Joint MMSE Precoding Design in Multi-user Two-Way MIMO Relay Systems

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Abstract—This paper addresses the problem of joint design of user precoders and receivers as well as the relay precoder based on the minimum mean-square-error (MMSE) criterion in the multi-user amplify-and-forward (AF) two-way multiple-input multiple-output (MIMO) relay systems with analog network coding (ANC). By decoupling the original highly nonconvex problem into three sub-problems, we propose an iterative algorithm to repeatedly optimize precoders and receivers. The convergence of the algorithm is ensured since the updated solution is optimal to each sub-problem. In addition, a novel channel switching technique at the relay node is implemented as means of exploring different local optimal solutions to further improve the system performance. Numerical simulation results demonstrate that the proposed iterative algorithm outperforms other algorithms in terms of both bit error rate (BER) and mean square error (MSE).

Keywords— two-way relay, multiple-input multiple-output (MIMO), analog network coding (ANC), minimum mean-square-error (MMSE), channel switching.

I. INTRODUCTION

Relay-assisted cooperative transmission offers significant benefits such as throughput enhancement and coverage extension for wireless communication systems. In two-way relay systems, two user nodes exchange information with the help of a relay node. Conventional relay protocols take four time slots to complete one round of information exchange. To improve spectral efficiency, several novel two-way relay protocols have been proposed in [1]-[2]. In [1], an XOR operation-based decode-and-forward (DF) protocol is proposed, where the relay decodes the received information from two user nodes in the first two time slots, respectively, and forwards the information combined on bit-level by means of XOR operation to both user nodes in the third time slot. Each user then performs XOR operation to obtain the needed information from the other user. Analog network coding (ANC) in [2] is another protocol that can be employed in two-way relay systems, in which information exchange between two users only requires two time slots. The two users simultaneously transmit data to the relay node over the same radio resource during the first time slot, and the relay node amplifies and forwards (AF) the superimposed received signal to both users in the second time slot. Each user can perform self-interference cancellation (SIC) before detecting the needed information. Due to its simplicity and efficiency, ANC protocol is more attractive in system design.

Furthermore, by equipping nodes with multiple antennas, multiple-input multiple-output (MIMO) technique can be incorporated into relay systems to provide diversity and multiplexing gains [3]. In [4], the two-way relay system with two single-antenna users and one multi-antenna relay is considered. The authors compute the optimal beamforming matrix at relay node to characterize the capacity region of the system. For two-way relay systems consisting of three multi-antenna nodes, the authors in [5] propose an iterative algorithm to jointly optimize the source and relay to maximize the sum-rate. Based on the minimum mean-square-error (MMSE) criterion, the joint source and relay optimal precoding design is also studied in [6], and an iterative algorithm as well as a heuristic channel parallelization algorithm is proposed therein. The two-way relay system with multi-pair of users is investigated in [7] to optimize the relay processing matrix based on both zero-forcing (ZF) and MMSE criteria. However, the authors in [7] only consider relay optimization and apply conventional beamforming techniques such as eigen beamforming at user nodes. Besides, they ignore the user’s ability to cancel self-interference and suppress all interference including self-interference through relay beamforming at the cost of diversity loss.

In this paper, we focus on the joint precoding design at user and relay nodes for multi-user AF two-way MIMO relay systems with analog network coding. Based on the MMSE criterion, we formulate an optimization problem to jointly design the user precoders and receivers as well as the relay precoder. It is generally difficult to obtain a closed-form solution for this highly nonconvex problem. Thus, we decouple the original problem into three sub-problems and propose an algorithm to iteratively solve the sub-problems. Since the iterative algorithm may converge to some local optimal solutions, a novel channel switching technique is proposed to search for different local optimal solutions to further improve system performance.

