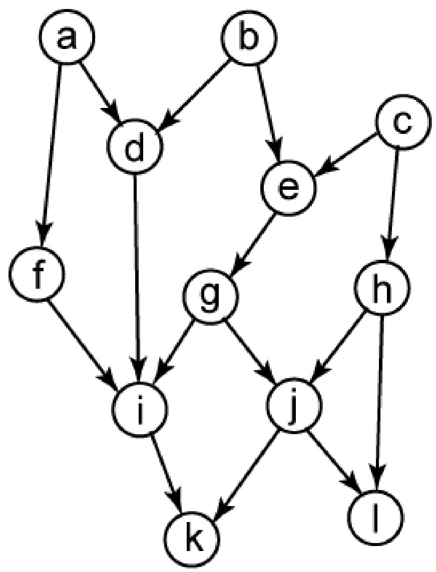
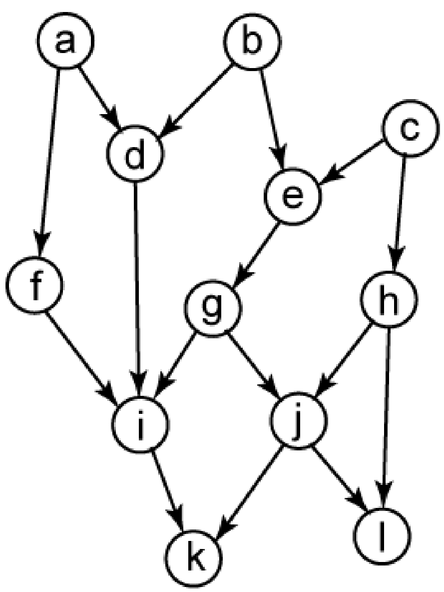
Rajaraman-Wong Algorithm

- Perform RW clustering on the following di-graph.
  - Inter-cluster delay = 3, node delay = 1
  - Size limit = 4
  - Topological order $T = [d,e,f,g,h,i,j,k,l]$ (not unique)



Max Delay Matrix

- All-pair delay matrix $\Delta(x,y)$
  - Max delay from output of the PIs to output of destination



	a	b	c	d	e	f	g	h	i	j	k	l
a	0	0	0	1	0	1	0	0	2	0	3	0
b	0	0	0	1	1	0	2	0	3	3	4	4
c	0	0	0	0	1	0	2	1	3	3	4	4
d	0	0	0	0	0	0	0	0	1	0	2	0
e	0	0	0	0	0	0	1	0	2	2	3	3
f	0	0	0	0	0	0	0	0	1	0	2	0
g	0	0	0	0	0	0	0	0	1	1	2	2
h	0	0	0	0	0	0	0	0	0	1	2	2
i	0	0	0	0	0	0	0	0	0	0	1	0
j	0	0	0	0	0	0	0	0	0	0	1	1
k	0	0	0	0	0	0	0	0	0	0	0	0
l	0	0	0	0	0	0	0	0	0	0	0	0

Label and Clustering Computation

- Compute *l*(*d*) and *cluster*(*d*)

First, $G_d = \{a, b, d\}$. By definition $l(a) = l(b) = 1$. Thus,

$$l_d(a) = l(a) + \Delta(a, d) = 1 + 1 = 2$$
$$l_d(b) = l(b) + \Delta(b, d) = 1 + 1 = 2$$

Then we have $S = \{a, b\}$ (recall that S contains $G_d \backslash \{d\}$ with their l_d values sorted in a decreasing order). Since both a and b can be clustered together with d while not violating the size constraint of 4, we form

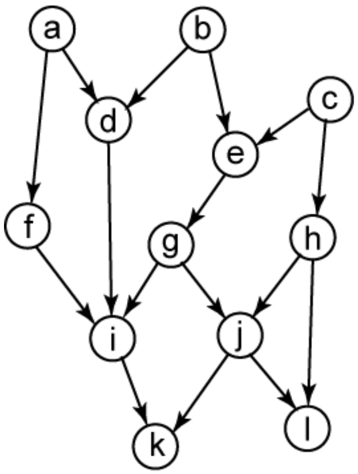
$$cluster(d) = \{a, b, d\}$$

Since both a and b are PI nodes, we see that

$$l_1 = \max\{l_d(a), l_d(b)\} = 2$$

Since S is empty after clustering, l_2 remains zero. Thus,

$$l(d) = \max\{l_1, l_2\} = 2$$

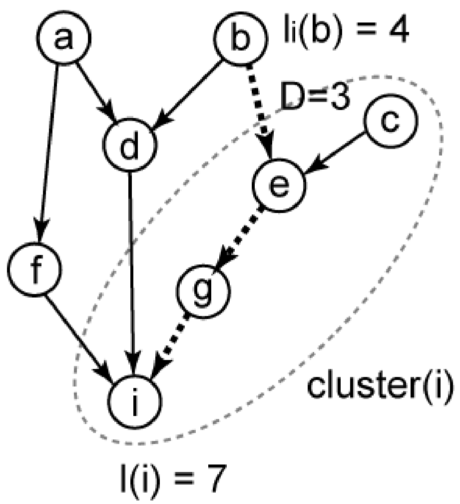


Label Computation

- Compute *l*(*i*) and *cluster*(*i*)

node i: $G_i = \{a, b, c, d, e, f, g, i\}$ (see Figure 1.3). Thus,

$$
\begin{aligned}
l_i(a) &= l(a) + \Delta(a, i) = 1 + 2 = 3 \\
l_i(b) &= l(b) + \Delta(b, i) = 1 + 3 = 4 \\
l_i(c) &= l(c) + \Delta(c, i) = 1 + 3 = 4 \\
l_i(d) &= l(d) + \Delta(d, i) = 2 + 1 = 3 \\
l_i(e) &= l(e) + \Delta(e, i) = 2 + 2 = 4 \\
l_i(f) &= l(f) + \Delta(f, i) = 2 + 1 = 3 \\
l_i(g) &= l(g) + \Delta(g, i) = 3 + 1 = 4
\end{aligned}
$$

