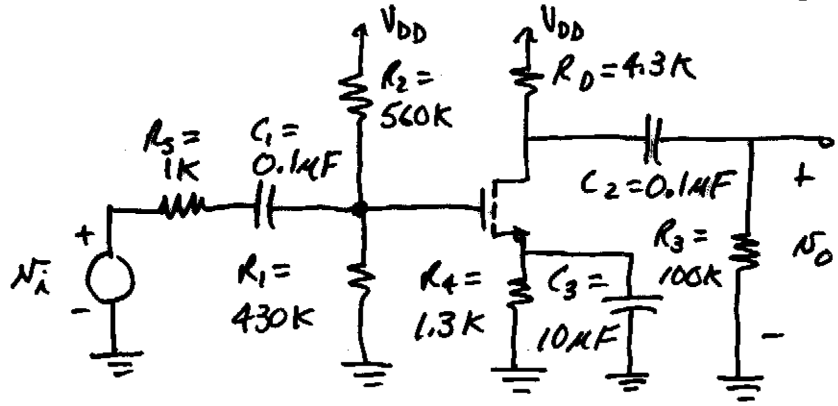
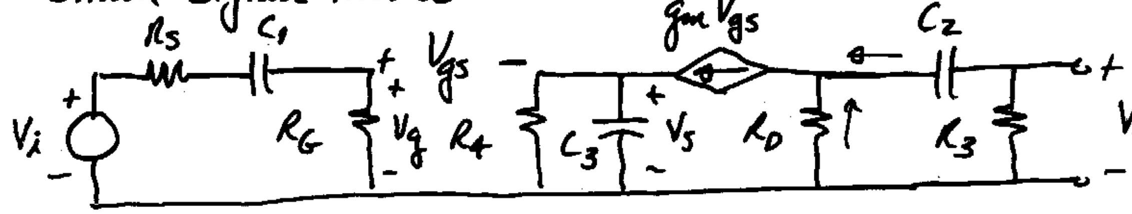
Example of Frequency Analysis using a MOSFET



$g_m = 1.23\,mS$

$r_o \approx \infty$

Small-signal model -



$R_G = R_1 \| R_2 = 243K$

1.) Direct analysis of low frequency poles and zeros.

$$\frac{V_o(s)}{V_i(s)} = \left(\frac{V_o}{V_{gs}}\right)\left(\frac{V_{gs}}{V_g}\right)\left(\frac{V_g}{V_i}\right)$$

$$V_o = -g_m V_{gs}\left(\frac{R_D}{R_D + R_3 + \frac{1}{sC_2}}\right)R_3 \rightarrow \frac{V_o}{V_{gs}} = \left(\frac{-g_m R_D R_3}{R_D + R_3}\right)\left(\frac{s}{s + \frac{1}{C_2(R_3 + R_D)}}\right)$$

$$V_{gs} = V_g - V_s = V_g - g_m V_{gs}\left(\frac{R_4 \frac{1}{sC_3}}{R_4 + \frac{1}{sC_3}}\right) = V_g - g_m R_4 V_{gs}\left(\frac{\frac{1}{R_4C_3}}{s + \frac{1}{R_4C_3}}\right)$$

$$V_{gs}\left[1 + g_m R_4\left(\frac{\frac{1}{R_4C_3}}{s + \frac{1}{R_4C_3}}\right)\right] = V_g$$

$$\frac{V_{gs}}{V_g} = \frac{s + \frac{1}{R_4C_3}}{s + \frac{1}{R_4C_3} + g_m R_4 \frac{1}{R_4C_3}} = \frac{s + \frac{1}{R_4C_3}}{s + \frac{(1 + g_m R_4)}{R_4C_3}}$$

