LETTER

Robust Joint Linear Precoding for AF MIMO Relay Broadcast Systems with Limited Feedback

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SUMMARY This letter proposes a robust joint linear precoding scheme based on the minimum mean squared error (MMSE) criterion for amplify-and-forward (AF) multiple-input multiple-output (MIMO) relay broadcast systems with limited feedback, where only the quantized channel direction information (CDI) of the forward channel is available for the base station (BS) and the relay station (RS). The proposed scheme employs an iterative algorithm which alternately optimizes the BS and RS precoders to jointly minimize the expected MSE conditioned on the quantized CDI.

key words: precoding, MIMO relay, MMSE, limited feedback

1. Introduction

In wireless cellular networks, relays can be deployed to extend network coverage and combat shadowing, and thus have attracted great attention from both industrial and academic areas. Generally, there are two major relaying protocols, decode-and-forward (DF) and amplify-and-forward (AF) [1]. For the DF relay, it decodes the received signal and forward the reencoded information, while for the AF relay, only linear processing is preformed to amplify and forward the received signal. Obviously, the AF relay is easier to implement and causes lower latency.

Recently, MIMO relay systems, where both the BS and the RS are equipped with multiple antennas to incorporate multiple-input multiple-output (MIMO) technologies, have been intensively studied. In a single-user scenario, the transceiver design problem, i.e. joint signal processing at the relay and destination, was addressed in [2] to maximize mutual information and in [3] to minimize the mean squared error of detected symbols at the destination, while in a multiuser system, joint linear precoding at the BS and RS was studied in [4]. However, all the aforementioned works have assumed perfect channel state information at transmitters (CSIT), which is not quite realistic in practice.

In this letter, we focus on the MIMO relay broadcast systems with limited feedback, where only the quantized channel direction information (CDI) [6] of the forward channel is available for the BS and RS. In [5], the authors investigated the robust precoding design, which took the channel quantization error into account, but only the RS precoding was considered therein. We further study the robust joint precoding design at the BS and RS based on the minimum mean squared error (MMSE) criterion. We propose an iterative algorithm which alternately optimizes the BS and RS precoders to jointly minimize the expected MSE conditioned on quantized CDI.

Notation: We use boldface uppercase letters and boldface lowercase letters to denote matrices and vectors, respectively. The notations \( E[\cdot], \text{tr}(\cdot), (\cdot)^H, (\cdot)^T, (\cdot)^{-1}, \|\cdot\|_F, \|\cdot\|_{\text{Frobenius}}, \|\cdot\|_{\text{Euclidean}}, \|\cdot\|_{\text{trace}} \) denote expectation, trace of a matrix, conjugate transpose, transpose, inverse, Frobenius norm, Euclidian norm, and real part of the argument, respectively. \( I_K \) stands for a \( K \times K \) identity matrix and \( \text{diag}(e_1, \ldots, e_K) \) denotes a diagonal matrix with diagonal elements \( e_1, \ldots, e_K \).

2. System Model

We consider a limited feedback AF MIMO relay broadcast system as shown in Fig. 1, where the BS and RS are equipped with \( M \) and \( N \) antennas, respectively, and \( K \) single-antenna users are served simultaneously. The transmission between the BS and users is assisted by the \( N \)-antenna RS since there is no direct link due to distance. In this letter, we consider only the half-duplex AF relaying protocol [1]. We assume that all channels experience quasi-static flat fading which remain unchanged during the two time slots required by this relaying protocol for one complete transmission. To support \( K \) independent data streams, it requires that \( M, N \geq K \). We assume \( M = K \) for simplicity. However, our results can be easily extended to the case \( M > K \) with antenna selection performed at the BS.

In the first time slot, the BS transmits the signal \( \mathbf{s} = \mathbf{P} \mathbf{x} \) to the RS, where \( \mathbf{P} \in \mathbb{C}^{M \times K} \) is the precoding matrix and \( \mathbf{x} = [x_1, x_2, \ldots, x_K]^T \) denotes \( K \) independent data streams with \( x_k \) intended for user \( k \). We assume \( \mathbf{x} \) satisfies \( E[\mathbf{x} \mathbf{x}^H] = I_K \).

