Joint User and Antenna Selection for Multiuser MIMO Downlink with Block Diagonalization

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Abstract: User selection is necessary for multiuser multiple-input multiple-output (MIMO) downlink systems with block diagonalization (BD) due to the limited free spatial transmit dimensions. The pure user selection algorithms can be improved by performing receive antenna selection (RAS) to increase sum rate. In this paper, a joint user and antenna selection algorithm, which performs user selection for sum rate maximization in the first stage and then performs antenna selection in the second stage, is proposed. The antenna selection process alternately drops one antenna with the poorest channel quality based on maximum determinant ranking (MDR) from the users selected during the first stage and activates one antenna with the maximum norm of projected channel from the remaining users. Simulation results show that the proposed algorithm significantly outperforms the algorithm only performing user selection as well as the algorithm combining user selection with MDR receive antenna selection in terms of sum rate.

Key words: multiuser, multiple-input multiple-output (MIMO), multi-antenna, block diagonalization (BD), joint user and antenna selection

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Nomenclatures

\[ A_u — \text{Set of active antennas of user } u \]
\[ A_\pi — \text{Set of active antennas of user } \pi \]
\[ C_{BD}^{(n)} — \text{Sum rate of the current combination of selected users and activated antennas at the } n\text{th loop} \]
\[ K — \text{Total number of users} \]
\[ \hat{K} = \text{card}(\Pi) — \text{denotes the cardinality of a set} \]
\[ L — \text{Number of users in } \Omega, L = \text{card}(\Omega) \]
\[ N_{\text{max}} — \text{The maximum number of antennas that can be dropped from the selected users in } \Pi \]
\[ N_u — \text{Number of antennas of user } u \]
\[ N_\pi — \text{Number of antennas of user } \pi \]
\[ N_{\text{min}} — \text{The minimum number of active antennas required by user } \pi \]
\[ R_u — \text{Set of remaining inactive antennas of user } u \]
\[ u_i = \text{Index of the } i\text{th user in } \Omega, u_i \in \{1, 2, \cdots, K\} \]
\[ \pi_i = \text{Index of the } i\text{th selected user in } \Pi, \pi_i \in \{1, 2, \cdots, K\} \]
\[ \Pi — \text{Set of users selected during the first stage} \]
\[ \Psi^{(n)} — \text{Set of all active antennas at the } n\text{th loop} \]
\[ \Omega — \text{Set of users not selected during the first stage} \]

0 Introduction

The multiple-input multiple-output (MIMO) technology promises spatial multiplexing and diversity gains[1] by deploying multiple antennas at the transmitter and receiver. Further, MIMO also provides a multiple access solution for the multiuser MIMO system, where the multi-antenna base station (BS) communicates with multiple uncooperative users over the same time and frequency resources through space division multiple access (SDMA).

In the multiuser MIMO system, generally no cooperation is allowed among different users in practice. Hence, each user’s channel suffers interference from other users. To handle this issue, interference elimination could be implemented at the user receiver side, but this will increase the complexity in user terminals. The alternate solution is to apply precoding at the transmitter side. The precoding techniques can be linear or nonlinear. The well-known nonlinear precoding is dirty-paper precoding (DPC)[2] and its simplified version, i.e., zero-forcing DPC[3]. Although DPC achieves
the capacity of MIMO broadcast channels, it incurs high complexity. Linear precoding techniques such as zero-forcing beamforming (ZFBF)\cite{4}, minimum mean squared error (MMSE) beamforming\cite{5} and block diagonalization (BD)\cite{6}, have much lower complexity and are relatively easier for implementation. When user terminals are equipped with multiple antennas, BD can be used to eliminate the inter-user interference.

The maximum number of users simultaneously supported by the BD technique is limited by the number of transmit antennas at the BS, that is, the total number of independent data streams of all active users should not be greater than the number of transmit antennas at the BS. However, usually in practice there are quite a large number of users served by one BS. Hence, the BS needs to implement user selection for scheduling. With BD, various user selection algorithms have been proposed in the literatures. The Ref. \cite{7} presented capacity-based and Frobenius norm-based low complexity user selection algorithms, but no antenna selection was considered. Reference \cite{8} presented an algorithm to select receive antennas one by one from all users to maximize the sum rate, but it was pointed out in Ref. \cite{9} that the algorithm tended to select antennas from too many users. Hence, probably only one antenna was activated for each selected user, which was not good for improving individual quality of service (QoS). Moreover, its complexity is high. Reference \cite{9} proposed another approach which first performed user selection and then carried out receive antenna selection to further improve the sum rate. However, the algorithm in Ref. \cite{9} did not make full use of the free transmit dimensions released from antenna selection.