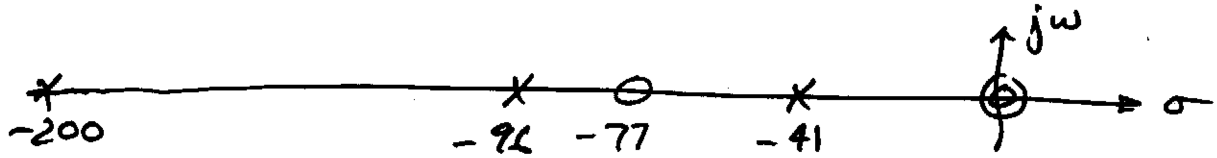
$\uparrow R_4 \| \frac{1}{g_m}$

$$\frac{V_g}{V_i} = \frac{R_G}{R_s + R_G + \frac{1}{sC_1}} = \left(\frac{R_G}{R_s + R_G}\right)\left(\frac{s}{s + \frac{1}{C_1(R_s + R_G)}}\right)$$

$\therefore \quad \omega_{p1} = -\frac{1}{C_1(R_S + R_G)} = \frac{-1}{(0.1\mu F)(244k)} = -41 \text{ rad/s}$

$\omega_{p2} = -\frac{1}{C_2(R_3 + R_D)} = \frac{-1}{(0.1\mu F)(104.3k)} = -96 \text{ rad/s}$

$\omega_{p3} = -76.9[1 + 1.23(1.3)] = -200 \text{ rad/s}$ $\quad z = \frac{-1}{R_4 C_3} = \frac{-1}{(4.3k)(10\mu F)} = -76.7 \text{ rad/s}$