![Fig. 1](image_url) Illustration of the limited feedback AF MIMO relay broadcast system.
The average transmit power at the BS is given as
\[ \mathbb{E} (\|s\|^2) = \text{tr}(PP^H) = P_s. \] (1)

The received signal at the RS is then given by
\[ y_r = H_1 s + n_1, \] (2)
where \( H_1 \in \mathbb{C}^{N_k \times M} \) denotes the backward channel from the BS to RS, and \( n_1 \in \mathbb{C}^{N_k \times 1} \) denotes the zero-mean complex Gaussian noise vector with covariance matrix \( R_{n_1} = \mathbb{E}(n_1n_1^H) = \sigma_n^2 I_N \).

In the second time slot, the RS simply amplifies the received signal \( y_r \), using precoding matrix \( G \in \mathbb{C}^{N \times N} \), and retransmits the signal \( \hat{s} = G y_r \), which satisfies the following transmit power constraint
\[ \mathbb{E} (\|\hat{s}\|^2) = \text{tr}\left[ G \left( H_1 P P^H H_1^H + \sigma_n^2 I_N \right) G^H \right] = P_r. \] (3)

The received signal at user \( k \) can be written as
\[ y_k = h_{2k}^H \hat{s} + n_{2k} \]
\[ = h_{2k}^H G H_1 P x + h_{2k}^H G n_1 + n_{2k}, \] (4)
where \( h_{2k} \in \mathbb{C}^{N \times 1} \) is the forward channel from the RS to user \( k \), and \( n_{2k} \in \mathbb{C}^{N \times 1} \) is the zero-mean complex Gaussian noise at the user side. By stacking the received signals of all users, we have
\[ y = H_2 G H_1 P x + H_2 G n_1 + n_2, \] (5)
where \( y = [y_1, \ldots, y_K]^T \), \( H_2 = [h_{21}, \ldots, h_{2K}]^H \), and \( n_2 = [n_{21}, \ldots, n_{2K}]^T \). We assume that the noise terms at all users are independent with the same power level, thus \( \mathbb{E}(n_2n_2^H) = \sigma_n^2 I_K \).

Both the BS and RS have perfect CSI of \( H_1 \), and user \( k \) has perfect knowledge of \( h_{2k} \), which can be realized through training as for example:

a) **In the first time slot, the BS transmits pilot signals.** \( H_1 \) is obtained at the RS;

b) **In the second time slot, the RS transmits pilot signals.** \( H_1 \) and \( h_{2k} \) are obtained at the BS and user \( k \), respectively.

Then each user quantizes its channel direction information (CDI) \( \hat{h}_{2k} = h_{2k} / \|h_{2k}\| \) to a vector \( \hat{h}_{2k} \) chosen from a codebook \( C_k \) which consists of \( 2^B \) \( N \)-dimensional unit norm vectors \( \{c_1, \ldots, c_{2^B}\} \) according to \( \hat{h}_{2k} = \arg \max_{c \in C_k} h_{2k}^H c \). The index of \( \hat{h}_{2k} \) is first fed back to the RS and then relayed to the BS with \( B \) bits.

### 3. Problem Formulation

We adopt the following MSE cost function [9] as our design metric:
\[ \varepsilon(P, G, \beta) = \mathbb{E} (\|x - \beta^* y\|^2) \]
\[ = K + \beta^2 \text{tr}(h_2 G H_1 P (h_2 G H_1 P)^H) + \beta^2 \text{tr}(\sigma_n^2 H_2 G (H_2 G)^H + \sigma_{1K}^2 I_K) \]
\[ - 2 \beta^2 \text{tr}(A (P^H H_1^H G^H H_2^H)), \] (6)

where \( \beta \in \mathbb{R}_+ \) is introduced to provide additional optimization flexibility and can be regarded as an implementation of automatic gain control (AGC). Note that the signal-to-interference-plus-noise ratio (SINR) of all users will not be affected by a particular choice of \( \beta \) once \( P \) and \( G \) are determined.

However, due to limited feedback of \( H_2 \), only \( \hat{H}_2 = [\hat{h}_{21}, \ldots, \hat{h}_{2K}]^H \) is available for the RS and the BS. Considering the quantization error and the lack of channel magnitude information for \( H_2 \), we use the average MSE cost function over \( H_2 \) conditioned on \( \hat{H}_2 \) as the new objective function, which we try to minimize by jointly designing \( P \) and \( G \). Mathematically, this robust design problem can be formulated as
\[ \min_{P, G, \beta} \mathbb{E}_{H_2, \hat{H}_2} \{\varepsilon(P, G, \beta)\} \] (7a)
\[ \text{s.t.} \quad \text{tr}(P P^H) = P_s \] (7b)
\[ \text{tr}\left[ G \left( H_1 P P^H H_1^H + \sigma_n^2 I_N \right) G^H \right] = P_r. \] (7c)

