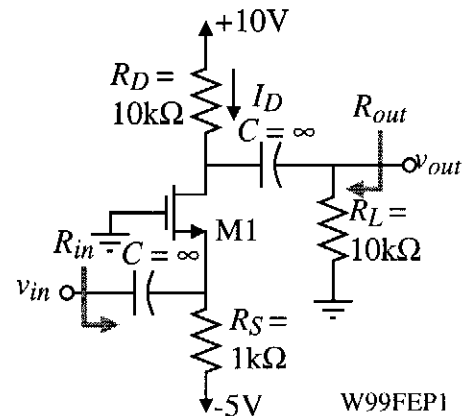## Homework Assignment No. 7 - Solutions

1.) If $K = 0.1\text{mA/V}^2$ and $V_t = 1\text{V}$ for the n-channel MOSFET shown, find the dc value of I_D, V_{GS}, and V_{DS}. If the dc value of $I_D = 1\text{mA}$ (not necessarily the answer above) find the value of the small-signal input resistance, R_{in}, voltage gain, v_{out}/v_{in}, and output resistance, R_{out}.
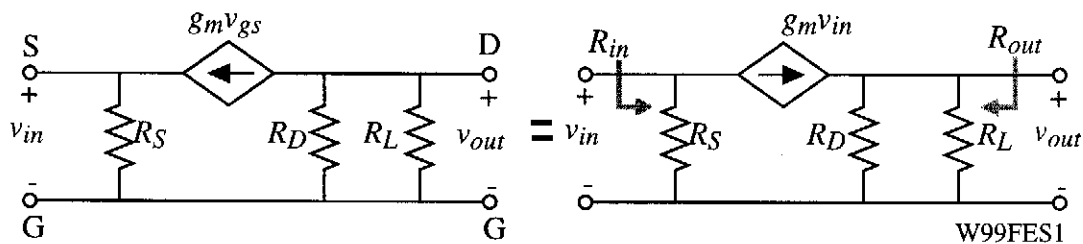

W99FEP1

Solution

(a.) The key equations are $I_D = K(V_{GS}-V_t)^2$ and $5 = V_{GS}+I_DR_S$. Substituting the first into the second gives, $5 = V_{GS} +0.1(V_{GS}-1)^2 \rightarrow 5 = V_{GS} + 0.1V_{GS}^2 - 0.2V_{GS} + 0.1$

or $\quad V_{GS}^2 + 8V_{GS} + 49 = 0 \rightarrow V_{GS} = -4\pm\frac{1}{2}\sqrt{64+4\cdot49} = -4\pm8.0623 = 4.0623\text{V}$

$I_D = 0.1\text{mA}(4.0623-1)^2 = 0.9377\text{mA}$ and $V_{DS} = 15\text{V} - I_D(11\text{k}\Omega) = 4.68\text{V}$

$\therefore \boxed{V_{GS} = 4.0623\text{V}, I_D = 0.9377\text{mA and } V_{DS} = 4.68\text{V}}$

(b.) The small signal model for this amplifier is shown below. r_{ds} has been neglected since λ was not given. The small signal parameter g_m is: $g_m = 2\sqrt{KI_D} = 0.632\text{mA/V}$

W99FES1

The voltage gain is found as,

$$v_{out} = +g_m(R_D\|R_L)v_{in} \rightarrow \frac{v_{out}}{v_{in}} = g_m(R_D\|R_L) = 0.632\cdot5 = 3.16\text{V/V}$$

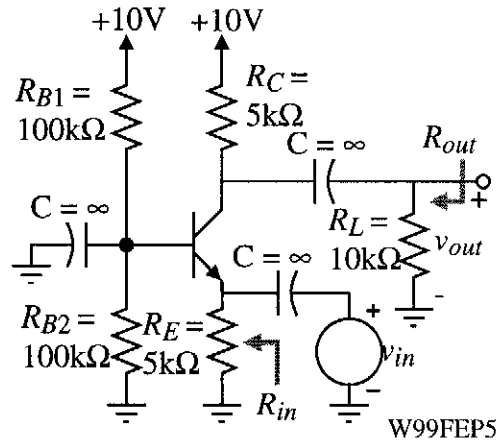$\therefore \boxed{\dfrac{v_{out}}{v_{in}} = 3.16\text{V/V}}$

