$R_{P}(x, y):=\frac{x \cdot y}{x+y} \quad$ Function for calculating parallel resistors.
$\mathrm{R}_{1}:=100000 \quad \mathrm{R}_{2}:=120000 \quad \mathrm{R}_{\mathrm{C}}:=4300 \quad \mathrm{R}_{\mathrm{E}}:=5600 \quad \mathrm{R}_{\mathrm{S}}:=5000 \quad \mathrm{R}_{\mathrm{L}}:=10000$
$\mathrm{V}_{\mathrm{p}}:=15 \quad \mathrm{~V}_{\mathrm{m}}:=-15 \quad \mathrm{~V}_{\mathrm{BE}}:=0.65 \quad \mathrm{~V}_{\mathrm{T}}:=0.025 \quad \beta:=99 \quad \alpha:=0.99$
$\mathrm{r}_{\mathrm{x}}:=20 \quad \mathrm{r}_{0}:=50000 \quad \mathrm{R}_{3}:=100$
$\mathrm{v}_{\mathrm{s}}:=1 \quad$ With $\mathrm{v}_{\mathrm{s}}=1$, the voltage gain is equal to $\mathrm{v}_{\mathrm{o}}$.


DC Bias Solution

$\mathrm{V}_{\mathrm{BB}}:=\frac{\mathrm{V}_{\mathrm{p}} \cdot \mathrm{R}_{2}+\mathrm{V}_{\mathrm{m}} \cdot \mathrm{R}_{1}}{\mathrm{R}_{1}+\mathrm{R}_{2}} \quad \mathrm{~V}_{\mathrm{BB}}=1.3636$
$\mathrm{R}_{\mathrm{BB}}:=\mathrm{R}_{\mathrm{P}}\left(\mathrm{R}_{1}, \mathrm{R}_{2}\right) \quad \mathrm{R}_{\mathrm{BB}}=5.4545 \cdot 10^{4}$
$I_{E}:=\frac{V_{B B}-V_{B E}-V_{m}}{\frac{R_{B B}}{1+\beta}+R_{E}} \quad I_{E}=2.557 \cdot 10^{-3}$
$\mathrm{V}_{\mathrm{C}}:=\mathrm{V}_{\mathrm{p}}-\alpha \cdot \mathrm{I}_{\mathrm{E}} \cdot \mathrm{R}_{\mathrm{C}} \quad \quad \mathrm{V}_{\mathrm{C}}=4.1151$
$\mathrm{V}_{\mathrm{B}}:=\mathrm{V}_{\mathrm{BE}}+\mathrm{I}_{\mathrm{E}} \cdot \mathrm{R}_{\mathrm{E}}+\mathrm{V}_{\mathrm{m}} \quad \mathrm{V}_{\mathrm{B}}=-0.0311$
$\mathrm{V}_{\mathrm{C}}-\mathrm{V}_{\mathrm{B}}=4.1461 \quad$ Thus active mode.
$r_{e}:=\frac{V_{T}}{I_{E}} \quad r_{e}=9.7773$

$\mathrm{v}_{\mathrm{tb}}:=\mathrm{v}_{\mathrm{s}} \cdot \frac{\mathrm{R}_{\mathrm{P}}\left(\mathrm{R}_{1}, \mathrm{R}_{2}\right)}{\mathrm{R}_{\mathrm{S}}+\mathrm{R}_{\mathrm{P}}\left(\mathrm{R}_{1}, \mathrm{R}_{2}\right)} \quad \mathrm{v}_{\mathrm{tb}}=0.916$
$\mathrm{R}_{\mathrm{tb}}:=\mathrm{R}_{\mathrm{P}}\left(\mathrm{R}_{\mathrm{S}}, \mathrm{R}_{\mathrm{P}}\left(\mathrm{R}_{1}, \mathrm{R}_{2}\right)\right) \quad \mathrm{R}_{\mathrm{tb}}=4.5802 \cdot 10^{3}$
$\mathrm{R}_{\text {te }}:=\mathrm{R}_{\mathrm{P}}\left(\mathrm{R}_{\mathrm{E}}, \mathrm{R}_{3}\right) \quad \mathrm{R}_{\text {te }}=98.2456$

