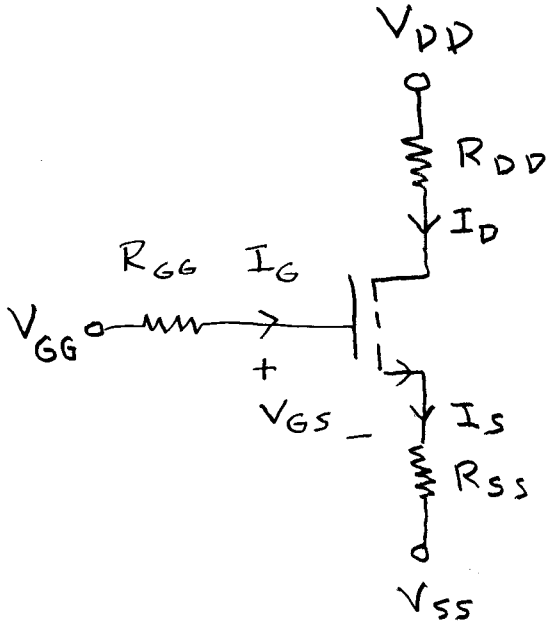
The MOSFET Bias Equation

Replace the dc circuits looking out of the drain, gate, and source leads with Thévenin equivalents



The loop equation between the V_{GG} and V_{SS} nodes is

$$V_{GG} - V_{SS} = I_G R_{GG} + V_{GS} + I_S R_{SS}$$

But $I_G = 0$ and $I_S = I_D$

$$\Rightarrow V_{GG} - V_{SS} = V_{GS} + I_D R_{SS}$$

To solve for V_{GS}, we use the equation

$$I_D = K(V_{GS} - V_{TO})^2$$

$$\Rightarrow \quad V_{GS} = \sqrt{\frac{I_D}{K}} + V_{TO}$$

$$\Rightarrow \quad V_{GG} - V_{SS} = \sqrt{\frac{I_D}{K}} + V_{TO} + I_D R_{SS}$$

$$\Rightarrow \quad I_D R_{SS} + \sqrt{\frac{I_D}{K}} + V_{TO} - (V_{GG} - V_{SS}) = 0$$

Let $I_D = x^2$ and $V_1 = V_{GG} - V_{SS} - V_{TO}$

$$\Rightarrow \quad x^2 R_{SS} + \frac{1}{\sqrt{K}} x - V_1 = 0$$

This is a quadratic equation with the solution

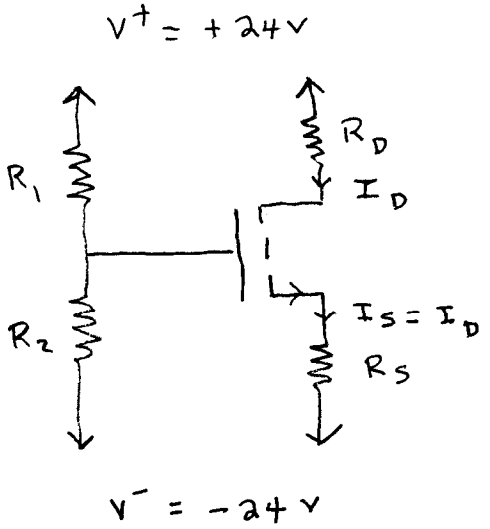
$$x = \frac{1}{2R_{SS}} \left[-\frac{1}{\sqrt{K}} + \sqrt{\frac{1}{K} + 4V_1 R_{SS}} \right]$$

$$= \frac{1}{2R_{SS}\sqrt{K}} \left[\sqrt{1 + 4KV_1 R_{SS}} - 1 \right]$$

$$= \sqrt{I_D}$$

$$\Rightarrow \quad I_D = \frac{1}{4KR_{SS}^2}\left[\sqrt{1+4KV_1R_{SS}} - 1\right]^2$$