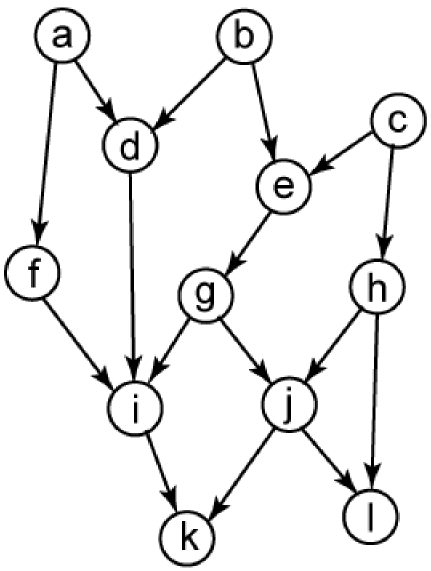


$S = \{g, e, c, b, a, d, f\}$, and we form $cluster(i) = \{i, g, e, c\}$.[1] Note that c is PI, so $l_1 = l_i(c) = 4$. Since $S = \{b, a, d, f\} \neq \emptyset$ after clustering, we have $l_2 = l_i(m(S)) + D = l_i(b) + D = 4 + 3 = 7$ (recall that $m(S)$ is the node in S with the maximum value of l_i value). Thus, $l(i) = \max\{l_1, l_2\} = 7$.

Labeling Summary

- Labeling phase generates the following information.
  - Max label = max delay= 8

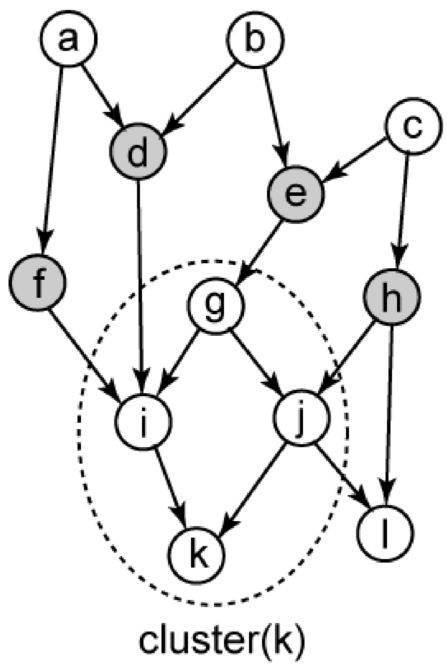
node	label	clustering
a	1	$\{a\}$
b	1	$\{b\}$
c	1	$\{c\}$
d	2	$\{a, b, d\}$
e	2	$\{b, c, e\}$
f	2	$\{a, f\}$
g	3	$\{b, c, e, g\}$
h	2	$\{c, h\}$
i	7	$\{c, e, g, i\}$
j	7	$\{b, e, g, j\}$
k	8	$\{g, i, j, k\}$
l	8	$\{e, g, j, l\}$



Clustering Phase

- Initially L = POs = $\{k,l\}$.

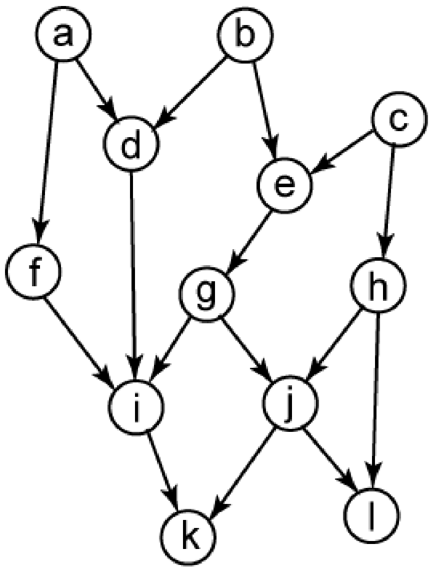
remove k from L, and add $cl(k)$ to $S = \{cl(k)\}$. According to Table 1.1, we see that $cl(k) = \{g, i, j, k\}$. Then, $I[cl(k)] = \{f, d, e, h\}$ as illustrated in Figure 1.4. Since S does not contain clusters rooted at f, d, e, and h, we have $L = \{l\} \cup \{f, d, e, h\} = \{l, f, d, e, h\}$.



Clustering Summary

- Clustering phase generates 8 clusters.
  - 8 nodes are duplicated

root	elements
k	$\{g, i, j, k\}$
l	$\{e, g, j, l\}$
f	$\{a, f\}$
d	$\{a, b, d\}$
e	$\{b, c, e\}$
h	$\{c, h\}$
b	$\{b\}$
c	$\{c\}$



Final Clustering Result

- Path *c-e-g-i-k* has delay 8 (= max label)