Optimal Maneuvering of Seismic Sensors for Localization of Subsurface Targets

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Abstract—We consider the problem of detecting and locating buried land mines and subsurface objects by using a maneuvering array that receives scattered seismic surface waves. We demonstrate an adaptive system that moves an array of receivers according to an optimal positioning algorithm based on the theory of optimal experiments. The goal is to minimize the number of distinct measurements (array movements) needed to localize mines. The adaptive localization algorithm has been tested using experimental data collected in a laboratory facility at Georgia Tech. The performance of algorithm is exhibited for cases with one or two targets and in the presence of common types of clutter like rocks found in the soil. It has also been tested for the case where the propagation properties of the medium vary spatially. In almost all test cases the mines were located exactly using three or four array movements. It is envisioned that future systems could incorporate this new method into a portable mobile mine-location system.

I. INTRODUCTION

Buried land mines and similar subsurface structures pose a huge threat to resettling civilians. It takes significant time and resources to clear out regions contaminated by mines, so it is important to develop efficient detection and localization systems to create a safer environment. Georgia Tech has built a laboratory to collect the real data needed to investigate buried land mine and subsurface target detection problems [1]. In this laboratory setting, detection schemes using seismic and electromagnetic waves have been tested [1]–[4].

The use of seismic waves to detect subsurface targets complements existing systems, which are usually based on Ground Penetrating Radar (GPR) and Electromagnetic Induction (EMI) sensing. The seismic waves are sensitive to differences in the mechanical properties of the soil and the mine while the GPR and EMI sensors sense differences between the electrical properties of the soil and the mine. The mechanical properties and structure of a mine are quite different from the soil and typical forms of clutter make its response to seismic waves somewhat different from that of most clutter objects. The dominant feature of the response of a buried mine is a soil-loaded resonance of the mine case and trigger mechanism [2], [3]. This resonance causes an enhanced and sustained motion of the soil above the mine thus enhancing the waves scattered from the mine. Hence, it is possible to use seismic imaging to discriminate land mines from common types of clutter such as rocks, wood, etc.

To detect a mine, a seismic wave launched from a source at a known location travels through the soil and interacts with underground objects. The resulting propagating waves in an elastic medium are of two main types: surface waves and body waves. This paper concentrates on reflected surface waves (Rayleigh waves) for locating the mine positions, because the Rayleigh waves carry the most of the returned energy.

Previous work on the seismic detection of land mines can be divided into two categories: methods that measure the seismic wave field directly above a mine and those that only measure a portion of the wave field away from the mine. For those in the former category, the wave field is measured over a two-dimensional region above a mine at several thousand points. This technique has been shown to be relatively resistant to clutter due to the strong resonate response directly above a land mine; however, making all these measurements is quite time consuming [3], [5], [6]. Methods to speed up these techniques using large arrays of sensors is ongoing [7], [8]. For methods in the latter category, the surface waves scattered from the land mine are measured at some distance away from the mine [9], [10]. This technique is more sensitive to clutter because the resonant response is more difficult to isolate in the scattered waves; however, it is potentially much faster because fewer measurements are required. In this paper, we develop a technique that has the potential to combine the strengths of the above techniques without the weakness. We show how a small array which starts at some distance from a buried land can be optimally maneuvered to find it. Finally, the array can be placed directly above the mine to verify the detection by direct measurement of the resonance above the mine.

(Some more work is need to integrate this into the paragraph.)

might also allow the source to maneuver but that case is not treated in the paper. With each new measurement, we want to maximize the information gained about the target. In our case, we use a small $3 \times 3$ array, so any one image has low resolution. However, as the array maneuvers, we can accumulate the measurements, and the cumulative imaging operation improves the resolution around the true mine location by increasing the effective aperture.