In this paper, the joint user and antenna selection in multiuser MIMO systems with BD are further studied to improve the sum rate. A joint selection algorithm which consists of two stages is proposed. In the first stage, the algorithm performs user selection based on the sum rate maximizing criterion; in the second stage, the algorithm performs antenna selection alternately in two groups of users, i.e., the users selected during the first stage and the remaining users. Specifically, the antenna selection process drops those antennas with poor channel quality from the selected users, and meanwhile, it activates the antennas with good channel quality from the remaining users. In such a way, the algorithm efficiently exploits the free transmit dimensions. To avoid activating antennas from too many users, the proposed algorithm also enforces a constraint on the maximum number of antennas that can be dropped from the selected users.

1 Multiuser MIMO System

1.1 System Model

We consider a multiuser MIMO system, where the BS and user $k$ are equipped with $M$ and $N_k$ antennas, respectively, and $K$ multi-antenna users are served. We assume that all user channels experience block flat fading, and the BS has perfect channel state information (CSI) of all channels. The received signal at user $k$ is

$$y_k = H_k W_k s_k + H_k \sum_{i=1,i\neq k}^{K} W_i s_i + n_k, \quad (1)$$

where, $H_k \in \mathbb{C}^{N_k \times M}$ denotes the channel matrix from the BS to user $k$; $s_k \in \mathbb{C}^{L_k \times 1}$ is the data vector intended for user $k$, $L_k$ is the number of data streams belong to user $k$; $W_k \in \mathbb{C}^{M \times N_k}$ is the corresponding linear precoding matrix; and $n_k \in \mathbb{C}^{N_k \times 1}$ is the zero-mean complex Gaussian noise vector. The second term in Eq. (1) represents the interference from other users. We assume that $E[s_k s_k^H] = I_{L_k}$ and $E[n_k n_k^H] = \sigma_n^2 I_{N_k}$ for all $k$, where $\sigma_n$ is the variance of the noise.

1.2 Block Diagonalization Preceding

Block diagonalization\cite{6} forces the inter-user interference to zero by choosing the precoding matrices that satisfy the following condition:

$$H_k W_i = 0, \quad \forall k \neq i, \quad 1 \leq k, i \leq K. \quad (2)$$

Hence, the precoding matrix $W_k$ of user $k$ must lie in the null space of the channel matrices of other users. The combined channel matrix of the users other than user $k$ is

$$\tilde{H}_k = [H_1^T \quad H_2^T \cdots \quad H_{k-1}^T \quad H_{k+1}^T \cdots \quad H_K^T]^T. \quad (3)$$

The prerequisite for the existence of the null space of $\tilde{H}_k$ is\cite{6}

$$M \geq \sum_{k=1}^{K} L_k. \quad (4)$$

Thus to ensure Eq. (4) is not violated, user selection is necessary when $\sum_{k=1}^{K} L_k > M$.

The maximum sum rate with BD can be expressed as

$$C_{BD} = \max \sum_{k=1}^{K} \text{tr} \left( \text{det} \left( I + \frac{H_k Q_k H_k^H}{\sigma_n^2} \right) \right), \quad (5)$$

where, $(\cdot)^H$ means the conjugate transpose of $(\cdot)$; $Q_k = W_k W_k^H$; and $P$ is the maximum transmit power at the BS. Water-filling method\cite{3} can be applied to optimize the power allocation and get the maximum sum rate.

2 Joint User and Antenna Selection Algorithm

We propose a joint user and antenna selection algorithm which aims at improving the sum rate with BD.
The algorithm consists of two stages. In the first stage, the algorithm performs user selection based on the sum rate maximizing criterion. Low complexity user selection methods such as capacity-based greedy algorithm and Frobenius norm-based algorithm proposed in Ref. [7] can be employed here. In the second stage, the algorithm performs antenna selection alternately in two groups of users, i.e., the users selected during the first stage and the remaining users. For the group of selected users, each time the antenna selection process drops one antenna with the poorest channel quality based on the maximum determinant ranking (MDR)\textsuperscript{[10]}\textsuperscript{[10]}. After dropping one antenna from the selected users’ group, the algorithm temporally selects an antenna with the maximum norm of channel projection on the null space of the combined channel of the users selected during the first stage and the activated antennas from the remaining users. However, if the sum rate actually decreases with this newly selected antenna activated, then it is not selected. The algorithm then repeats the antenna selection process until reaching the maximum number of antennas that can be dropped from the selected users’ group. To ensure that the best combination of users and antennas with the maximum sum rate enabled by this algorithm is chosen as the final selection result, in each step of the second stage, the intermediate combinations as well as their corresponding sum rates are registered. The algorithm compares all the registered results and outputs the best combination. Note that the group of selected users only includes the users selected during the first stage.