$$
\begin{aligned}
& r^{\prime} \mathrm{e}:=\frac{\mathrm{R}_{\mathrm{tb}}+\mathrm{r}_{\mathrm{x}}}{1+\beta}+\mathrm{r}_{\mathrm{e}} \quad \mathrm{r}^{\prime} \mathrm{e}^{=55.7788} \\
& \mathrm{R}_{\mathrm{tc}}:=\mathrm{R}_{\mathrm{P}}\left(\mathrm{R}_{\mathrm{C}}, \mathrm{R}_{\mathrm{L}}\right) \quad \mathrm{R}_{\mathrm{tc}}=3.007 \cdot 10^{3} \\
& \mathrm{r}_{\mathrm{ic}}:=\frac{\mathrm{r}_{0}+\mathrm{R}_{\mathrm{P}}\left(\mathrm{r}_{\mathrm{\prime}}, \mathrm{R}_{\mathrm{te}}\right)}{1-\frac{\alpha \cdot \mathrm{R}_{\mathrm{te}}}{\mathrm{r}^{\prime} \mathrm{e}^{+}+\mathrm{R}_{\mathrm{te}}}} \quad \quad \mathrm{r}_{\mathrm{ic}}=1.3577 \cdot 10^{5} \\
& i_{\text {csc }}:=\frac{v_{\text {tb }}}{r^{\prime}{ }_{e}+R_{P}\left(R_{\text {te }}, r_{0}\right)} \cdot\left(\alpha-\frac{R_{\text {te }}}{R_{\text {te }}+r_{0}}\right) \quad i_{\text {csc }}=5.8835 \cdot 10^{-3} \\
& \mathrm{v}_{\mathrm{o}}:=-\mathrm{i}_{\mathrm{csc}} \cdot \mathrm{R}_{\mathrm{P}}\left(\mathrm{R}_{\mathrm{C}}, \mathrm{R}_{\mathrm{P}}\left(\mathrm{r}_{\mathrm{ic}}, \mathrm{R}_{\mathrm{L}}\right)\right) \quad \mathrm{v}_{\mathrm{o}}=-17.3084
\end{aligned}
$$

$A_{\mathrm{v}}:=\mathrm{v}_{\mathrm{o}}$
$A_{V}=-17.3084$
This is the voltage gain.

Circuit for $\mathrm{r}_{\text {out }}$.


$$
\mathrm{r}_{\text {out }}:=\mathrm{R}_{\mathrm{P}}\left(\mathrm{R}_{\mathrm{C}}, \mathrm{r}_{\text {ic }}\right) \quad \mathrm{r}_{\text {out }}=4.168 \cdot 10^{3}
$$

Circuit for $\mathrm{r}_{\text {in }}$.

$r_{i b}:=r_{x}+(1+\beta) \cdot\left(r_{e}+R_{P}\left(R_{t e}, r_{0}+R_{t c}\right)\right)-\frac{\beta \cdot R_{t e} \cdot R_{t c}}{R_{t c}+r_{0}+R_{t e}}$
$r_{i b}=1.0253 \cdot 10^{4}$
$\mathrm{r}_{\text {in }}:=\mathrm{R}_{\mathrm{P}}\left(\mathrm{r}_{\mathrm{ib}}, \mathrm{R}_{\mathrm{P}}\left(\mathrm{R}_{1}, \mathrm{R}_{2}\right)\right) \quad \mathrm{r}_{\text {in }}=8.6309 \bullet 10^{3}$

Exact AC solution based on the pi model that is the subject of a homework problem.