Most methods for optimally maneuvering of sensors use some form of the Fisher information matrix (from which the Cramer-Rao bound is obtained). One applicable theory for optimal sensor placement is the “theory of optimal experiments” [11], which predicts the results of experiments based on information-theoretic concepts. Fisher information is used as the design criterion in the theory of optimal experiments.
Various measures of Fisher information are possible, giving different design criteria, e.g., the D-criterion, A-criterion, or E-criterion, etc. An example of using the method of optimal experiments for sensor placement can be found in [12], [13], which deals with the movement of sensors used in direction-of-arrival (DOA) estimation to localize the source. The optimality criterion is based on maximizing the determinant of the Fisher information matrix and minimizing the trace of the Cramer-Rao lower bound (CRLB). In this case, the optimal observer (sensor) path is determined to localize a moving source. In [14], a single moving sensor is used to localize a vapor emitting source by estimating the location of the source and its CR bound at each step. The sensor position at \( n + 1 \) is calculated so as to minimize the CR bound on the location errors, given the measurement up to and including time \( n + 1 \). In [15], a D-optimal design is used for optimal sensor placement to solve an inverse problem. A recent example of D-optimal experiment design involves moving an electromagnetic induction (EMI) sensor to locate buried targets [16].

The method presented in this paper will also use D-optimal design to maneuver a seismic array to locate buried targets. The organization of the paper is as follows. Section II describes the steps in the proposed algorithm including the 2-D spectrum analysis technique for separating waves, the data model and imaging method, and performance bounds on target location estimates. Also, the D-optimal algorithm for maneuvering the seismic array is derived. Sections III to VII describe the results of applying the new algorithm to experimental data collected in a laboratory setting for scenarios with single, multiple targets, and clutter.

II. PROPOSED ALGORITHM FOR OPTIMAL MANEUVERING

The proposed optimal maneuvering algorithm is an iteration that involves three main steps: The first step is identification of the seismic wave components and separation of the reflected wave from the incident wave; the second is near-field imaging with a propagation model to estimate the target location; and the third is the optimal maneuver calculation to reposition the receiving array for the next iteration. During the first step, different seismic wave components have to be identified and separated, because the imaging in the second step must be done with only reflected waves. The system uses an active source that is also in the vicinity of the receivers, so the array will also record a very strong forward wave. For example, the raw collected data at four time instants from a TS-50 (anti-personnel) land mine buried at a depth of 1 cm. is shown in Fig. 1. This figure shows the strong forward seismic wave approaching the mine during the first two frames, and reflecting from the mine in the third. In the last frame the weak reflection from the mine can be seen clearly.

If the target is far away, we can remove the forward pulse with a time gate, but when the target is nearby, the two waves tend to overlap in time. A robust frequency-domain algorithm based on Prony’s method is used to first identify different wave components and then separate them [17], [18]. This analysis technique requires a linear array of sensors to collect the space-time data, and ten or fifteen sensors will suffice. At present, we use a \( 3 \times 10 \) array and perform the wave separation on each ten-element linear subarray. Then a \( 3 \times 3 \) subarray of sensors is retained for the imaging and optimal maneuvering steps.

After the waves have been separated, the next step is to image the targets to locate their positions. The sensor array and source form an active array system. The data model that applies to this case can be derived from the model used in classical passive array design. After forming the data model, we derive an imaging algorithm that works in the frequency domain. Even though the received sensor data is not narrowband, the nature of the seismic waves suggests that frequency domain processing is more suitable for two reasons. First, soil is an example of a highly dispersive medium, where propagation velocity varies with frequency. Second, targets at various depths can be imaged by varying the frequency content of the probing pulse. Since the propagation velocity varies with frequency, we formulate a propagation model and steering vectors that can be applied to this case.

One goal is to design the system so that the array can be placed on a mobile platform, which can maneuver as it senses the environment. Therefore, the size of the array has to be small, which means a small aperture. This small aperture will result in low resolution or higher uncertainty about the position estimate of the target. One way to increase resolution is to increase the effective aperture by moving the array and forming a synthetic aperture. Hence, the algorithm that determines the next optimal position for the array must also accumulate its measurements from several positions.

This is how the algorithm will work. At first, using the 2-D sensor array with known relative positions, an initial estimate of the target location is made by using the imaging algorithm. Then, the variance of the location estimate is calculated by using the Fisher information matrix (FIM). Based on the expected value of the FIM, the next optimal array position is determined by using the theory of optimal experiments [11], [16]. The search for the next optimal array position maximizes...
the determinant of the Fisher information matrix, which is called D-optimal design. The two steps involved in the maneuver strategy for a mobile array of sensors are shown in Figs. 2(a) and 2(b).