Using quantization cell approximation [8], \( H_2 \) can be decomposed as
\[ H_2 = D(I_K - \Theta)^{1/2} \hat{H}_2 + D \Theta^{1/2} U, \] (8)
where \( D = \text{diag}(\|h_{21}\|, \ldots, \|h_{2K}\|, \Theta = \text{diag}(\sin^2 \theta_1, \ldots, \sin^2 \theta_K), \text{and } U \text{ is a matrix whose column vectors are orthogonal to those of } \hat{H}_2. \) The expectation of \( \sin^2 \theta_k \) can be calculated according to its cumulative distribution function [8] as \( \mathbb{E}(\sin^2 \theta_k) = (N - 1 - \delta) / N, \) where \( \delta = 2^{-B/(N-1)}. \) Let \( A = D(I_K - \Theta)^{1/2} \) and \( B = D \Theta^{1/2}. \) Applying Lemma 1 of [7], we have
\[ \mathbb{E}(A^H A) = (N - (N - 1) \delta) I_K \] (9)
\[ \mathbb{E}(B^H B) = (N - 1) I_K \] (10)
\[ \mathbb{E}(U^H U) = \frac{K}{N - 1} I_N - \frac{1}{N - 1} \hat{H}_2^H \hat{H}_2. \] (11)

Substituting (8) into (6) and using equations (9) to (11), we can calculate the objective function (7a) as
\[ \mathbb{E}_{H_2, \hat{H}_2} \{\varepsilon(P, G, \beta)\} \]
\[ = K + \beta^2 N(1 - \delta) \text{tr}(\hat{H}_2^H G H_1 P P^H H_1^H G H_2^H) \]
\[ + \beta^2 K \delta \text{tr}(G H_1 P P^H H_1^H G^H) \]
\[ + \beta^2 \sigma_n^2 N(1 - \delta) \text{tr}(H_2 G G^H H_2^H) \]
\[ + \beta^2 K \delta \sigma_n^2 \text{tr}(G G^H) + \beta^2 K \sigma_{1K}^2 \]
\[ - 2 \beta^2 \sigma A \text{tr}(P P^H H_1^H G^H H_2^H), \] (12)

where \( \sigma = \mathbb{E}(\cos \theta_k) \mathbb{E}(\|h_{2k}\|). \)

### 4. Robust Joint MMSE Precoding

The optimization problem (7) is convex, but to obtain its closed-form solution is nontrivial. In the following, we first decompose (7) into two sub-problems, and derive their
closed-form solutions, respectively, based on which we then propose an iterative algorithm to compute the optimal robust joint precoders.

4.1 Optimal RS Precoding

Given any BS precoding matrix $P$, the optimal robust RS precoding matrix $G$ for the sub-problem

$$
\min_{G, \beta} \mathbb{E}_{H_i, H_i^*} \left[ \epsilon(G, \beta) \right]
$$

s.t. $\text{tr} \left[ G \left( H_i P P^H H_i^* + \sigma_i^2 I_N \right) G^H \right] = P_t$

(13)

can be readily derived based on the results in [5] as

$$
G = \sqrt{P_t / \text{tr} \left[ Q \left( H_i P P^H H_i^* + \sigma_i^2 I_N \right) Q^H \right]} Q.
$$

(14)

where

$$
Q = \left( \hat{H}_1^H \hat{H}_2^H + \mu_0 I_N \right)^{-1} \hat{H}_2^H P P^H H_i^* \left( H_i P P^H H_i^* + \sigma_i^2 I_N \right)^{-1},
$$

and

$$
\mu_0 = \frac{1}{K \delta P + K \sigma_f^2}.
$$

Note that the power constraint (7b) in (7) is removed here since it is irrelevant with $G$. Obviously, the optimal $G$ depends on $P$. However, the optimal $P$ remains unsolved.

