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## The Common-Base Amplifier

## **Basic Circuit**

Fig. 1 shows the circuit diagram of a single stage common-base amplifier. The object is to solve for the small-signal voltage gain, input resistance, and output resistance.



Figure 1: Common-base amplifier.

## **DC** Solution

(a) Replace the capacitors with open circuits. Look out of the 3 BJT terminals and make Thévenin equivalent circuits as shown in Fig. 2.

$$V_{BB} = \frac{V^+ R_2 + V^- R_1}{R_1 + R_2} \qquad R_{BB} = R_1 ||R_2$$
$$V_{EE} = V^- \qquad R_{EE} = R_E \qquad V_{CC} = V^+ \qquad R_{CC} = R_C$$

(b) Make an "educated guess" for  $V_{BE}$ . Write the loop equation between the  $V_{BB}$  and the  $V_{EE}$  nodes. To solve for  $I_C$ , this equation is

$$V_{BB} - V_{EE} = I_B R_{BB} + V_{BE} + I_E R_{EE} = \frac{I_C}{\beta} R_{BB} + V_{BE} + \frac{I_C}{\alpha} R_{EE}$$

(c) Solve the loop equation for the currents.

$$I_C = \alpha I_E = \beta I_B = \frac{V_{BB} - V_{EE} - V_{BE}}{R_{BB}/\beta + R_{EE}/\alpha}$$

(d) Verify that  $V_{CB} > 0$  for the active mode.

$$V_{CB} = V_C - V_B = (V_{CC} - I_C R_{CC}) - (V_{BB} - I_B R_{BB}) = V_{CC} - V_{BB} - I_C (R_{CC} - R_{BB}/\beta)$$



Figure 2: DC bias circuit.

## **Small-Signal or AC Solutions**

It will be assumed that the base spreading resistance  $r_x$  is non zero. This is a resistance in series with the base lead in the small signal models.

(a) Redraw the circuit with  $V^+ = V^- = 0$  and all capacitors replaced with short circuits as shown in Fig. 3.



Figure 3: Signal circuit.

(b) Calculate  $g_m$ ,  $r_{\pi}$ ,  $r_e$ , and  $r_0$  from the DC solution.

$$g_m = \frac{I_C}{V_T}$$
  $r_\pi = \frac{V_T}{I_B}$   $r_e = \frac{V_T}{I_E}$   $r_0 = \frac{V_A + V_{CE}}{I_C}$ 

(c) Solve for  $r_{in}$  and  $r_{out}$ .

$$r_{in} = R_1 ||R_2||r_{ie} \qquad r_{ie} = \frac{r_\pi}{1+\beta} \stackrel{\text{or}}{=} r_e$$

$$r_{out} = r_{ic} \| R_C$$
  $r_{ic} = r_0 \left( 1 + \frac{\beta \times R_s \| R_E}{r_{\pi} + R_s \| R_E} \right) + r_{\pi} \| R_s \| R_E$ 

(d) Replace the circuits looking out of the base and emitter with Thévenin equivalent circuits as shown in Fig. 4.

$$v_{tb} = 0 \qquad R_{tb} = 0 \qquad v_{te} = v_s \frac{R_E}{R_s + R_E} \qquad R_{te} = R_s ||R_E$$

$$= \int_{\overline{I}} \frac{r_o ut}{\sqrt{I}} \int_{\overline{I}} \frac{r_o ut}{\sqrt{I}} \int_{\overline{I}} \frac{v_o}{\sqrt{I}} \int_{\overline{I}} \frac{r_i e}{\sqrt{I}} \int_{\overline{I}} \frac{r_i e}{\sqrt{I}} \int_{\overline{I}} \frac{v_i e}{$$

Figure 4: Signal circuit with Thévenin emitter circuit.

(e) Replace the BJT in Fig. 4 with the Thévenin emitter circuit and the Norton collector circuit as shown in Fig. 5.



Figure 5: Emitter and collector equivalent circuits.

