© Copyright 2010. W. Marshall Leach, Jr., Professor, Georgia Institute of Technology, School of Electrical and Computer Engineering.

## The Common-Base Amplifier

## Basic Circuit

Fig. 1 shows the circuit diagram of a single stage common-base amplifier. The object is to solve for the small-signal voltage gain, input resistance, and output resistance.


Figure 1: Common-base amplifier.

## DC Solution

(a) Replace the capacitors with open circuits. Look out of the 3 BJT terminals and make Thévenin equivalent circuits as shown in Fig. 2.

$$
\begin{gathered}
V_{B B}=\frac{V^{+} R_{2}+V^{-} R_{1}}{R_{1}+R_{2}} \quad R_{B B}=R_{1} \| R_{2} \\
V_{E E}=V^{-} \quad R_{E E}=R_{E} \quad V_{C C}=V^{+} \quad R_{C C}=R_{C}
\end{gathered}
$$

(b) Make an "educated guess" for $V_{B E}$. Write the loop equation between the $V_{B B}$ and the $V_{E E}$ nodes. To solve for $I_{C}$, this equation is

$$
V_{B B}-V_{E E}=I_{B} R_{B B}+V_{B E}+I_{E} R_{E E}=\frac{I_{C}}{\beta} R_{B B}+V_{B E}+\frac{I_{C}}{\alpha} R_{E E}
$$

(c) Solve the loop equation for the currents.

$$
I_{C}=\alpha I_{E}=\beta I_{B}=\frac{V_{B B}-V_{E E}-V_{B E}}{R_{B B} / \beta+R_{E E} / \alpha}
$$

(d) Verify that $V_{C B}>0$ for the active mode.

$$
V_{C B}=V_{C}-V_{B}=\left(V_{C C}-I_{C} R_{C C}\right)-\left(V_{B B}-I_{B} R_{B B}\right)=V_{C C}-V_{B B}-I_{C}\left(R_{C C}-R_{B B} / \beta\right)
$$



Figure 2: DC bias circuit.

## Small-Signal or AC Solutions

It will be assumed that the base spreading resistance $r_{x}$ is non zero. This is a resistance in series with the base lead in the small signal models.
(a) Redraw the circuit with $V^{+}=V^{-}=0$ and all capacitors replaced with short circuits as shown in Fig. 3.


Figure 3: Signal circuit.
(b) Calculate $g_{m}, r_{\pi}, r_{e}$, and $r_{0}$ from the DC solution.

$$
g_{m}=\frac{I_{C}}{V_{T}} \quad r_{\pi}=\frac{V_{T}}{I_{B}} \quad r_{e}=\frac{V_{T}}{I_{E}} \quad r_{0}=\frac{V_{A}+V_{C E}}{I_{C}}
$$

(c) Solve for $r_{\text {in }}$ and $r_{\text {out }}$.

$$
r_{i n}=R_{1}\left\|R_{2}\right\| r_{i e} \quad r_{i e}=\frac{r_{\pi}}{1+\beta} \stackrel{\text { or }}{=} r_{e}
$$

$$
r_{\text {out }}=r_{i c}\left\|R_{C} \quad r_{i c}=r_{0}\left(1+\frac{\beta \times R_{s} \| R_{E}}{r_{\pi}+R_{s} \| R_{E}}\right)+r_{\pi}\right\| R_{s} \| R_{E}
$$

(d) Replace the circuits looking out of the base and emitter with Thévenin equivalent circuits as shown in Fig. 4.

$$
v_{t b}=0 \quad R_{t b}=0 \quad v_{t e}=v_{s} \frac{R_{E}}{R_{s}+R_{E}} \quad R_{t e}=R_{s} \| R_{E}
$$



Figure 4: Signal circuit with Thévenin emitter circuit.
(e) Replace the BJT in Fig. 4 with the Thévenin emitter circuit and the Norton collector circuit as shown in Fig. 5.