Fig. 2. Algorithm steps illustrated: (a) The source generates a probing pulse. Waves scattered from the target are collected by the receiving array. At step 1, the target position \( z_i \) is estimated when the array center is at \( \zeta_i \). (b) Estimate the next array position \( \zeta_{i+1} \) by using the target estimate \( z_i \) and the constrained cumulative Fisher Information Matrix measure along a circle.

A. Spectrum Analysis Technique for Wave Separation

The parametric model is based on a technique first developed for borehole sonic logging applications [19]. The collected data \( s(x, t) \), which is a function of space and time, has a 2-D Fourier representation in the \((k, \omega)\) domain

\[
s(x, t) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{S}(k, \omega) e^{i(kx - \omega t)} \, dk \, d\omega, \tag{1}
\]

where \( x \) is the spatial position, \( k \) is the spatial wave number, \( \omega \) is the temporal frequency, and \( \hat{S}(k, \omega) \) is the 2-D Fourier transform. By taking a Fourier transform of the space-time data across \( t \) only, we obtain

\[
S(x, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{S}(k, \omega) e^{i(kx)} \, dk, \tag{2}
\]

At each temporal frequency \( \omega \), exponential modeling is done across the spatial dimension to get a model consisting of a sum of exponentials that represents propagating waves. In effect, we approximate the integral in (2) with a sum

\[
S(x, \omega) \approx \sum_{p=1}^{P} a_p(\omega) e^{i\kappa_p(\omega)x}, \tag{3}
\]

where \( P \) is the model order. The pole-zero modeling technique used is based on the Iterative Quadratic Maximum Likelihood (IQML) algorithm, which is also called the Steiglitz-McBride extension of Prony’s method [18], [20]–[22].

The poles determine the exponent \( \kappa_p(\omega) = k_p(\omega) + j\alpha_p(\omega) \) whose real part is the wave number \( k_p(\omega) \) and whose imaginary part is the attenuation \( \alpha_p(\omega) \). Wave number can be converted to phase velocity via

\[
v_p(\omega) = \omega / k_p(\omega),
\]

and then we can plot the magnitude of \( a_p \) versus frequency and velocity. This type of plot is a 2-D velocity-frequency spectrum from which it easy to obtain the dispersion curves for the various modes that make up the signal. The complex amplitudes \( a_p \) are used to determine the strength of different wave components.

Furthermore, the individual modes of \( s(x, t) \) can be identified and grouped according to velocity \( v_p(\omega) \) and frequency. Once we have sorted out a single mode in the velocity-frequency domain, the waveform for that mode can be reconstructed in the space-time domain by using the model

\[
s(x, t) = \sum_i a(\omega_i) e^{i(\alpha(\omega_i)x + f(\omega_i)t + k(\omega_i)x)} \tag{4}
\]

This processing has been applied to data collected in the laboratory and the field, and used to extract waves reflected from buried targets [18]. Consider the setup shown in Fig. 3(a), where a linear array lies between the source and the target. The target in this case is a VS-1.6 AT land mine buried at a depth of 8 cm. The array consists of ten sensors (ground contacting accelerometers) with an inter-sensor spacing of 3 cm. The resulting velocity-frequency spectrum is shown in Fig. 3(b). The analysis easily separates the forward and reflected waves on the basis of positive and negative velocities, or equivalently, positive and negative wave numbers. Once these waves are identified in Fig. 3(b), their individual parameters can be extracted, followed by reconstruction in the space-time domain using (4). The extracted forward and reverse waves at the first sensor are shown in Fig. 4, showing how well this method is able to separate and reconstruct these waves.

B. Target Location Estimates and Performance Bounds

Once we have extracted the reflected wave(s) we can address the problem of finding the target location(s) as a near-field array processing problem.

1) Data Model for active sensing: Consider a single seismic source illuminating \( K \) targets, and an array of \( P \) seismic receivers, where the source, targets and receivers are coplanar.
Since we model the soil as a dispersive medium with frequency-dependent velocity, we prefer to do the processing in the frequency domain.