The output resistance is simply $R_D\|R_S$ and is $\boxed{R_{out} = 5\text{k}\Omega}$

The input resistance is found as,

$$i_{in} = \frac{v_{in}}{R_S} + g_mv_{in} \quad \rightarrow \quad R_{in} = \frac{v_{in}}{i_{in}} = \frac{1}{R_S+\dfrac{1}{g_m}} = \frac{1}{g_m}\|R_S = \frac{1\text{k}\Omega}{1+0.632} = 613\Omega$$

$\therefore \boxed{R_{in} = 613\Omega}$

2.) (a.) If ß = 100 and V_T = 25mV of the NPN transistor shown, solve for the dc value of collector current. (b.) If this collector current is 1mA (so if you found I_C incorrectly, you will not be penalized on the rest of the problem) find the small signal values of the input resistance, R_{in}, output resistance, R_{out}, and voltage gain, v_{out}/v_{in}.
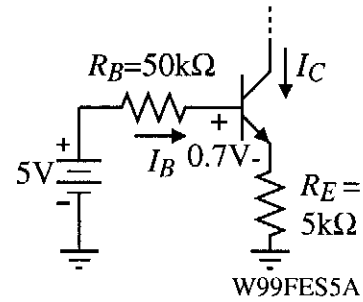
Solution

(a.) The circuit used to calculate the collector current is shown. The base current is

$$I_B = \frac{5 - 0.7}{50k\Omega + 101 \cdot 5k\Omega} = 7.75\mu A$$

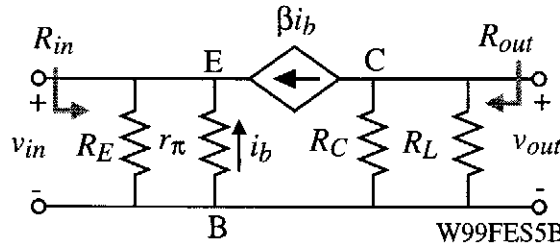$$\therefore \quad \boxed{I_C = 100 \cdot I_B = 0.75mA}$$

(b.) The small signal parameters are

$g_m = \dfrac{1mA}{25mV} = 0.04mA/V$ and $r_\pi = 101 \cdot 25\Omega = 2.525k\Omega$.

The small-signal model for the above circuit is given as

$$i_{in} = \frac{v_{in}}{R_E} - (1+\beta)i_b = \frac{v_{in}}{R_E} + \frac{(1+\beta)v_{in}}{r_\pi} \quad \rightarrow \quad \boxed{R_{in} = \frac{v_{in}}{i_{in}} = R_E \| \frac{1}{g_m} = 24.87\Omega}$$

$$v_{out} = -\beta i_b(R_C\|R_L) = -\beta(R_C\|R_L)\frac{-v_{in}}{r_\pi} \quad \rightarrow \quad \boxed{\frac{v_{out}}{v_{in}} = \frac{100 \cdot 3.33}{2.525} = 132V/V}$$

Finally, $\boxed{R_{out} = R_C\|R_L = 3.33k\Omega}$

14.29

(a) $r_\pi = \dfrac{100(0.025\,V)}{12.5\mu A} = 200\,k\Omega$ | $g_m = 40(12.5\mu A) = 0.5\,mS$

$A_{Vth} = \dfrac{g_m R_L}{1 + g_m R_{th}} = \dfrac{0.5\,mS(100\,k\Omega)}{1 + 0.5\,mS(50\,\Omega)} = 48.8$ | $R_{IN} = \dfrac{r_\pi}{\beta_o + 1} = 1.98\ k\Omega$

$R_{OUT} = r_o\left(1 + g_m R_{th}\right) = \dfrac{60\,V}{12.5\mu A}\left[1 + 0.5\,mS(50\,\Omega)\right] = 4.92\ M\Omega$ | $A_{Ith} = \alpha_o = 0.990$

(b) $A_{Vth} = \dfrac{0.5\,mS(100\,k\Omega)}{1 + 0.5\,mS(2.2\,k\Omega)} = 23.8$ | $R_{IN} = 1.98\ k\Omega$