We summarize the proposed algorithm in the following.

**Stage 1  User selection**

The low complexity capacity-based algorithm or Frobenius norm-based algorithm proposed in Ref. [7] can be employed.

We get the group of selected users with all their antennas activated:

\[
\Pi = \{\pi_1, \pi_2, \ldots, \pi_{K}\}, \quad A_{\pi_i} = \{1, 2, \ldots, N_{\pi_i}\}, \quad \Omega = \{1, 2, \ldots, K\} - \Pi, \quad R_{u_i} = \{1, 2, \ldots, K\}, \quad A_{u_i} = \emptyset, \quad \psi^{(0)} = \{A_{\pi_i}|\pi_i \in \Pi\} \cup \{A_{u_i}|u_i \in \Omega\}.
\]

The sum rate is calculated according to Eq. (5), denoted as \(R^{(0)}\).

To avoid activating antennas from too many users during Stage 2, we enforce a constraint on the maximum number of antennas that can be dropped from the group of selected users:

\[
N_{\text{max}} = \sum_{i=1}^{K}(N_{\pi_i} - N_{\pi_i}^\text{min}) \tag{6}
\]

**Stage 2  Antenna selection**

Set loop count \(n = 1\).

**Step 1** Drop one antenna with the poorest channel quality based on the MDR\textsuperscript{[10]} from users in \(\Pi\).

Let \(H(A_{\pi_i})\) and \(H(A_{u_i})\) denote the current channel matrices corresponding to active antennas of users in \(\Pi\) and \(\Omega\), respectively. The combined channel is represented as

\[
\hat{H}_{\psi} = [H^T(A_{\pi_1}), H^T(A_{\pi_2}), \ldots, H^T(A_{\pi_K}), H^T(A_{u_1}), H^T(A_{u_2}), \ldots, H^T(A_{u_N})]^T.
\]

Then the antenna to drop is

\[
(\pi_j, a) = \arg \max_{a \in A_{\pi_j}, \pi_j \in \Pi} \left[\left(\hat{H}_{\psi} \hat{H}_{\psi}^H\right)^{-1}\right]_{\pi_j}^a, \tag{8}
\]

where \((\pi_j, a)\) denotes the \(a\)th antenna of user \(\pi_j\), and \([\left(\hat{H}_{\psi} \hat{H}_{\psi}^H\right)^{-1}]_{\pi_j}^a\) denotes the diagonal element of \(\left(\hat{H}_{\psi} \hat{H}_{\psi}^H\right)^{-1}\) corresponding to antenna \((\pi_j, a)\).

Then \(A_{\pi_j} = A_{\pi_j} - \{a\}\), and the sum rate after dropping the antenna is denoted as \(C_{\text{drop}}\).

**Step 2** Temporally selects one antenna from the users in \(\Omega\) with the maximum norm of channel projection on the null space of the combined channel \(H_{\psi}\).

1. Apply Gram-Schmidt orthogonalization procedure to \(H_{\psi}\) to obtain the row space base \(W_{\psi}\).
2. Calculate the projection of channels for antennas in \(R_{u_j}\) on the null space of \(H_{\psi}\) as

\[
\hat{h}_{u_j}^r = h_{u_j}^r - h_{u_j}^* W_{\psi}^H W_{\psi}, \tag{9}
\]

where \(h_{u_j}^*\) denotes the channel of the \(r\)th antenna of user \(u_j\).

3. Select the antenna with the largest Frobenius norm of the projected channel:

\[
(u_j, r) = \arg \max_{r \in R_{u_j}, u_j \in \Omega} \|\hat{h}_{u_j}^r\|_F, \tag{10}
\]

where \((u_j, r)\) denotes the \(r\)th antenna of user \(u_j\).
4. Calculate the sum rate with the newly selected antenna activated, denoted as \(C_{\text{sel}}\). If \(C_{\text{drop}} > C_{\text{BD}}\); then

\[
C_{\text{BD}} = C_{\text{sel}}, \quad A_{u_j} = A_{u_j} \cup \{r\}, \quad R_{u_j} = R_{u_j} - \{r\}, \quad \psi^{(n)} = \psi^{(n-1)} \cup \{(u_j, r)\} - \{(\pi_j, a)\}.
\]
Otherwise, do not select antenna \((u_j, r)\). Then \(C^{(n)}_{\text{BD}} = C^{\text{drop}}_{\text{BD}}\) and \(\Psi^{(n)} = \Psi^{(n-1)} - \{(\tau_j, a)\}\).