The received seismic data at frequency $\omega$ can be written as

$$y(\omega) = G(\omega)D(\omega)g_1(\omega) + n(\omega),$$

where $g_1(\omega)$ is a $K \times 1$ vector that models the propagation from the single seismic source to the targets, $D(\omega)$ is a $K \times K$ diagonal matrix whose elements are scattering coefficients from the $K$ targets, $G(\omega)$ is a $P \times K$ matrix that represents the propagation from the targets back to the receiver array, and $n(\omega)$ is additive noise [23]–[26]. The elements of the propagation matrices are given by the 2-D Green’s function. Since only the reflected signals are of interest, the active system in (5) can be simplified to the following equivalent passive system

$$y(\omega) = G(\omega)s(\omega) + n(\omega),$$

where $D(\omega)g_1(\omega)$ has been replaced by a $K \times 1$ signal vector $s(\omega)$ that represents the reflected signals from targets. Equation (6) has the same mathematical form as the narrow-band data model [27] used in conventional array signal processing and this similarity will be exploited while calculating the maximum likelihood location estimate.

In the seismic problem, the elements of the propagation matrix $G(\omega)$ are given by the illuminating Green’s vector (steering vector) [23]–[25], [28],

$$g(z, x, \omega) = [\tilde{g}(z, x_1, \omega), \ldots, \tilde{g}(z, x_P, \omega)]^T,$$

where $z$ is the target position, $x_i$ the $i^{th}$ sensor position in the 2-D plane, and $\tilde{g}$ the 2-D Green’s function, whose analytical form is

$$\tilde{g}(r, r’, \omega) = \frac{i}{4} H_0^{(1)} \left( \frac{\omega}{v(\omega)} |r - r’| \right),$$

where $H_0^{(1)}$ is the zero-order Hankel function of the first kind, and $v(\omega)$ is the frequency-dependent Rayleigh wave velocity.

Spectrum analysis of the surface waves introduced in Section II-A can be used to determine the velocity vs. frequency [18]. To minimize confusion when we refer to existing array processing literature results, we change the notation for the propagation matrix from $G$ to $A$. The final form of the data model becomes [27]:

$$y(\omega) = A(\zeta, z, \omega)s(\omega) + n(\omega),$$

where $y(\omega) \in \mathbb{C}^{P \times 1}$ is the noisy array output vector, $n(\omega) \in \mathbb{C}^{K \times 1}$ is complex additive noise, and $s(\omega) \in \mathbb{C}^{K \times 1}$ is the signal vector. The steering matrix $A(\zeta, z, \omega)$ has elements given by the Green’s function (7), which depends on the array center position $\zeta$ and the (unknown) target position $z$. Consequently, our objective is to determine the target position $z$ given the received array data $y(\omega)$.

2) Target Location Estimation: Let the data vector $Y = \left[ y^T(\omega_1), \ldots, y^T(\omega_N) \right]^T$, $Y \in \mathbb{C}^{P \times N}$, be formed by aggregating the Fourier transform of $y$ at $N$ frequencies, $\omega_i$. Under the assumption of independent, identically distributed (i.i.d.) Gaussian noise, the likelihood function (a probability density) for the current received data [29] is:

$$p(Y) = \prod_{l=1}^N \frac{1}{\pi\sigma_n^2} \exp \left\{ -\frac{1}{\sigma_n^2} \|y(\omega_l) - A(\omega_l)s(\omega_l)\|^2 \right\}$$

From (10), we obtain the negative log-likelihood function

$$L^- = NP \log(\pi\sigma_n^2) + \frac{1}{\sigma_n^2} \sum_{l=1}^N \|y(\omega_l) - A(\omega_l)s(\omega_l)\|^2.$$ \hspace{1cm} (11)

and the Maximum-Likelihood (ML) estimate is determined by minimizing $L^-$. In (11), both the target signal and the noise variance are unknown, but the gradient of $L^-$ separates so that we can first solve for the noise variance from the derivative of $L^-$ with respect to $\sigma_n^2$:

$$\hat{\sigma}_n^2 = \frac{1}{NP} \sum_{l=1}^N \|y(\omega_l) - A(\omega_l)s(\omega_l)\|^2.$$ \hspace{1cm} (12)