$R_{OUT} = \dfrac{60\,V}{12.5\mu A}\left[1 + 0.5\,mS(2.2\,k\Omega)\right] = 10.1\ M\Omega$

14.31

$V_{SG} = 12 - (33\,k\Omega)I_{SD}$ | $V_{SG} = 12 - \dfrac{\left(3.3x10^4\right)\left(2x10^{-4}\right)}{2}\left(V_{SG} - 1\right)^2$

$V_{SG} = 2.68\,V$ & $I_{SD} = \dfrac{\left(2x10^{-4}\right)}{2}\left(V_{SG} - 1\right)^2 = 282\mu A$

$V_{SD} = 24 - I_{SD}(33\,k\Omega + 24\,k\Omega) = 7.93\ V$ - Saturation region operation is correct.

$g_m = \sqrt{2\left(2x10^{-4}\right)\left(2.82x10^{-4}\right)} = 3.36x10^{-4}\,S$ | $\mathbf{v}_{th} = \dfrac{33\,k\Omega}{0.5\,k\Omega + 33\,k\Omega}\mathbf{v}_s = 0.985\mathbf{v}_s$

$R_{th} = 0.5\,k\Omega\|33\,k\Omega = 493\,\Omega$

$R_L = 24\,k\Omega\|100\,k\Omega = 19.4\,k\Omega$ | $A_V = 0.985\dfrac{3.36x10^{-4}\,S(19.4\,k\Omega)}{1 + 3.36x10^{-4}\,S(493\,\Omega)} = 5.51$

$A_I = \dfrac{33\,k\Omega}{33\,k\Omega + \dfrac{1}{g_m}}\dfrac{24\,k\Omega}{24\,k\Omega + 100\,k\Omega} = 0.178$ | $R_{IN} = 33\,k\Omega\left\|\dfrac{1}{g_m}\right. = 2.73\,k\Omega$ | $R_{OUT} = R_D = 24\,k\Omega$

$\mathbf{v}_{sg} = \mathbf{v}_s\dfrac{R_{IN}}{R_S + R_{IN}} \le 0.2(V_{SG} - 1)$ | $\mathbf{v}_s\dfrac{2.73\,k\Omega}{0.5\,k\Omega + 2.73\,k\Omega} \le 0.2(2.68 - 1) \to \mathbf{v}_s \le 0.398\ V$

14.33

$I_B = \dfrac{(9 - 0.7)V}{100\,k\Omega + (50 + 1)82\,k\Omega} = 1.94\mu A$ | $I_C = 96.9\ \mu A$

$V_{CE} = 18 - 82000I_E - 39000I_C = 6.12\ V$ | Forward - active region is correct.

$g_m = 40I_C = 3.88\,mS$ | $r_\pi = \dfrac{\beta_o}{g_m} = 12.9\,k\Omega$ | $r_o = \dfrac{(50 + 6.12)V}{96.9\mu A} = 579\,k\Omega$ - neglected

$\mathbf{v}_{th} = \mathbf{v}_s\dfrac{82\,k\Omega}{0.5\,k\Omega + 82\,k\Omega} = 0.994\mathbf{v}_s$ | $R_{th} = 0.5\,k\Omega\|82\,k\Omega = 497\,\Omega$ | $R_L = 39\,k\Omega\|100\,k\Omega = 28.1\,k\Omega$

$A_V = 0.994\dfrac{50(28.1\,k\Omega)}{12.9\,k\Omega + 51(0.497\,k\Omega)} = 36.5$ | $R_{IN} = 82\,k\Omega\left\|\dfrac{r_\pi}{\beta_o + 1}\right. = 253\,\Omega$

$A_I = A_V\dfrac{R_S + R_{IN}}{R_3} = 36.5\dfrac{500\,\Omega + 253\,\Omega}{100\,k\Omega} = 0.275$ | $R_{OUT} = R_C = 39.0\ k\Omega$

$\mathbf{v}_{eb} = \mathbf{v}_s\dfrac{R_{IN}}{R_S + R_{IN}} \le 5.00\,mV$ | $0.336\mathbf{v}_s \le 5.00\,mV$ | $\mathbf{v}_s \le 14.9\ mV$