Example 1



$R_1 = 5\ M\Omega$

$R_2 = 2\ M\Omega$

$R_D = 10\ k\Omega$

$R_S = 3\ k\Omega$

$K = 0.001\ A/V^2$

$V_{TO} = 1.75\ V$

Solve for I_D

$$V_{GG} = V^+ \frac{R_2}{R_1+R_2} + V^- \frac{R_1}{R_1+R_2} = -16\ V$$

$$R_{GG} = R_1 \| R_2 = 1.43\ M\Omega$$

$$V_{SS} = V^- = -24\ V$$

$$R_{SS} = R_S = 3\ k\Omega$$

$$V_1 = V_{GG} - V_{SS} - V_{TO} = 6.25\ V$$

$$I_D = \frac{1}{4KR_S}\left[\sqrt{1+4KV_1R_S} - 1\right]^2 = 1.655\ mA$$

For the MOSFET to be in the active mode, $V_{DS} > V_{GS} - V_{TO}$

$$V_{GS} = V_G - V_S = V_{GG} - (V^- + I_D R_S)$$

$$= 3.036 \text{ V}$$

$$V_{GS} - V_{TO} = 1.286 \text{ V}$$

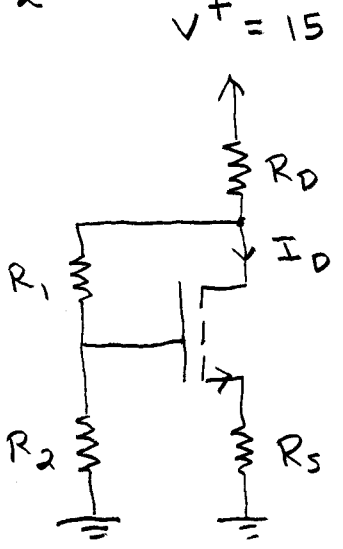
$$V_{DS} = (V^+ - I_D R_D) - (V^- + I_D R_S)$$

$$= V^+ - V^- - I_D(R_D + R_S)$$

$$= 26.49 \text{ V}$$

$\Rightarrow V_{DS} > V_{GS} - V_{TO} \Rightarrow$ Active Mode

Example 2



$R_D = 2.5 \text{ k}\Omega$

$R_S = 1 \text{ k}\Omega$

$R_1 = 1 \text{ M}\Omega$

$R_2 = 1 \text{ M}\Omega$

$K = 0.001 \text{ A/V}^2$

$V_{TO} = 1.5 \text{ V}$

$$V_{GG} = V^{+} \frac{R_2}{R_D + R_1 + R_2} - I_D \frac{R_D}{R_D + R_1 + R_2}$$

$$= 7.491 - 1199\ I_D$$

$$R_{SS} = R_S = 1\,k\Omega \qquad V_{SS} = 0$$

From the derivation of the bias equation, we have

$$I_D R_{SS} + \sqrt{\frac{I_D}{K}} + V_{TO} - (V_{GG} - V_{SS}) = 0$$

$$\Rightarrow \quad 2199\ I_D + 31.62\sqrt{I_D} - 5.991 = 0$$

Let $I_D = x^2$

$$\Rightarrow \quad 2199\ x^2 + 31.62\ x - 5.991 = 0$$

$$\Rightarrow \quad x = 0.04550$$

$$\Rightarrow \quad I_D = x^2 = 2.07\ mA$$

For the MOSFET to be in the active mode $V_{DS} > V_{GS} - V_{TO}$.