$$
\begin{aligned}
& \mathrm{r}_{\pi}:=(1+\beta) \cdot \mathrm{r}_{\mathrm{e}} \quad \mathrm{R}^{\prime}:=\mathrm{R}_{\mathrm{tb}}+\mathrm{r}_{\mathrm{x}}+\mathrm{r}_{\pi} \\
& \mathrm{A}_{\mathrm{v}}:=-\frac{\frac{\beta}{\mathrm{R}^{\prime}}}{\frac{\frac{1}{\mathrm{r}_{0}}+\frac{\beta}{\mathrm{R}^{\prime}}}{\frac{1}{\mathrm{r}_{0}}+\frac{1}{\mathrm{R}_{\mathrm{tc}}}} \frac{\frac{1+\beta}{\mathrm{R}^{\prime}}}{\mathrm{R}^{\prime}}+\frac{1}{\mathrm{R}_{\mathrm{te}}}+\frac{1}{\mathrm{r}_{0}}}{\frac{1}{\mathrm{r}_{0}}}_{\frac{1}{\mathrm{r}_{0}}+\frac{\beta}{\mathrm{R}^{\prime}}}^{\frac{1+\beta}{\mathrm{R}^{\prime}}+\frac{1}{\mathrm{R}_{\mathrm{te}}}+\frac{1}{\mathrm{r}_{0}}} \quad \mathrm{~A}_{\mathrm{v}}=-18.895 \quad \text { This is } \mathrm{v}_{\mathrm{o}} / \mathrm{v}_{\mathrm{tb}} . \\
& \mathrm{v}_{\mathrm{o}}:=\mathrm{A}_{\mathrm{v}} \cdot \frac{\mathrm{R}_{\mathrm{P}}\left(\mathrm{R}_{1}, \mathrm{R}_{2}\right)}{\left.\mathrm{R}_{\mathrm{S}}+\mathrm{R}_{\mathrm{P}} \mathrm{R}_{1}, \mathrm{R}_{2}\right)}
\end{aligned} \quad \mathrm{G}_{\mathrm{mb}} \quad \mathrm{v}_{\mathrm{o}}=-17.3084 \quad \begin{aligned}
& \text { Same gain as calculated using } \mathrm{G}_{\mathrm{mb}} \\
& \text { and } \mathrm{r}_{\text {ic }} \text { above. }
\end{aligned}
$$

The following solution is based on the $\mathrm{r}_{0}$ approximations where $\mathrm{r}_{0}$ is neglected in calculating $\mathrm{i}_{\mathrm{csc}}$ but not neglected in calculating $\mathrm{r}_{\mathrm{ic}}$.

$$
\begin{aligned}
& \mathrm{G}_{\mathrm{mb}}:=\frac{\alpha}{\mathrm{r}_{\mathrm{e}}+\mathrm{R}_{\mathrm{te}}} \\
& \mathrm{i}_{\mathrm{csc}}:=\mathrm{v}_{\mathrm{tb}} \cdot \mathrm{G}_{\mathrm{mb}} \quad \quad \mathrm{i}_{\mathrm{csc}}=5.8878 \cdot 10^{-3}
\end{aligned}
$$

$$
\left.\mathrm{v}_{\mathrm{o}}:=-\mathrm{i} \mathrm{csc} \cdot \mathrm{R}_{\mathrm{P}}\left(\mathrm{R}_{\mathrm{C}}, \mathrm{R}_{\mathrm{P}} \mathrm{r}_{\mathrm{ic}}, \mathrm{R}_{\mathrm{L}}\right)\right) \mathrm{v}_{\mathrm{o}}=-17.3211
$$

$$
\begin{array}{lll}
\mathrm{A}_{\mathrm{v}}:=\mathrm{v}_{\mathrm{o}} & \mathrm{~A}_{\mathrm{v}}=-17.3211 & \begin{array}{l}
\text { This is the voltage gain. It is } 0.075 \% \text { lower } \\
\text { than the exact solutions found above. }
\end{array}
\end{array}
$$

$$
\mathrm{r}_{\text {out }}:=\mathrm{R}_{\mathrm{P}}\left(\mathrm{r}_{\text {ic }}, \mathrm{R}_{\mathrm{C}}\right) \quad \mathrm{r}_{\text {out }}=4.168 \cdot 10^{3}
$$

$$
r_{i b}:=r_{x}+(1+\beta) \cdot\left(r_{e}+R_{t e}\right) \quad r_{i b}=1.0822 \cdot 10^{4}
$$

$$
\mathrm{r}_{\text {in }}:=\mathrm{R}_{\mathrm{P}}\left(\mathrm{r}_{\mathrm{ib}}, \mathrm{R}_{\mathrm{P}}\left(\mathrm{R}_{1}, \mathrm{R}_{2}\right)\right) \quad \mathrm{r}_{\text {in }}=9.0305 \cdot 10^{3}
$$