Figure 5: Emitter and collector equivalent circuits.
(e) Solve for $i_{c(s c)}$.

$$
i_{c(s c)}=i_{c}^{\prime}=-G_{m} v_{t e} \quad G_{m}=\frac{1}{\frac{r_{\pi}}{\beta}+R_{t e}} \stackrel{\text { or }}{=} \frac{1}{\frac{1}{g_{m}}+R_{t e}}
$$

(f) Solve for $v_{o}$.

$$
v_{o}=-i_{c(s c)} \times-\left(r_{i c}\left\|R_{C}\right\| R_{L}\right)
$$

(g) The flow graph is show in Figure 6. The voltage gain is given by

$$
A_{v}=\frac{v_{o}}{v_{s}}=\frac{R_{E}}{R_{s}+R_{E}} \times-G_{m} \times-\left(r_{i c}\left\|R_{C}\right\| R_{L}\right)
$$



Figure 6: Flow graph for the voltage gain.

Example 1 For the CB amplifier in Fig. 1, it is given that $R_{s}=100 \Omega, R_{1}=120 \mathrm{k} \Omega, R_{2}=100 \mathrm{k} \Omega$, $R_{C}=4.3 \mathrm{k} \Omega, R_{E}=5.6 \mathrm{k} \Omega, R_{L}=20 \mathrm{k} \Omega, V^{+}=15 \mathrm{~V}, V^{-}=-15 \mathrm{~V}, V_{B E}=0.65 \mathrm{~V}, \beta=99$, $\alpha=0.99, V_{A}=100 \mathrm{~V}$, and $V_{T}=0.025 \mathrm{~V}$. Solve for $A_{v}, r_{\text {in }}$, and $r_{\text {out }}$.

Solution. Because the dc bias circuit is the same as for the common-emitter amplifier example, the dc bias values, $r_{e}, g_{m}, r_{\pi}$, and $r_{0}$ are the same. They are

$$
\begin{gathered}
I_{C}=2.092 \mathrm{~mA} \quad I_{E}=2.113 \mathrm{~mA} \quad I_{B}=21.13 \mu \mathrm{~A} \\
r_{0}=\frac{V_{A}+V_{C E}}{\alpha I_{E}}=52.18 \mathrm{k} \Omega \quad g_{m}=\frac{I_{C}}{V_{T}}=\frac{2.092}{25}=\frac{1}{11.95} \mathrm{~S} \\
r_{\pi}=\frac{V_{T}}{I_{B}}=\frac{\beta V_{T}}{I_{C}}=\frac{99 \times 25}{2.113}=1.183 \mathrm{k} \Omega \quad r_{e}=\frac{V_{T}}{I_{E}}=11.83 \Omega
\end{gathered}
$$

In the signal circuit, the Thévenin voltage and resistance seen looking out of the emitter are given by

$$
v_{t e}=\frac{R_{E}}{R_{s}+R_{E}} v_{s}=0.9825 v_{s} \quad R_{t e}=R_{s} \| R_{E}=98.25 \Omega
$$

The Thévenin resistances seen looking out of the collector is

$$
R_{t c}=R_{C} \| R_{L}=3.539 \mathrm{k} \Omega
$$

Next, we calculate $G_{m}, r_{i c}$, and $r_{i e}$.

$$
\begin{gathered}
G_{m}=\frac{1}{\frac{r_{x}+r_{\pi}}{\beta}+\frac{R_{t e}}{\alpha}}=\frac{1}{111.4} \mathrm{~S} \\
r_{i c}=r_{0}\left(1+\frac{\beta \times R_{t e}}{r_{x}+r_{\pi}+R_{t e}}\right)+\left(r_{x}+r_{\pi}\right) \| R_{t e}=442.3 \mathrm{k} \Omega \quad r_{i e}=r_{e}=12.03 \Omega
\end{gathered}
$$

The voltage gain is given by

$$
A_{v}=\frac{v_{o}}{v_{s}}=\frac{v_{t e}}{v_{s}} \times \frac{i_{c}^{\prime}}{v_{t e}} \times \frac{v_{o}}{i_{c}^{\prime}}=\frac{R_{E}}{R_{s}+R_{E}} \times-G_{m} \times-\left(r_{i c} \| R_{t c}\right)=30.97
$$

The input and output resistances are

$$
r_{i n}=R_{1}\left\|R_{2}\right\| r_{i b}=11.81 \Omega \quad r_{\text {out }}=r_{i c} \| R_{C}=4.259 \mathrm{k} \Omega
$$