(e) Solve for  $i_{c(sc)}$ .

$$i_{c(sc)} = i'_{c} = -G_{m}v_{te}$$
  $G_{m} = \frac{1}{\frac{r_{\pi}}{\beta} + R_{te}} \stackrel{\text{or}}{=} \frac{1}{\frac{1}{g_{m}} + R_{te}}$ 

(f) Solve for  $v_o$ .

$$v_o = -i_{c(sc)} \times - (r_{ic} \| R_C \| R_L)$$

(g) The flow graph is show in Figure 6. The voltage gain is given by

$$A_v = \frac{v_o}{v_s} = \frac{R_E}{R_s + R_E} \times -G_m \times -(r_{ic} \| R_C \| R_L)$$

$$v_s \underbrace{ \begin{array}{ccc} \frac{R_E}{R_s + R_E} & v_{te} - G_m & i'_c & -r_{ic} ||R_C||R_L \\ \end{array}}_{V_s \underbrace{ \begin{array}{ccc} \end{array}} v_s \underbrace{ \end{array}} v_o$$

Figure 6: Flow graph for the voltage gain.

**Example 1** For the CB amplifier in Fig. 1, it is given that  $R_s = 100 \Omega$ ,  $R_1 = 120 \text{ k}\Omega$ ,  $R_2 = 100 \text{ k}\Omega$ ,  $R_C = 4.3 \text{ k}\Omega$ ,  $R_E = 5.6 \text{ k}\Omega$ ,  $R_L = 20 \text{ k}\Omega$ ,  $V^+ = 15 \text{ V}$ ,  $V^- = -15 \text{ V}$ ,  $V_{BE} = 0.65 \text{ V}$ ,  $\beta = 99$ ,  $\alpha = 0.99$ ,  $V_A = 100 \text{ V}$ , and  $V_T = 0.025 \text{ V}$ . Solve for  $A_v$ ,  $r_{in}$ , and  $r_{out}$ .

Solution. Because the dc bias circuit is the same as for the common-emitter amplifier example, the dc bias values,  $r_e$ ,  $g_m$ ,  $r_{\pi}$ , and  $r_0$  are the same. They are

$$\begin{split} I_C &= 2.092 \,\mathrm{mA} \qquad I_E = 2.113 \,\mathrm{mA} \qquad I_B = 21.13 \,\mu\mathrm{A} \\ r_0 &= \frac{V_A + V_{CE}}{\alpha I_E} = 52.18 \,\mathrm{k\Omega} \qquad g_m = \frac{I_C}{V_T} = \frac{2.092}{25} = \frac{1}{11.95} \,\mathrm{S} \\ r_\pi &= \frac{V_T}{I_B} = \frac{\beta V_T}{I_C} = \frac{99 \times 25}{2.113} = 1.183 \,\mathrm{k\Omega} \qquad r_e = \frac{V_T}{I_E} = 11.83 \,\Omega \end{split}$$

In the signal circuit, the Thévenin voltage and resistance seen looking out of the emitter are given by

$$v_{te} = \frac{R_E}{R_s + R_E} v_s = 0.9825 v_s \qquad R_{te} = R_s ||R_E = 98.25 \,\Omega$$

The Thévenin resistances seen looking out of the collector is

$$R_{tc} = R_C \| R_L = 3.539 \,\mathrm{k}\Omega$$

Next, we calculate  $G_m$ ,  $r_{ic}$ , and  $r_{ie}$ .

$$G_m = rac{1}{rac{r_x + r_\pi}{eta} + rac{R_{te}}{lpha}} = rac{1}{111.4}\,\mathrm{S}$$

$$r_{ic} = r_0 \left( 1 + \frac{\beta \times R_{te}}{r_x + r_\pi + R_{te}} \right) + (r_x + r_\pi) \, \|R_{te} = 442.3 \, \mathrm{k\Omega} \qquad r_{ie} = r_e = 12.03 \, \Omega$$

The voltage gain is given by

$$A_{v} = \frac{v_{o}}{v_{s}} = \frac{v_{te}}{v_{s}} \times \frac{i_{c}'}{v_{te}} \times \frac{v_{o}}{i_{c}'} = \frac{R_{E}}{R_{s} + R_{E}} \times -G_{m} \times -(r_{ic} \| R_{tc}) = 30.97$$

The input and output resistances are

$$r_{in} = R_1 \| R_2 \| r_{ib} = 11.81 \,\Omega \qquad r_{out} = r_{ic} \| R_C = 4.259 \,\mathrm{k}\Omega$$