**Step 3** If \(n < N_{\text{max}}\), then \(n = n + 1\) and repeat Steps 1 and 2 of Stage 2. Otherwise, quit the algorithm and output the best antenna selection result \(\Psi_{\text{opt}}\) as

\[
P = \arg \max_{0 \leq n \leq N_{\text{max}}} C^{(n)}_{\text{BD}},
\]

\[
\Psi_{\text{opt}} = \Psi(P).
\]

Compared to Ref. [9], the major computation complexity increase comes from Step 2 of Stage 2. Assuming each user has \(N\) antennas, calculating the projection of one antenna’s channel on the null space of \(\hat{H}_\psi\) needs \(O(M^2)\) complex multiplication, selecting an antenna needs calculating \(O(K N)\) times of projections, and totally \(O(M)\) times of selection are needed. Hence, the total complexity increase is \(O(M^3 K N)\) complex multiplication.

### 3 Simulation Results

In this section, we provide simulation results of the proposed joint user and antenna selection (JUAS) algorithm. For comparison, we have also simulated the algorithm combining user selection with MDR receive antenna selection (US with MDR RAS) in Ref. [9] and the algorithm only performing user selection (US only) in Ref. [7].

We assume that all user channels experience Rayleigh flat fading, and all entries of channel matrices follow identical independent Gaussian distribution with zero mean and unit variance. All results are averaged over 2500 random channel realizations.

Figure 1 shows the sum rate versus the number of users with \(M = 16, N_k = 4\) for all users and \(N_{\text{min}} = 2\). The total number of users is 50. The capacity-based user selection is applied in all algorithms[7]. The JUAS algorithm significantly outperforms the other algorithms, and the performance gap becomes wider when SNR increases.

![Fig. 1 Sum rate versus the number of users](image)

Figure 2 shows the sum rate versus the SNR with \(M = 8, N_k = 4\) for all users and \(N_{\text{min}} = 2\). The signal-to-noise ratio (SNR) is fixed at 10 dB. The norm-based user selection is applied in all algorithms[7]. The JUAS algorithm has a sum rate increase of 6% when SNR=0dB, and such gain increases with the SNR, reaching the maximum increase of 13% at 12 dB. Then, the increase magnitude goes down with the SNR, but still has a 10% increase at 20 dB. Hence, the JUAS algorithm efficiently improves the sum rate performance in a wide SNR range. Meanwhile, the gain for the US with MDR RAS algorithm is below 4%, and goes straight down with increasing SNR, less than 1% at high SNRs.

![Fig. 2 Sum rate versus SNR](image)

To get a clearer view on the sum rate improvement for BD systems by applying the JUAS algorithm, we further show the sum rate gain over the algorithm only performing user selection in percentage in Fig. 3 with the same simulation configurations as in Fig. 2. The JUAS algorithm has a sum rate increase of 6% when SNR=0dB, and such gain increases with the SNR, reaching the maximum increase of 13% at 12 dB. Then, the increase magnitude goes down with the SNR, but still has a 10% increase at 20 dB. Hence, the JUAS algorithm efficiently improves the sum rate performance in a wide SNR range. Meanwhile, the gain for the US with MDR RAS algorithm is below 4%, and goes straight down with increasing SNR, less than 1% at high SNRs.

![Fig. 3 Sum rate gain over the algorithm only performing US](image)
4 Conclusion

We have proposed a joint user and antenna selection algorithm for the multiuser MIMO downlink with BD. In the user selection stage, the existing user selection algorithms such as capacity-based and norm-based algorithms can be employed, and in the antenna selection stage, the algorithm alternately drops one antenna with the poorest channel quality based on the maximum determinant ranking from the users selected during the first stage and activates one antenna with the maximum norm of projected channel from the remaining users. Compared with the US with MDR RAS algorithm, the proposed algorithm only incurs a complexity increase of $O(M^3KN)$ complex multiplication. We have shown through simulation that the JUAS algorithm achieves higher sum rate than the US only as well as the US with MDR RAS algorithms.

References