Notations: Boldface lowercase letters denote vectors, while boldface uppercase letters denote matrices. $E[\cdot]$ denotes the expectation of the random variables. $\otimes$ is the Kronecker operation of matrices. $\text{vec}(\cdot)$ and $\text{mat}(\cdot)$ are the matrix...
vectorization operation and the corresponding inverse operation. \(\text{tr}[]\), \(A^{-1}\), \(A^\dagger\) and \(A^\ddagger\) stand for the trace, inverse, transpose and conjugate transpose operation of matrix \(A\), respectively. \(\mathbb{C}^{m \times n}\) represents the space of \(m \times n\) matrices with complex entries. \(I_M\) denotes an \(M \times M\) identity matrix. \(0_{M \times N}\) stands for the \(M \times N\) all zero matrix. The real part of a complex value is denoted as \(Re\{\cdot\}\).

II. SYSTEM MODEL

There are one relay node and 2K user nodes in the multi-user amplify-and-forward two-way relay system shown in Fig. 1. User 2K \(\cdots\) 1 and user 2K, \(k = 1, \ldots, K\), form a pair of users , and they exchange information with each other via the assistance of the two-way relay. In the first time slot, all of the 2K users simultaneously transmit signal to the relay node. In the second time slot, the relay node processes the received signal with the relay precoder and forwards the precoded signal to all user nodes. Each user first cancels self-interference from the received signal and then detects the needed data with its own receiver.

Assuming that the relay node is equipped with \(M\) antennas and user node \(k\), \(k = 1, \ldots, 2K\), is equipped with \(N_k\) antennas. Each user transmits \(L_k\) independent data streams, where \(L_k \leq N_k\). To support transmission of all data streams, the relay antenna number \(M\) should satisfy \(M \geq L = \min\{L_k, \forall k = 1, \ldots, 2K\}\), where \(L = \sum_{k=1}^{2K} L_k\).

In the first time slot, user \(k\), \(k = 1, \ldots, 2K\), transmits \(s_k = A_k d_k\) to relay subject to the transmit power constraint
\[
E[s_k^* s_k] = \text{tr} [\sigma_d^2 A_k^* A_k] \leq P_k,
\]
where \(A_k \in \mathbb{C}^{N_k \times L_k}\) is the user precoder and \(d_k \in \mathbb{C}^{L_k \times 1}\) is the data vector of user \(k\) with \(E[d_k d_k^*] = \sigma_d^2 I_{L_k}\). The transmit signals of all users can be expressed more concisely as
\[
s = Ad,\]
where \(s = [s_1^T, \ldots, s_{2K}^T]^T\) is the combined multiuser transmit signal vector, \(d = [d_1^T, \ldots, d_{2K}^T]^T\) is the combined multiuser data symbol vector with \(E/dd^*\) = \(\sigma_d^2 I_L\), and the combined multiuser transmit precoder is
\[
A = \begin{bmatrix}
A_1 & 0_{N_1 \times L_2} & \cdots & 0_{N_1 \times L_{2K}} \\
0_{N_2 \times L_1} & A_2 & \cdots & 0_{N_2 \times L_{2K}} \\
\vdots & \vdots & \ddots & \vdots \\
0_{N_{2K} \times L_1} & 0_{N_{2K} \times L_2} & \cdots & A_{2K}
\end{bmatrix}.
\]

The channel matrix between user \(k\) and the relay node is denoted as \(H_k \in \mathbb{C}^{M \times N_k}\). Then the signal received at the relay node can be written as
\[
r = Hs + n_r = HAd + n_r,
\]
where \(H = [H_1, \ldots, H_{2K}]\) is the combined multiuser channel matrix from all users to the relay node and \(n_r \in \mathbb{C}^{M \times 1}\) represents the zero-mean additive white Gaussian noise (AWGN) at the relay node with \(E[n_r n_r^*] = \sigma_n^2 I_M\).

In the second time slot, the relay node processes the received signal \(r\) with relay precoder \(F \in \mathbb{C}^{M \times M}\) and then forwards \(x = Fr\) to all users subject to transmit power constraint \(P_r\),

\[
E[x^*x] = \text{tr} [F \sigma_n^2 HAA^* + \sigma_d^2 I_{M}] \leq P_r.
\]

The received signal at user \(k\) can be written as
\[
y_k = G_k x + n_k,
\]
where \(n_k \in \mathbb{C}^{N_k \times 1}\) is the zero-mean AWGN with \(E[n_k n_k^*] = \sigma_n^2 I_{N_k}\), and \(G_k \in \mathbb{C}^{N_k \times M}\) represents the channel matrix from the relay node to user \(k\).

