

What Value of Q_{TC} Minimizes the Lower Cutoff Frequency of a Closed-Box Loudspeaker System?

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The Butterworth second-order alignment is usually considered to be the closed-box alignment that minimizes the lower system cutoff frequency. This is true if the enclosure and its filling introduce no losses. In this case, the mechanical quality factor of the system changes by the same factor that the electrical quality factor changes when the box volume is varied. For an unfilled enclosure, the box losses usually limit the mechanical quality factor to a value in the range $7 \leq Q_{MC} \leq 10$. For a filled enclosure, the mechanical quality factor is lower, usually in the range $2 \leq Q_{MC} \leq 5$.

In the design of a closed-box system, a value of Q_{MC} must be assumed a priori in order to calculate the box volume. Once the system is assembled, its value can be measured to check or verify the assumed value. If desired, the filling can be adjusted to obtain the assumed value. Otherwise, the box volume must be changed. To calculate the box volume, the desired value of the total quality factor Q_{TC} must be specified. For a Butterworth alignment, the value is $Q_{TC} = 1/\sqrt{2}$. To minimize the lower cutoff frequency, the value of Q_{TC} must be lower than $1/\sqrt{2}$ when box and filling losses cannot be neglected. To determine the value of Q_{TC} as a function of the assumed value of Q_{MC} , a Mathcad simulation has been used to plot the lower cutoff frequency as a function of Q_{TC} for seven values of Q_{MC} in the range $2 \leq Q_{MC} \leq 5$. A polynomial curve fit to the data yielded the following equation for Q_{TC} as a function of Q_{MC} :

$$Q_{TC} = 0.1994 + 0.2822Q_{MC} - 0.06219Q_{MC}^2 + 0.0048Q_{MC}^3$$

The figure below shows a plot of the above equation for Q_{TC} as a function of Q_{MC} . The points which were used in the polynomial curve fit are shown as circles. A typical value for Q_{MC} for a filled box might be $Q_{MC} = 3.5$. For this value, the equation gives $Q_{TC} = 0.63$. This is lower than the the value for a Butterworth alignment by a factor of 0.89 or by 11%. Because the value of Q_{TC} which minimizes the lower -3 dB cutoff frequency is lower than the value for a Butterworth alignment, it follows that the box size is larger. It can be concluded that the optimum box volume which minimizes the lower -3 dB cutoff frequency is larger than the volume for a Butterworth alignment if enclosure losses cannot be neglected.

