuit for Example 2. (b) Equivalent input circuit.

Solution. The input resistance is given by the ratio of the source voltage to the source current. By superposition, we can write

$$v_S = i_S \left(R_1 + R_2\right) + G_m v_1 R_2$$

where v_1 is given by

$$v_1 = i_S R_1$$

Substitute the expression for v_1 into the expression for v_S to obtain

$$v_S = i_S \left(R_1 + R_2\right) + G_m i_S R_1 R_2 = i_S \left(R_1 + R_2 + G_m R_1 R_2\right)$$

Thus the input resistance is given by

$$R_{in} = \frac{v_S}{i_S} = R_1 + R_2 + G_m R_1 R_2 = 3\,\text{k}\Omega + 2\,\text{k}\Omega + 0.1 \times 3\,\text{k}\Omega \times 2\,\text{k}\Omega = 605\,\text{k}\Omega$$

The equivalent circuit seen looking into the input is shown in Fig. 6(b).

Example 3 *Solve for the open circuit output voltage and the short circuit output current for the circuit in Fig. 7(a).*

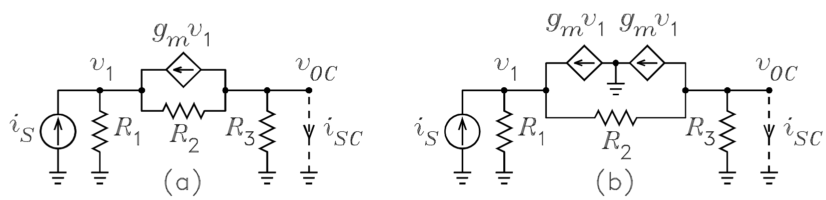

Figure 7: Circuit for Example 3

Solution. The circuit contains a floating current source. To make superposition simpler to apply, this source can be broken into two series sources as shown in Fig. 7(b). The node between the sources is shown connected to ground. Although no current flows from this node to ground when both sources are active, a current does flow when either is zeroed. However, by superposition, the sum of these currents must be zero. Because the currents flow into the ground node, the voltages or current in the circuit are not affected. By superposition, the open circuit output voltage is given by

$$\begin{aligned} v_{OC} &= (i_S + g_m v_1) \frac{R_1}{R_1 + R_2 + R_3} R_3 - g_m v_1 (R_1 + R_2) \| R_3 \\ &= i_S \frac{R_1 R_3}{R_1 + R_2 + R_3} - g_m v_1 \frac{R_2 R_3}{R_1 + R_2 + R_3} \end{aligned}$$

The voltage v_1 is given by

$$\begin{aligned} v_1 &= (i_S + g_m v_1) R_1 \| (R_2 + R_3) - g_m v_1 \frac{R_3}{R_1 + R_2 + R_3} R_1 \\ &= i_S R_1 \| (R_2 + R_3) + g_m v_1 \frac{R_1 R_2}{R_1 + R_2 + R_3} \end{aligned}$$

Solution for v_1 yields

$$v_1 = i_S \frac{R_1 \| (R_2 + R_3)}{1 - g_m \dfrac{R_1 R_2}{R_1 + R_2 + R_3}}$$

When this is used in the equation for v_{OC}, we obtain

$$v_{OC} = i_S \frac{R_3}{R_1 + R_2 + R_3} \left[R_1 - g_m R_2 \frac{R_1 \| (R_2 + R_3)}{1 - g_m \dfrac{R_1 R_2}{R_1 + R_2 + R_3}} \right]$$

By superposition, the short circuit output current is given by

$$i_{SC} = (i_S + g_m v_1) \frac{R_1}{R_1 + R_2} - g_m v_1 = i_S \frac{R_1}{R_1 + R_2} - g_m v_1 \frac{R_2}{R_1 + R_2}$$

The voltage v_1 is given by

$$v_1 = (i_S + g_m v_1) R_1 \| R_2$$

This can be solved for v_1 to obtain

$$v_1 = i_S \frac{R_1 \| R_2}{1 - g_m (R_1 \| R_2)}$$

When this is used in the equation for i_{SC}, we obtain

$$i_{SC} = i_S \frac{1}{R_1 + R_2} \left[R_1 - g_m R_2 \frac{(R_1 \| R_2)}{1 - g_m (R_1 \| R_2)} \right]$$

Example 4 *For the circuit of Fig. 8(a), it is given that* $i_S = 5\,\text{mA}$, $R_1 = 4\,\text{k}\Omega$, $R_2 = 2\,\text{k}\Omega$, $R_3 = 3\,\text{k}\Omega$, *and* $A_i = 3$. *Solve for the Norton equivalent circuit seen looking into the output terminals and the voltage at the output if a load resistor of value* $R_L = 6\,\text{k}\Omega$ *is connected from the output to ground.*

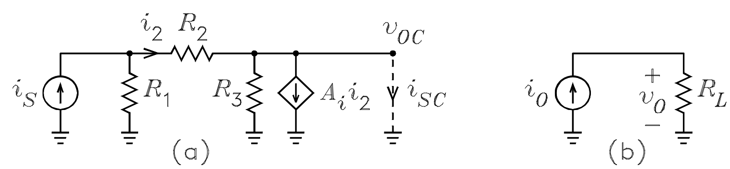

Figure 8: Circuit for Example 4.