Assuming that channel state information (CSI) is perfectly known, each user first cancels self-interference and gets the signal \(\hat{y}_k = y_k - G_k F H_k A_k d_k\).

Then the receiver \(B_k \in \mathbb{C}^{L_k \times N_k}\) is used to detect data as follows
\[
\hat{d}_k = B_k (y_k - G_k F H_k A_k d_k) = B_k G_k F H_k A_k d_k + B_k G_k F n_r + B_k n_k - B_k G_k F H_k A_k d_k.
\]

Let \(P_k\) be blockdiag\(\{0_{L_1 \times L_1}, 0_{L_2 \times L_2}, \ldots, 0_{L_{2K} \times L_{2K}}\}\) and \(\bar{P}_k = I_L - P_k\). We can rewrite \(\hat{d}_k\) as
\[
\bar{d}_k = B_k G_k F H_k A_k d_k + B_k G_k F n_r + B_k n_k.
\]

Denote that \(Q_k = [0_{L_1 \times L_1}, 0_{L_2 \times L_2}, \ldots, I_{L_k}, \ldots, 0_{L_{2K} \times L_{2K}}]\) and let \(\bar{k}\) be index of the user paired with user \(k\). The MSE at user \(k\) can be calculated as
\[
J_k = E\left\{[\bar{d}_k - d_k]^2\right\} = E\left\{||\bar{d}_k - Q_k d||^2\right\} = E\left\{[B_k G_k F H_k A_k d_k + B_k G_k F n_r + B_k n_k - Q_k d]^2\right\}.
\]

III. JOINT MMSE PRECODING DESIGN

In this paper, we will jointly design the user precoders \(A_k\) and receivers \(B_k\), \(k = 1, \ldots, 2K\), as well as the relay precoder \(F\) to minimize the total MSE of detected data at all user nodes. The problem can be formulated as
\[
\min_{A_k, B_k, F, k=1,\ldots,2K} J = \sum_{k=1}^{2K} J_k
\]
can be given as

\[ \text{s.t. } \text{tr}[\sigma_A^2 A_k^* A_k] \leq P_k \]
\[ \text{tr}[F(\sigma_A^2 HAA' H^* + \sigma_I^2 I_m) F^*] \leq P_r. \]  

The above problem is difficult to solve due to its nonconvexity. We thus decouple the problem into three sub-problems which are convex and can be efficiently solved, and then an iterative algorithm is proposed to compute the precoders and receivers.

### A. User Receiver Design

The first sub-problem is proposed to optimize receiver \( B_k \) at each user node, given the relay precode \( F \) and user precoders \( A_k, k = 1, \ldots, 2K \). The original problem is reduced to the following unconstrained convex optimization problem

\[ \min_{B_k, k=1, \ldots, 2K} J = \sum_{k=1}^{2K} J_k. \]  

where \( J_k = \sum_{k=1}^{2K} J_k \text{ s.t. tr}[FR_F F^*] \leq P_r \)

where

\[ J_k = \sigma_d^2 \text{tr}[Q_k^* Q_k] + \text{tr}[G_k^* B_k G_k^* FR_{sk} F^*] \]
\[ -\sigma_d^2 \text{tr}[HAP_k Q_k^* B_k G_k^*] - \sigma_d^2 \text{tr}[G_k^* B_k^* Q_k^* P_k A' A' H^* F^*] \]
\[ + \sigma_d^2 \text{tr}[B_k^* B_k^*]. \]  

\[ R_{sk} = \sigma_d^2 HAP_k A' A' H^* + \sigma_d^2 I_m, \text{ and } R_k = \sigma_d^2 HAA' H^* + \sigma_d^2 I_m. \]  