Approximate Solution 1 using the equation $i_{c}=g_{m} \cdot\left(v_{b}-v_{e}\right)$

$$
\mathrm{i}_{\mathrm{c}}=\mathrm{g}_{\mathrm{m}} \cdot\left(\mathrm{v}_{\mathrm{b}}-\mathrm{v}_{\mathrm{e}}\right)
$$

Assume $r_{x}:=0$ and $r_{0}$ is infinity, i.e. an open circuit.

$$
\begin{aligned}
& g_{m}:=\frac{\alpha \cdot I_{E}}{V_{T}} \quad g_{m}=0.1013 \quad r_{i b}:=(1+\beta) \cdot\left(r_{e}+R_{P}\left(R_{E}, R_{3}\right)\right) \quad r_{i b}=1.0802 \cdot 10^{4} \\
& v_{b}:=v_{s} \cdot \frac{R_{P}\left(r_{i b}, R_{P}\left(R_{1}, R_{2}\right)\right)}{R_{S}+R_{P}\left(r_{i b}, R_{P}\left(R_{1}, R_{2}\right)\right)} \quad v_{b}=0.6433 \quad r_{\pi}:=(1+\beta) \cdot r_{e} \quad r_{\pi}=977.7264 \\
& r_{i b}:=r_{\pi}+(1+\beta) \cdot R_{t e} \quad r_{i b}=1.0802 \bullet 104
\end{aligned}
$$

$$
A_{V}:=\frac{-R_{P}\left(R_{1}, R_{2}\right)}{R_{S}+R_{P}\left(R_{1}, R_{2}\right)} \cdot \frac{r_{i b}}{R_{t b}+r_{i b}} \cdot \frac{g_{m} \cdot R_{t c}}{1+\frac{g_{m} \cdot R_{t e}}{\alpha}} \quad A_{V}=-17.7277
$$

$$
\mathrm{r}_{\text {in }}:=\mathrm{R}_{\mathrm{P}}\left(\mathrm{r}_{\mathrm{ib}}, \mathrm{R}_{\mathrm{P}}\left(\mathrm{R}_{1}, \mathrm{R}_{2}\right)\right) \quad \mathrm{r}_{\text {in }}=9.0166 \cdot 10^{3}
$$

$$
\mathrm{r}_{\text {out }}:=\mathrm{R}_{\mathrm{C}} \quad \mathrm{r}_{\text {out }}=4.3 \cdot 10^{3}
$$

Approximate Solution 2 using the equation $\mathrm{i}_{\mathrm{c}}=\beta \cdot \mathrm{i}_{\mathrm{b}}$

Again, assume $r_{x}:=0$ and $r_{0}$ is infinity, i.e. an open circuit.

$$
\mathrm{A}_{\mathrm{V}}:=\frac{-\mathrm{R}_{\mathrm{P}}\left(\mathrm{R}_{1}, \mathrm{R}_{2}\right)}{\left.\mathrm{R}_{\mathrm{S}}+\mathrm{R}_{\mathrm{P}} \mathrm{R}_{1}, \mathrm{R}_{2}\right)} \cdot \frac{\beta \cdot \mathrm{R}_{\mathrm{tc}}}{\mathrm{R}_{\mathrm{tb}}+\mathrm{r}_{\mathrm{ib}}} \quad \mathrm{~A}_{\mathrm{V}}=-17.7277
$$

This is the same as in Approximate Solution 1 and is simpler.

$$
\begin{aligned}
& \mathrm{r}_{\text {in }}:=\mathrm{R}_{\mathrm{P}}\left(\mathrm{r}_{\text {ib }}, \mathrm{R}_{\mathrm{P}}\left(\mathrm{R}_{1}, \mathrm{R}_{2}\right)\right) \quad \mathrm{r}_{\text {in }}=9.0166 \cdot 10^{3} \\
& \mathrm{r}_{\text {out }}:=\mathrm{R}_{\mathrm{C}} \quad \mathrm{r}_{\text {out }}=4.3 \cdot 10^{3}
\end{aligned}
$$