4.2 Optimal BS Precoding

To solve for the optimal $P$, one may substitute the expression of (14) into the original problem (7), which makes $P$ the only variable. However, such manipulation renders the problem extremely complex and intractable. Instead, we seek to derive the optimal $P$ for a given $G$ through solving the following sub-problem

$$
\min_{P, \beta} \mathbb{E}_{H_i, H_i^*} \left[ \epsilon(P, \beta) \right]
$$

s.t. $\text{tr} \left( P P^H \right) = P_s$

(15)

The power constraint (7c) in (7) is dropped here for convenience, but later in Sect. 4.3 we will show that the power constraint violation can be avoided with the proper optimization process. This is a convex optimization problem, and its Lagrangian function [10] is given as

$$
L(P, \beta, \lambda_i) = \mathbb{E}_{H_i, H_i^*} \left[ \epsilon(P, \beta) \right] + \lambda_i \left( \text{tr} \left( P P^H \right) - P_s \right).
$$

(16)

By setting the derivations of (16) to zero, we have

$$
\left( \frac{\partial L(P, \beta, \lambda_i)}{\partial P} \right)^T = \beta^{-2} N(1 - \delta) H_i^H G^H H_i^* H_i^* H_i G H_i P
$$

$$
+ \beta^{-2} K \delta H_i^H G H_i H_i^* H_i \hat{H}_2^H \hat{H}_2^H \hat{H}_2 G H_i P
$$

$$
+ \lambda_i P = 0,
$$

(17)

and

$$
\frac{\partial L(P, \beta, \lambda_i)}{\partial \beta} = -2 \beta^{-3} \left[ N(1 - \delta) \text{tr} \left( \hat{H}_2 G H_i P P^H H_i^* G^H H_i^* \right) \right.
$$

$$
+ K \delta \text{tr} \left( G H_i P P^H H_i^* G^H H_i^* \right)
$$

$$
+ \sigma_i^2 N(1 - \delta) \text{tr} \left( GG^H \right) + K \delta \sigma_f^2 \text{tr} \left( G G^H \right) + K \sigma_f^2 \right]
$$

$$
+ 2 \beta^{-2} \sigma_r \text{tr} \left( \left[ P^H H_i^* G^H H_i^* \right]^2 \right) = 0.
$$

(18)

Then it follows from (17) that

$$
P = \beta \sigma_r W H_i^* G^H H_i^* \hat{H}_2^H,
$$

(19)

where

$$
W = \left[ H_i^H G^H \left( N(1 - \delta) \hat{H}_2^H \hat{H}_2^H + K \delta \sigma_r \right) G H_i + \gamma_1 I_M \right]^{-1}
$$

and $\gamma_1 = \lambda_1 \beta^2$. Note that $W$ is a Hermitian matrix and only depends on $\gamma_1$. From (18), we also have

$$
\beta \sigma_r \text{tr} \left( \left[ P^H H_i^* G^H H_i^* \right]^2 \right) - K \delta \text{tr} \left( G H_i P P^H H_i^* G^H H_i^* \right)
$$

$$
- N(1 - \delta) \text{tr} \left( H_i G H_i P H_i^* \left( H_i P P^H H_i^* + \sigma_i^2 I_N \right)^{-1} \right)
$$

$$
= \xi,
$$

(20)

where $\xi = \sigma_i^2 N(1 - \delta) \text{tr} \left( \hat{H}_2 G H_i \left( H_i P P^H H_i^* + \gamma_1 I_N \right)^{-1} \right) + K \delta \sigma_f^2$. Substituting (19) into the left hand side of (20) gives

$$
\gamma_1 (\beta \sigma_r) \text{tr} \left( H_i G H_i W H_i^* \left( H_i P P^H H_i^* \right)^{-1} \right) = \xi,
$$

(21)

and applying the BS power constraint (1) to (19) gives

$$
(\beta \sigma_r)^2 \text{tr} \left( H_i G H_i W H_i^* \left( H_i P P^H H_i^* \right)^{-1} \right) = P_s.
$$

(22)

Since $W$ is a Hermitian matrix, i.e. $W = W^H$, we can substitute (22) into (21) and get

$$
\gamma_1 = \xi / P_s.
$$

(23)

Thus $W$ is now determined. Also from (22), we have

$$
\beta = \beta_{eff} / \sigma_r,
$$

(24)

where $\beta_{eff} = \sqrt{P_s / \text{tr} \left( H_i G H_i W H_i^* \left( H_i P P^H H_i^* \right)^{-1} \right)}$. Hence, combining (19), (23) and (24), we can give the optimal robust BS precoding matrix $P$ for a given $G$ as

$$
P = \beta_{eff} W H_i^* G^H H_i^* \hat{H}_2^H.
$$

(25)

4.3 Iterative Algorithm

From (14) and (25), we can see $P$ and $G$ are functions of each other, and thus we propose an iterative algorithm which alternately optimizes $P$ and $G$ as follows:

- **Step 1:** Initialization
  We start by applying an identity precoding matrix at the BS with equal power allocation, i.e. $P^{(0)} = \sqrt{P_s / M I_M}$, and use (14) to calculate $G^{(0)}$. Set $n = 0$.

