Norton Collector Circuit

The Norton equivalent circuit seen looking into the collector can be used to solve for the response of the common-emitter and common-base stages. Fig. 1(a) shows the bjt with Thévenin sources connected to its base and emitter. With the collector grounded, the collector current is called the short-circuit output current or $i_{c(sc)}$. The current source in the Norton collector circuit has this value. To solve for this current, we use the simplified T model in Fig. 1(b). The current $i_{c(sc)}$ can be solved for by superposition of v_{tb} and v_{te}.

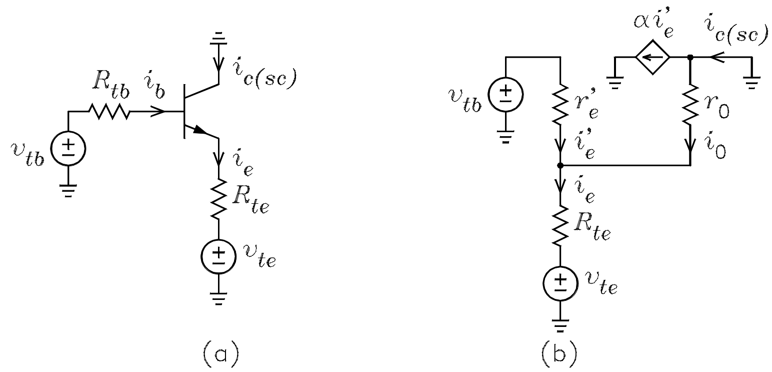

Figure 1: (a) BJT with Thevenin sources connected to the base and the emitter. (b) Simplified T model.

With v_{te} and $\alpha i'_e$ sources set to zero, it follows from Fig. 1(b) that

$$i_{c(sc)} = -\frac{v_{tb}}{r'_e + R_{te}\|r_0} \frac{R_{te}}{R_{te} + r_0} \qquad i'_e = \frac{v_{tb}}{r'_e + R_{te}\|r_0} \tag{1}$$

With the v_{tb} and $\alpha i'_e$ sources set to zero, we have

$$i_{c(sc)} = -\frac{v_{te}}{R_{te} + r'_e\|r_0} \frac{r'_e}{r_0 + r'_e} \qquad i'_e = -\frac{v_{te}}{R_{te} + r'_e\|r_0} \frac{r_0}{r_0 + r'_e} \tag{2}$$

With the v_{tb} and v_{te} sources set to zero, we have

$$i_{c(sc)} = \alpha i'_e \qquad i'_e = 0 \tag{3}$$

By the principle of superposition, these equations can be combined to obtain the total solution given by

$$\begin{aligned} i_{c(sc)} &= -\frac{v_{tb}}{r'_e + R_{te}\|r_0} \frac{R_{te}}{R_{te} + r_0} - \frac{v_{te}}{R_{te} + r'_e\|r_0} \frac{r'_e}{r'_e + r_0} + \alpha i'_e \\ &= \frac{v_{tb}}{r'_e + R_{te}\|r_0} \left(\alpha - \frac{R_{te}}{R_{te} + r_0} \right) - \frac{v_{te}}{R_{te} + r'_e\|r_0} \frac{\alpha r_0 + r'_e}{r_0 + r'_e} \end{aligned} \tag{4}$$

This equation is of the form

$$i_{c(sc)} = G_{mb} v_{tb} - G_{me} v_{te} \tag{5}$$

where

$$G_{mb} = \frac{1}{r'_e + R_{te} \| r_0} \left(\alpha - \frac{R_{te}}{r_0 + R_{te}} \right) = \frac{\alpha}{r'_e + R_{te} \| r_0} \frac{r_0 - R_{te}/\beta}{r_0 + R_{te}} \tag{6}$$

$$G_{me} = \frac{1}{R_{te} + r'_e \| r_0} \frac{\alpha r_0 + r'_e}{r_0 + r'_e} = \frac{\alpha}{R_{te} + r'_e \| r_0} \frac{r_0 + r'_e/\alpha}{r_0 + r'_e} \tag{7}$$

Figure 2(a) shows the simplified T model with $v_{tb} = v_{te} = 0$ and a test source connected to the collector. The resistance seen looking into the collector is given by $r_{ic} = v_t/i_c$. The resistor in the collector Norton equivalent circuit has this value. To solve for r_{ic}, we can write

$$i_c = \alpha i'_e + i_0 = -\alpha i_0 \frac{R_{te}}{r'_e + R_{te}} + i_0 = \frac{v_t}{r_0 + r'_e \| R_{te}} \left(1 - \frac{\alpha R_{te}}{r'_e + R_{te}} \right) \tag{8}$$

where current division has been used to express i'_e as a function of i_0. It follows that r_{ic} is given by

$$r_{ic} = \frac{v_t}{i_c} = \frac{r_0 + r'_e \| R_{te}}{1 - \alpha R_{te} / \left(r'_e + R_{te} \right)} \tag{9}$$

The Norton equivalent circuit seen looking into the collector is shown in Fig. 2(b).

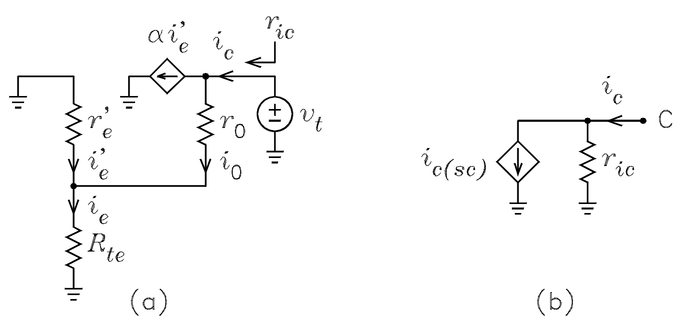

Figure 2: (a) Circuit for calculating r_{ic}. (b) Norton collector circuit.

For the case $r_0 \gg R_{te}$ and $r_0 \gg r'_e$, we can write

$$i_{c(sc)} = G_m \left(v_{tb} - v_{te} \right) \tag{10}$$

where

$$G_m = \frac{\alpha}{r'_e + R_{te}} \tag{11}$$

The value of $i_{c(sc)}$ calculated with this approximation is simply the value of $\alpha i'_e$, where i'_e is calculated with r_0 considered to be an open circuit. The term "r_0 approximation" is used in the following when r_0 is neglected in calculating $i_{c(sc)}$ but not neglected in calculating r_{ic}.