This sub-problem is convex with respect to \( F \), and hence the Lagrangian multiplier method can be used to derive the optimal solution. The Lagrangian function can be constructed as

\[ L(F, \lambda) = J + \lambda(\text{tr}[FR_F F^*] - P_r), \]

where \( \lambda \geq 0 \). Its derivative with respect to \( F \) is

\[ \frac{\partial L}{\partial F} = \sum_{k=1}^{2K} [(R_{sk} F^* G_k^* B_k G_k/G_k)^T] \]
\[ - \sum_{k=1}^{2K} \sigma_d^2 (HAP_k Q_k^* B_k G_k^* T) + \lambda (R_k F^*)^T. \]  

The Karush-Kuhn-Tucker (KKT) conditions [8] of the problem can be given as

\[ \frac{\partial L}{\partial F} = 0 \]

From (19) and (20), we have

\[ \sum_{k=1}^{2K} R_{sk} FR_{sk} + \lambda FR_F = R_r, \]
where \( R_r = \sum_{k=1}^{2K} \sigma_d^2 G_k^* B_k^* Q_k^* P_k A' A' H^* + \) and \( R_r = G_k^* B_k^* R_k G_k \).

From (23), we can obtain the expression of the optimal solution as

\[ F_{opt} = \text{mat} \{ [\sum_{k=1}^{2K} R_{sk}^* \otimes R_{rk} + \lambda R_{sk}^* \otimes I_m ]^{-1} \text{vec}(R_r) \}. \]  

where the nonnegative Lagrangian multiplier \( \lambda \) should be chosen to satisfy (21) and (22).

Using (24), it is easy to verify that the function \( g = \text{tr}[FR_F F^*] \) is a monotonic decreasing function with respect to \( \lambda \). Moreover, it can be shown that

\[ 0 \leq \lambda \leq \sqrt{\frac{\text{tr}[R_r (R_r^*)^{-1} R_r (R_r^*)^{-1} R_r]}{P_r}}. \]

Thus, we can conveniently search for the optimal \( \lambda \) using bisection method within the bounds on \( \lambda \).

### C. User Precoder Design

Given relay precode \( F \) and user receivers \( B_k, k = 1, \ldots, 2K \), we can formulate the third sub-problem to design user precoders \( A_k \). The sub-problem can be expressed as

\[ \min_{A_k, k=1, \ldots, 2K} J = \sum_{k=1}^{2K} J_k \text{ s.t. tr}[\sigma_A^2 A_k^* A_k] \leq P_k \]
\[ \sigma_d^2 \sum_{k=1}^{2K} \text{tr}[H_k^* F^* F_k^* A_k A_k^*] \leq P_r - \sigma_d^2 \text{tr}[F^* F], \]

where

\[ J_k = \sigma_d^2 \sum_{i=k}^{2K} \text{tr}[A_i H_i^* F^* F_k^* B_k B_k^* G_i F_i A_i] \]
\[ - \sigma_d^2 \text{tr}[B_k G_k F_k^* A_k] - \sigma_d^2 \text{tr}[A_k H_k^* F^* F_k B_k^* G_k F_k] \]
\[ + \sigma_d^2 \text{tr}[I_m^*] + \sigma_d^2 \text{tr}[F_i^* F_k^* B_k^* B_k G_k F_k] \]
\[ + \sigma_d^2 \text{tr}[B_k B_k^*]. \]

The objective function can be rewritten as

\[ J = \sum_{k=1}^{2K} J_k = \sum_{k=1}^{2K} \text{tr}[R_{ak} A_k A_k^*] \]
\[ -2 \sum_{i=k}^{2K} \text{Re} \{ \text{tr}[R_{bk} A_k] \} + R_c, \]

where \( R_{ak} = \sum_{i=k}^{2K} \sigma_d^2 H_i^* F^* G_i^* B_k^* B_k G_k F_k, \]
\[ R_{bk} = \sigma_d^2 B_k G_k F_k \]
\[ \text{and } R_c = \sigma_d^2 \text{tr}[I_m^*] + \sigma_d^2 \text{tr}[B_k B_k^* G_k F_k]. \]