- **Step 2:** Iteration
  a. Use (25) to calculate $P^{(n+1)}$ with $G^{(n)}$;
  b. Use (14) to calculate $G^{(n+1)}$ with $P^{(n+1)}$;
  c. $n = n + 1$.

- **Step 3:** Termination
  The algorithm terminates either when $G^{(n)}$ converges, i.e. $\frac{||G^{(n+1)} - G^{(n)}||_F}{||G^{(n)}||_F} \leq \eta_{th}$, or when $n \geq N_{max}$, where $\eta_{th}$ is
a predefined threshold and $N_{\text{max}}$ is the maximum iteration number. Note that $\mathbf{P}^{(n)}$ also converges when $\mathbf{G}^{(n)}$ converges.

In each iteration, $\mathbf{G}^{(n)}$ is always updated after $\mathbf{P}^{(n)}$, which ensures that the final output of $\mathbf{G}$ satisfies the power constraint (7c), and thus the relaxation in sub-problem (15) does not incur infeasible solutions. Since the updated $\mathbf{G}^{(n)}$ and $\mathbf{P}^{(n)}$ always lead to a better solution with decreased MSE cost, $\mathbf{G}^{(n)}$ and $\mathbf{P}^{(n)}$ finally converge to the global optimal solution considering the convexity of (7). As for implementation, both the BS and RS can employ the same algorithm to obtain $\mathbf{P}$ and $\mathbf{G}$ in a distributed manner.

5. Simulation Results

In this section, we provide simulation results for a limited feedback AF MIMO relay broadcast system to compare the performance of our proposed robust joint MMSE precoding (R-JMMSE) scheme with the robust relay-only MMSE precoding (R-ROMMSE) scheme in [5] and the non-robust joint MMSE precoding (NR-JMMSE) scheme in [4] based on CDI feedback. The joint MMSE precoding (JMMSE) scheme with perfect CSI is also simulated for reference.

We assume both backward and forward channels experience quasi-static flat Rayleigh fading, and all entries of $\mathbf{H}_1$ and $\mathbf{H}_2$ have identical and independent complex Gaussian distribution with zero-mean and unit variance. The random vector quantization (RVQ) codebook [6] is used for CDI quantization of $\mathbf{H}_2$. We define the SNRs for the backward link and the forward link as $\text{SNR}_1 = \frac{P}{M_1}$ and $\text{SNR}_2 = \frac{P}{N_2}$, respectively. In all cases, we set $\text{SNR}_1 = \text{SNR}_2$. All results are averaged over 10,000 random channel realizations, and 1,000 BPSK symbols are transmitted for each data stream per channel realization. In addition, for the iteration algorithm of the R-JMMSE scheme, we set the convergence threshold $\eta_P = 0.01$ and the maximum iteration number $N_{\text{max}} = 20$.

Figure 2 shows the average bit error rate (BER) performance of all schemes for $B = 8$ and 16 bits with $M = N = K = 4$. It can be seen from Fig. 2 that the proposed R-JMMSE scheme achieves superior BER performance compared with the other schemes. In high SNR regions, the R-JMMSE scheme has a considerable gain over the NR-JMMSE scheme. However, its gain over the R-ROMMSE scheme diminishes while SNR increases, and finally both schemes reach the same BER floor. This is due to the fact that the system performance is limited by the CDI quantization error at high SNR. When $B$ increases, the BER performance of all schemes improves. Furthermore, the gain of the R-JMMSE scheme over the R-ROMMSE scheme also increases with $B$, because more joint processing gain can be exploited with smaller channel uncertainties.

6. Conclusion

We proposed a robust joint MMSE precoding scheme for AF MIMO relay broadcast systems with limited CDI feedback of the forward channel. The optimization problem was formulated to minimize the expected MSE conditioned on the quantized CDI. After solving two sub-problems which were more tractable, we then proposed an iterative algorithm to jointly optimize the BS and RS precoders. Simulation results showed that the proposed R-JMMSE scheme achieved better BER performance than the NR-JMMSE scheme as well as the R-ROMMSE scheme.

References