Using (12) we can then solve for the signal from the derivative of $L^-$ with respect to $\hat{s}(\omega_l)$

$$\hat{s}(\omega_l) = (A^H(\omega_l)A(\omega_l))^{-1} A^H(\omega_l)y(\omega_l).$$ \hspace{1cm} (13)

where $A^H$ is complex conjugate transpose of $A$. Substituting (12) and (13) in (11), one can determine the ML cost function to minimize as a function of $z$

$$J(z) = \sum_{l=1}^N \left\{ \left( I - A(\omega_l) (A^H(\omega_l)A(\omega_l))^{-1} A^H(\omega_l) \right)y(\omega_l) \right\}^2$$

$$= \sum_{l=1}^N \text{trace} \left\{ P_A^+(\omega_l)R_y(\omega_l) \right\},$$ \hspace{1cm} (14)

where $P_A^+(\omega_l) = I - A(\omega_l)(A^H(\omega_l)A(\omega_l))^{-1} A^H(\omega_l)$ is the projection onto the null space of $A^H(\omega_l)$ and $R_y(\omega_l) = y(\omega_l)y^H(\omega_l)$ is the single snapshot covariance matrix estimate at $\omega_l$. The target location estimate is then obtained by minimizing the cost function $J(z)$, i.e., $z = \arg \min J(z)$.

An example of estimating the target position for the TS50 data in Fig. 1 is shown with the surface plot in Fig. 5. The surface plot (of the inverse of $J(z)$) was obtained by using (14) and evaluating this cost function at each point $z$ over a 2-D grid. The target position estimate is the minimum indicated with a square.
3) **Cramer-Rao Lower Bound for the Estimate of $\mathbf{z}$**: The Cramer-Rao lower bound (CRLB) is an information theoretic inequality which provides a lower bound for the variances of the unbiased estimators. If an estimator achieves the CRLB, then it is also a solution of the likelihood equation. The Cramer-Rao lower bound is the inverse of the Fisher information matrix (FIM). Assuming that the variance of the additive noise in (5) is known, the log-likelihood function for a single target can be written as:

$$L(\mathbf{z}, \mathbf{w}) = -\frac{1}{\sigma_n^2} \sum_{i=1}^{N} \| \mathbf{y}(\omega_i) - \mathbf{a}(\mathbf{z}, \omega_i) s(\omega_i) \|^2,$$

where $\mathbf{a}(\mathbf{z}, \omega)$ is the propagation (steering) vector from the array center $\mathbf{z}$ to the target position $\mathbf{w}$. The $(i, j)^{th}$ element of the FIM $\mathbf{F}$ is given by the partial derivative of (15) with respect to the $i^{th}$ and $j^{th}$ parameters of the vector $\mathbf{z}$ [29]:

$$\mathbf{F}_{i,j}(\mathbf{z}, \mathbf{w}) = E_{\mathbf{y}} \left( \frac{\partial^2 L(\mathbf{z}, \mathbf{w})}{\partial z_i \partial z_j} \right)$$

$$= -\frac{2}{\sigma_n^2} \sum_{i=1}^{N} \mathbb{E} \left[ \left( \frac{\partial \mathbf{a}(\mathbf{z}, \omega_i)}{\partial z_i} \right) H \frac{\partial \mathbf{a}(\mathbf{z}, \omega_i)}{\partial z_j} \right],$$

where $E_{\mathbf{y}}\{\cdot\}$ denotes the expected value. The partial derivative of the steering vector is calculated with respect to the target coordinates for a fixed array center.

**C. Movement of the Seismic Array via Optimal Experiments**

Previously, we described how to determine the target position estimate and the corresponding FIM which represents the uncertainty about the estimate as a function of the array center position $\mathbf{z}$. Suppose that we are at step $i$ with a target location estimate $\mathbf{z}_i$, and now we are interested in determining the next optimal array center position candidate $\mathbf{z}_{i+1}$. Our approach is to select the new position to reduce the expected uncertainty in the estimated target coordinates by minimizing the determinant of the CRLB, or, equivalently, by maximizing the determinant of the FIM as a function of the array center $\mathbf{z}$. In the literature of optimal experiments, this technique is called D-optimal design [11] (see [16] for an application to magnetic sensors).