$$V_D = V^+ \frac{R_1 + R_2}{R_D + R_1 + R_2} - I_D R_D \| (R_1 + R_2)$$

$$= 10.02 \text{ V}$$

$$V_S = I_D R_S = 2.07 \text{ V}$$

$$V_{DS} = V_D - V_S = 7.95 \text{ V}$$

$$V_{GS} = \sqrt{\frac{I_D}{K}} + 1.5 = 3.026 \text{ V}$$

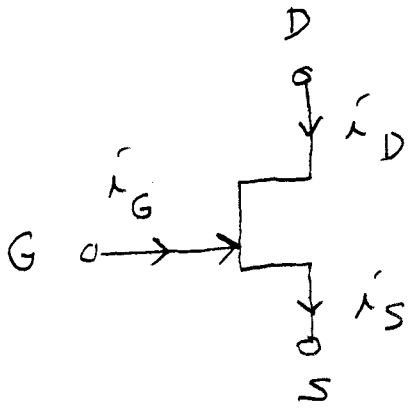
$$V_{GS} - V_{TO} = 1.526 \text{ V}$$

$\Rightarrow V_{DS} > V_{GS} - V_{TO} \Rightarrow$ Active mode

The Junction Field Effect Transistor or The JFET

The JFET differs from the MOSFET in that the gate is separated from the channel by

a pn junction instead of an insulating or dielectric layer. The pn junction must be reverse biased, otherwise gate current will flow. The circuit symbol for the n-channel JFET is



The drain current is given by

$$i_D = \beta_0 (1 + \lambda v_{DS})(v_{GS} - V_{T0})^2$$

If we let

$$\beta = \beta_0 (1 + \lambda v_{DS})$$

the drain current can be written

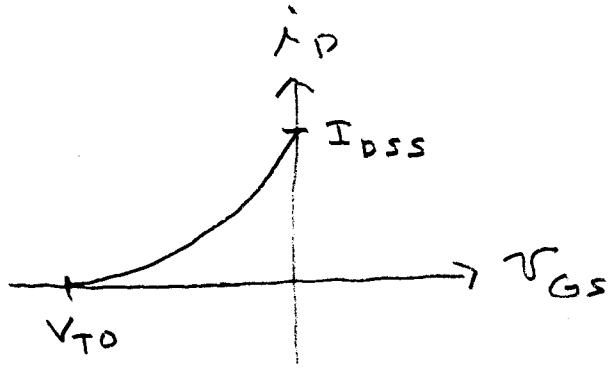
$$i_D = \beta (v_{GS} - V_{TO})^2$$

β is the transconductance parameter. Its zero bias value is β_0. V_{TO} is the threshold voltage.

For the JFET to be in the active mode, we must have

$$v_{DS} > v_{GS} - V_{TO}$$

The JFET is a depletion mode device, Thus V_{TO} is negative. The transfer characteristics are a plot of i_D versus v_{GS} for v_{DS} = constant.



For $v_{GS} > 0$, gate current flows. Thus v_{GS} is constrained to be in the range $V_{TO} \leq v_{GS} \leq 0$.

The value of i_D for $v_{GS} = 0$ is denoted I_{DSS}. This is called the drain to source saturation current.

$$I_{DSS} = \beta V_{TO}^2$$

$$\Rightarrow \beta = \frac{I_{DSS}}{V_{TO}^2}$$

$$\Rightarrow i_D = \frac{I_{DSS}}{V_{TO}^2} (v_{GS} - V_{TO})^2$$

$$= I_{DSS} \left(\frac{v_{GS}}{V_{TO}} - 1 \right)^2$$

$$= I_{DSS} \left(1 - \frac{v_{GS}}{V_{TO}} \right)^2$$

The latter equation is a commonly used equation for i_D. Note that I_{DSS} varies with v_{DS}

according to the equation

$$I_{DSS} = \beta V_{TO}^2 = \beta_o (1 + \lambda V_{DS}) V_{TO}^2$$

If K in the MOSFET equations is replaced with β, we obtain the corresponding JFET equation. The hybrid-π and T models are the same. For the JFET

$$g_m = 2\sqrt{\beta I_D} = \frac{-2}{V_{TO}} \sqrt{I_D I_{DSS}}$$

$$r_o = \frac{\frac{1}{\lambda} + V_{DS}}{I_D}$$

$$r_\pi = \frac{1}{g_m}$$

Note that $g_m > 0$ because $V_{TO} < 0$.

The JFET bias equation is the same as the MOSFET bias equation with K replaced with β.