Solution. By superposition, the open-circuit output voltage is given by

$$v_{OC} = i_S \frac{R_1}{R_1 + R_2 + R_3} R_3 - A_i i_2 \left[(R_1 + R_2) \| R_3 \right]$$

The current i_2 is given by

$$i_2 = i_S \frac{R_1}{R_1 + R_2 + R_3} + A_i i_2 \frac{R_3}{R_1 + R_2 + R_3}$$

Solution yields

$$i_2 = i_S \frac{R_1}{R_1 + R_2 + R_3} \frac{1}{1 - \dfrac{A_i R_3}{R_1 + R_2 + R_3}} = i_S \frac{R_1}{R_1 + R_2 + (1 - A_i) R_3}$$

When this equation is used in the equation for v_{OC}, we obtain

$$v_{OC} = i_S \left[\frac{R_1 R_3}{R_1 + R_2 + R_3} - A_i \left[(R_1 + R_2) \| R_3 \right] \frac{R_1}{R_1 + R_2 + (1 - A_i) R_3} \right]$$

For the element values given, it follows that $v_{OC} = -\infty$. This means that the output resistance of the circuit is ∞, i.e. the circuit looks like a current source to any load.

By superposition, the short-circuit output current is given by

$$i_{SC} = i_S \frac{R_1}{R_1 + R_2} - A_i i_2$$

The current i_2 is given by

$$i_2 = i_S \frac{R_1}{R_1 + R_2}$$

When this is used in the equation for i_{SC}, we obtain

$$i_{SC} = i_S \left[\frac{R_1}{R_1 + R_2} - A_i \frac{R_1}{R_1 + R_2} \right] = i_S \frac{(1 - A_i) R_1}{R_1 + R_2} = -\frac{4}{3} i_S$$

Fig. 8(b) shows the circuit with R_L connected to its output. The output voltage is given by

$$v_O = -\frac{4}{3} \times 5\,\text{mA} \times 6\,\text{k}\Omega = -40\,\text{V}$$

Amplifier Representations

Figure 9 shows the diagram of an amplifier. The source is represented by a Thévenin equivalent circuit. In general, the input resistance R_{in} is a function of the load resistance R_L and the output resistance R_{out} is a function of the source resistance R_S.

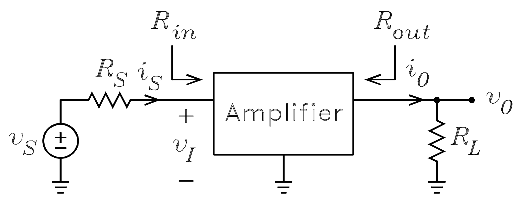

Figure 9: Amplifier model.

The output circuit of the amplifier can be represented by either a Thévenin equivalent circuit or a Norton equivalent circuit using one of the four dependent sources described above. The four equivalent circuits are summarized below.

VCVS Model

Figure 10(a) shows the amplifier model with the output represented by a voltage-controlled voltage source. The output voltage is given by

$$v_O = A_v v_I \frac{R_L}{R_{out} + R_L} = A_v \left(v_S \frac{R_{in}}{R_S + R_{in}} \right) \frac{R_L}{R_{out} + R_L}$$

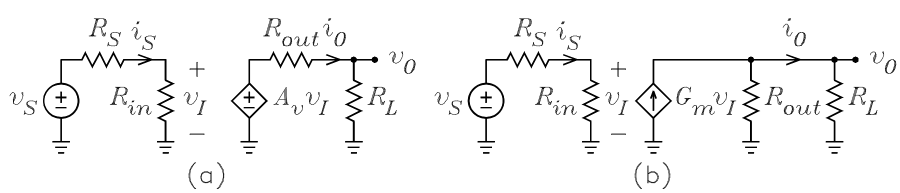

Figure 10: (a) Voltage controlled voltage source amplifier. (b) Voltage controlled current source amplifier.

VCCS Model

Figure 10(b) shows the amplifier model with the output represented by a voltage-controlled current source. The output voltage is given by

$$v_O = G_m v_I \left(R_{out} \| R_L \right) = G_m \left(v_S \frac{R_{in}}{R_S + R_{in}} \right) \left(R_{out} \| R_L \right)$$

CCVS Model

Figure 11(a) shows the amplifier model with the output represented by a current-controlled voltage source. The output voltage is given by

$$v_O = R_m i_S \frac{R_L}{R_{out} + R_L} = R_m \left(\frac{v_S}{R_S + R_{in}} \right) \frac{R_L}{R_{out} + R_L}$$

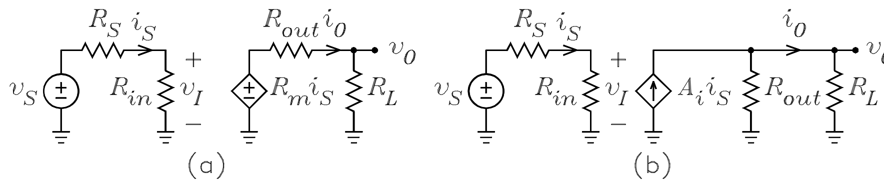

Figure 11: (a) Current controlled voltage source amplifier. (b) Current controlled current amplifier.

CCCS Model

Figure 11(b) shows the amplifier model with the output represented by a current-controlled current source. The output voltage is given by

$$v_O = A_i i_S \frac{R_L}{R_{out} + R_L} = A_i \left(\frac{v_S}{R_S + R_{in}} \right) \frac{R_{out} R_L}{R_{out} + R_L}$$