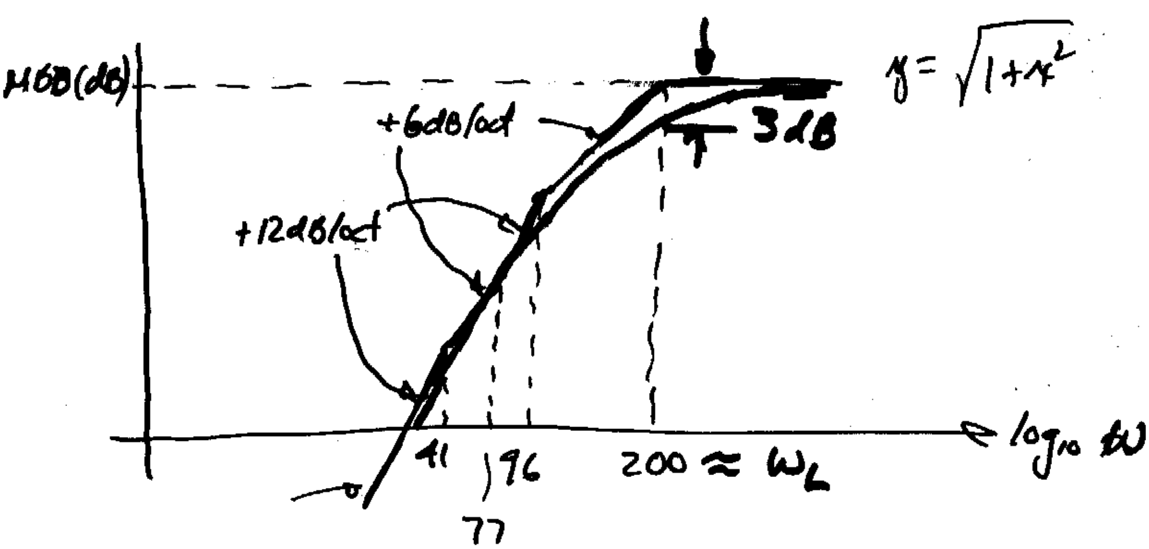


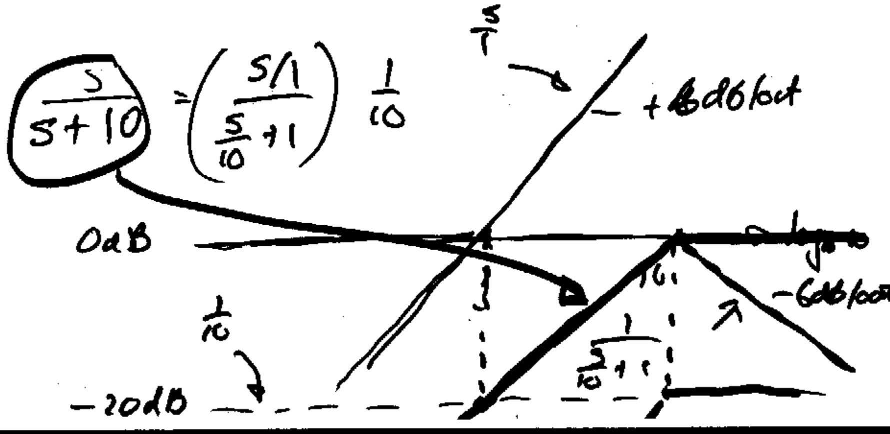
$f_L = ?$

1.) Dominant root $\rightarrow$ $\omega_L = 200$ rad/s $\rightarrow f_L = \frac{200}{6.28} \approx \underline{30 \text{ Hz}}$

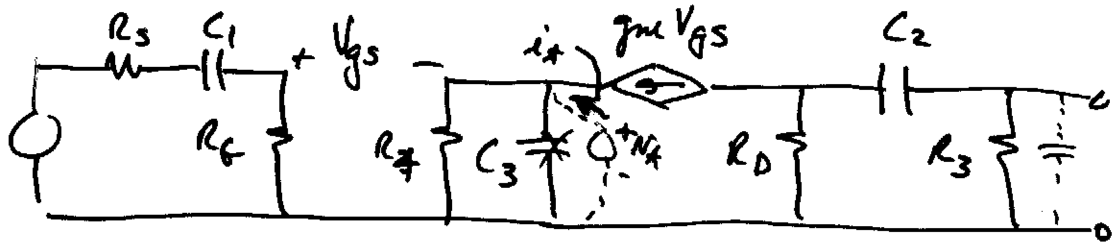
2.) $f_L = \frac{1}{2\pi}\sqrt{200^2 + 41^2 + 96^2 - 2(76.9^2)} = \underline{31.5 \text{ Hz}}$

3.) Bode plot





Find ω_L by s.c. time constants –



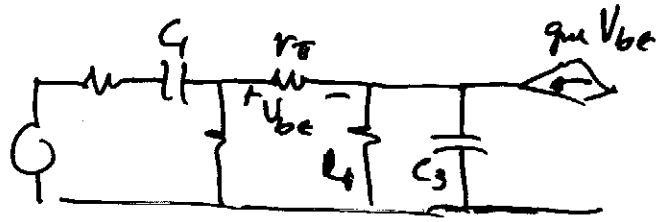
$\omega_L \simeq \sum \frac{1}{R_{is} C_i}$ $R_{1s} = R_s + R_G = 243K$ $R_{2s} = R_D + R_3 = 104.3K$

$R_{3s} = \frac{1}{g_m} \| R_4 = \frac{v_x}{i_x} = \frac{v_x}{i_x} = 0.5K$

$\omega_L = \frac{1}{(244K)(0.1\mu F)} + \frac{1}{(104.3K)(0.1\mu F)} + \frac{1}{0.5K(10\mu F)} = (0.04 + 0.096 + 0.2)k$

$= 337 \text{ rad/s} \rightarrow f_L \simeq \underline{\underline{53.6 Hz}}$ (ignores zeros)