This is a typical quadratically constrained quadratic program (QCQP) problem, and the convex optimization tool packet in [9] can be used to efficiently solve this sub-problem.
D. Summary

The iterative precoding design algorithm based on MMSE criterion is summarized as follows:

**Algorithm 1:** Iterative precoding design algorithm

1. Initialization:
   \[ A_k = \sqrt{\frac{p_r}{L_k}} \left[ I_{L_k} \right]_k, \quad k = 1, \ldots, 2K, \quad F = \sqrt{\frac{p_r}{L_k}} I_M, \]

2. Iteration:
   a) Given \( A_k, k = 1, \ldots, 2K \), and \( F \), use (15) to obtain the receiver \( B_k \) for each user node \( k \);
   b) Given \( A_k \) and \( B_k, k = 1, \ldots, 2K \), compute the relay precoder \( F \) using (24);
   c) Given \( B_k, k = 1, \ldots, 2K \), and \( F \), solve the problem (26) to obtain user precoders \( A_k, k = 1, \ldots, 2K \);

3. Termination: The iteration terminates when the total MSE in (11) converges or the maximum number of iterations are reached.

Since the updated solution is optimal to each sub-problem, the total MSE decreases after each step of iteration, and thus the iteration algorithm always converges. However, due to the nonconvexity of the joint design problem, the algorithm may converge to some local optimal solutions. We next propose a novel channel switching technique to search for different local optimal solutions.

IV. CHANNEL SWITCHING

We propose a novel channel switching technique which can be incorporated into relay precoder design to further improve system performance. As illustrated in Fig. 2, we introduce one permutation matrix \( \Gamma \in \mathbb{C}^{M \times M} \) which switches the columns of the channel \( G \), where \( G = [G_{1}^{T}, ..., G_{L}^{T}]^{T} \) is the combined multiuser channel matrix from relay node to all user nodes. By utilizing this channel switching technique, the equivalent channel of the system can be expressed as

\[ \tilde{G} = G\Gamma. \]

(29)

Then Algorithm 1 is employed to find a local optimal solution for the switched channels.

There are \( M! \) different permutation matrices for \( \Gamma \), denoted as \( \Gamma^{(p)}, p = 1, \ldots, M! \). Each permutation matrix can be described with a permutation vector \( \omega^{(p)} \) over \([1, 2, \ldots, M]\) and the corresponding permutation matrix \( \Gamma^{(p)} \) can be expressed as

\[ \Gamma^{(p)}(m, n) = \begin{cases} 1, & \text{when } \omega^{(p)}(m) = n \\ 0, & \text{otherwise}. \end{cases} \]

(30)

The permutation matrix \( \Gamma \) can actually be absorbed into the relay precoder as

\[ \tilde{F} = \Gamma F. \]

(31)

Since \( \Gamma \) is indeed a unitary matrix, the relay transmit power does not change, i.e.

\[ \text{tr}[\tilde{F}R_x\tilde{F}^T] = \text{tr}[FR_xF^T] \leq p_r. \]

(32)

Note that if the permutation matrix is applied to switch the columns of channel \( H \), the equivalent relay precoder becomes \( \tilde{F} = FT \). Hence, transmit power at the relay node will be changed and the power constraint in (5) may be violated.

Since the number of different patterns of \( \Gamma \) is finite, we exhaustively search for the best permutation matrix that leads the iterative algorithm to converge to the local optimal solution with the minimum total MSE.