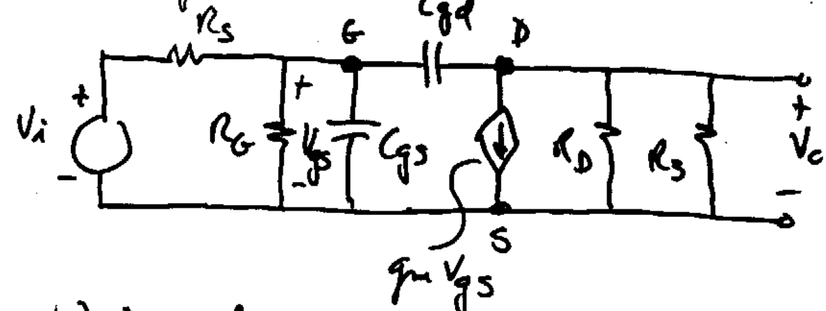
BJT



$s^2 + A_1 s + A_0 \longrightarrow p_1 \ p_2$

<u>High Frequency Response</u>

Small-signal model –

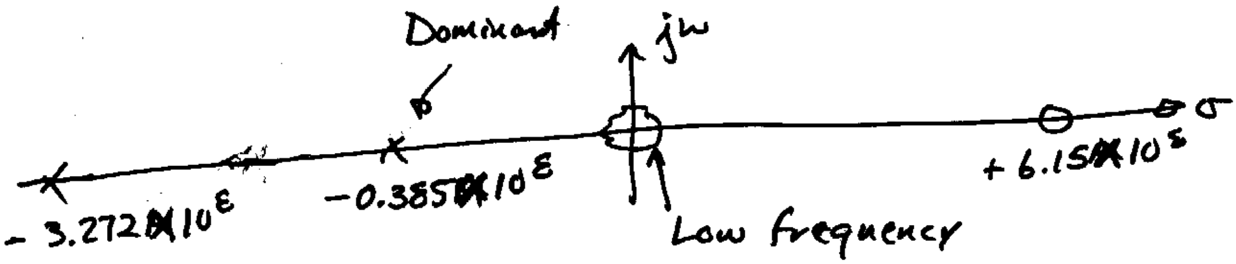


$C_{gd} = 2pF$
$C_{gs} = 10pF$

1.) Direct analysis
Two sheets later –

$$\frac{V_o}{V_i} = \frac{-10^{-3}\left[1.23\times10^{-3} - 2\times10^{-12} s\right]}{20\times10^{-24}\left[s^2 + 3.628\times10^{8} + 1.165\times10^{16}\right]}$$

$p_1 = -0.385\times10^{8}$ rads/sec , $p_2 = -3.272\times10^{8}$ rad/s
$z_1 = +6.15\times10^{8}$ rad/sec



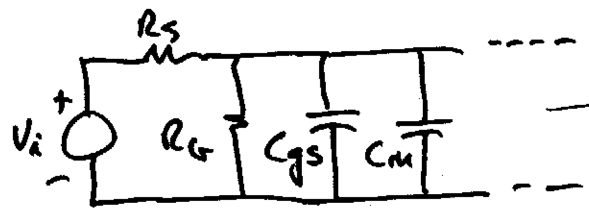
$\omega_H \approx 385\ M\ rad/s \rightarrow \underline{5.67 MHz}$

Miller multiplication -

Assume that $\frac{1}{\omega C_{gd}} >> R_D || R_3$

$C_M = C_{gd}(1-K) \qquad K = \frac{V_o}{V_{gs}} \hat{=} -g_m R_D||R_3 = -5.07 \frac{V}{V}$

$C_M = 2pF(6.07) = 12pF$



$\omega_H = \frac{1}{(R_s||R_G)(C_{gs}+C_M)} = 45.3 \times 10^6 \frac{rad}{s}$

$f_H = \underline{7.22 MHz}$

O.C. Time Constant

$R_{cgs} = R_s||R_G = 0.996k \qquad R_{cgds} = (R_s||R_G)\left[1+g_m(R_D||R_3)\right] + R_D||R_3$

$= 10.168k$

$\omega_H = \frac{1}{(0.996K)10pF + 10.168K(2pF)} = 33.33 \times 10^6\ rad/s$

$f_H \approx \underline{5.3MHz}$

$\omega_T = \frac{g_m}{C_{gs}+C_{gd}}$