The modified algorithm with channel switching is summarized below:

**Algorithm 2:** Iterative algorithm with channel switching

1. Initialization: \( M_{\text{SE}_\text{min}} = \infty, A_k = 0, k = 1, \ldots, 2K, B_k = 0, k = 1, \ldots, 2K \); set \( p = 1 \);

2. Exhaustive search:
   a) For permutation matrix \( \Gamma^{(p)}, 1 \leq p \leq M! \), use Algorithm 1 to obtain the precoders and receivers \( \{F_{local}, A_{k,local}, B_{k,local}, k = 1, \ldots, 2K\} \) and output the total MSE \( M_{SE_{local}} \) when Algorithm 1 converges;
   b) If \( M_{SE_{local}} < M_{SE_{min}} \)
      (i) \( M_{SE_{min}} = M_{SE_{local}}, F = F_{local}, A_k = A_{k,local}, B_k = B_{k,local}, k = 1, \ldots, 2K \);
      (ii) go to step (c);
   Otherwise, directly go to step (c);
   c) if \( p < M! \), \( p = p + 1 \) and go to step (a); otherwise, quit algorithm and output the best local optimal solution \( F, A_k \) and \( B_k, k = 1, \ldots, 2K \).

V. SIMULATION RESULTS

In this section, numerical results are provided to evaluate the proposed algorithms. The antenna configurations at the relay node and user nodes are \( M = 4 \) and \( N_k = 2, k = 1, \ldots, 2K \), and each user can only transmit one data stream, i.e. \( L_k = 1 \). The total number of user pairs is \( K = 2 \). The power constraints at all user nodes are identical, i.e. \( P_k = P_s, \forall k = 1, \ldots, 2K \). The average signal-to-noise ratio (SNR) for the first time slot and the second time slot of the two-way communication system are defined as \( \text{SNR}_1 = P_s/\sigma_1^2 \) and \( \text{SNR}_2 = P_s/\sigma_2^2 \), respectively. It is assumed that \( \text{SNR}_1 = \text{SNR}_2 \) in our simulation.
All channels experience flat Rayleigh fading and all entries of channel matrices are independent identically distributed zero mean complex Gaussian with unit variance. Moreover, we assume the system works in time-division duplex (TDD) mode, and the channels in the first time slot and second time slot are reciprocal. All channels are semi-static which remain unchanged during the two time slots. All results are averaged over 10,000 independent channel realizations and 10,000 QPSK symbols are transmitted for each data stream in each realization.

We compare our proposed algorithms with the MMSE relay precoding algorithm in [7]. Note that the authors in [7] only optimize the relay precoder and apply some conventional beamforming techniques at the user nodes, and we simulate the case when eigen beamforming is used for transmit and receive beamforming in their algorithm. To illustrate the benefits of joint precoding, we also simulate the relay-only precoding algorithm by fixing the user precoders as $A_k = \frac{I_K}{\sqrt{k}}\begin{bmatrix} 1 & 1 & \ldots & 1 \\ 0_{(K-k)\times K} \\ \end{bmatrix}$, $k = 1, \ldots, 2K$, and skipping step (c) in the iteration process of the proposed Algorithm 1.

Fig. 3 presents the average BER results versus SNR. The proposed Algorithm 1 significantly outperforms the MMSE relay precoding algorithm with eigen beamforming in [7].

Indeed, from the slopes of the curves in the high SNR region, it can be observed that our proposed algorithm achieves more diversity gain. This is because the algorithm in [7] requires some extra degrees of freedom at relay node to suppress self-interference, while this self-interference is canceled by the user node itself in our algorithm. It is also seen that joint precoding using Algorithm 1 achieves superior performance compared to the relay-only precoding algorithm. Furthermore, the modified algorithm with channel switching (Algorithm 2) shows the best performance among all algorithms, because it explores more local optimal solutions than Algorithm 1. The MSE results are shown in Fig. 4 with the same simulation assumptions. The proposed Algorithm 1 outperforms MMSE relay precoding algorithm with eigen beamforming again, and the proposed Algorithm 2 still has the best performance.

VI. CONCLUSION

In this paper, we have studied the joint MMSE precoding design for the multi-user two-way AF MIMO relay communication systems. Considering the complexity and nonconvexity of the problem, the original problem is decoupled into three convex sub-problems, and an iterative algorithm is proposed to cyclically solve the three sub-problems. In addition, channel switching at relay node can be utilized as a way to explore different local optimal solutions to further improve system performance. The simulation results verify the superior BER and MSE performances of the proposed joint precoding algorithms compared with other algorithms.

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