Let $q$ represent the determinant of the FIM. The cumulative effect of the measurements up to step $i$ can be written as:

$$q(\{\mathbf{z}_1, \ldots, \mathbf{z}_i\}) = |\mathbf{F}(\mathbf{z}_1, \ldots, \mathbf{z}_i)| = \sum_{j=1}^{i} \mathbf{F}(\mathbf{z}_j),$$

where $| \cdot |$ stands for determinant and $\mathbf{F}(\mathbf{z}_i)$ represents the FIM at step $i$. The logarithmic increase due to the additional measurements at step $i + 1$ is given by

$$\delta_q(\mathbf{z}_{i+1}) = \ln q(\{\mathbf{z}_1, \ldots, \mathbf{z}_i, \mathbf{z}_{i+1}\}) - \ln q(\{\mathbf{z}_1, \ldots, \mathbf{z}_i\})$$

$$= \ln | I + \mathbf{F}(\mathbf{z}_{i+1}) B_i^{-1} |,$$

where $I$ is an identity matrix, and $B_i = \sum_{j=1}^{i} \mathbf{F}(\mathbf{z}_j)$. Therefore, to maximize the expected information gain, the next optimal array center should be determined by

$$\mathbf{z}_{i+1} = \arg \max_{\mathbf{z}} \left\{ \ln | I + \mathbf{F}(\mathbf{z}_{i+1}) B_i^{-1} | \right\}. \quad (18)$$

1) **Constraints**: In this optimization problem (18), there are hidden constraints that come from the configuration of the seismic measurement system. First of all, we need to make sure that the receiving array is always between the source and the targets to receive adequate reflected waves. At the back of the target, the scattered waves are weak and are mixed with the very strong forward probing pulse, so it is difficult to extract the reflected wave using the Prony analysis. One way to impose this condition is to restrict the movement of the array center to be less than a fixed step size of $r$. In effect, this means that we calculate the maximum of (18) on a circle of radius $r$, with the center of the circle at the previous optimum array center position.

Another way to constraint the array movement would be to add a penalty term as in [16]:

$$\Psi(\mathbf{z}) = \delta_q(\mathbf{z}_{i+1}) - \nu \sqrt{ \left( \mathbf{z}_{i+1} - \mathbf{z}_i \right)^T \Sigma^{-1} \left( \mathbf{z}_{i+1} - \mathbf{z}_i \right) },$$

where $\nu \geq 0$ is the penalty factor that must be chosen relative to the size of $\delta_q(\mathbf{z}_{i+1})$, and $\Sigma$ is a diagonal matrix, whose diagonal elements are chosen to ensure smooth movement of the array from previous position. However, this approach constrains the step size indirectly, depending on the choice of $\nu$, and does not guarantee that the array will stay between the source and the target, so it has not been used here.

An example of the circle constraint is shown in Fig. 6(a), where a circle of radius of 25 cm is used. Based on the initial target location estimate of the TS-50 land mine in Fig. 5, the unconstrained optimal array position would be determined by using (18) evaluated at each grid point where the new array center could be located. The circle constraint restricts this evaluation to the small subset of points on the circle in Fig. 6(a), so it is also much more efficient than the unconstrained approach or the penalty function method.

Once the next optimum array position is determined and the array is moved to the new position, a new batch of data is collected. This new data is appended to the existing data, and the new target position estimate, as well as the next optimum array movement, are determined by using the cumulative data. Further steps are shown in Fig. 6(b). With each successive step the target position estimate is improved in the sense that there is a decrease in the uncertainty ellipse of the estimate. Intuitively, the explanation is that the cumulative estimation is effectively increasing the aperture.

**III. PROCESSING OF EXPERIMENTAL DATA**

Several experiments have been conducted in order to characterize the behavior of this maneuvering-array system when finding single and multiple targets, with and without clutter objects. This section summarizes those trials. The data was recorded in a laboratory setting, where mines (and rocks) buried in a sandbox are used as targets [1]. Typical raw data that would be collected has already been shown in Figs. 1 and 4. A shaker is used as the seismic source to produce an input signal that is a differentiated Gaussian pulse centered at
450 Hz with a bandwidth that is significant from 100 to 1500 Hz. A $3 \times 10$ array of ground contacting accelerometers is used as the receiving array [30]. The rectangular seismic array consists of three lines each containing ten uniformly spaced sensors. In all of the experiments described below the system was run in an “on-line real-time” mode. The movement of the array, the firing of the source, and the data acquisition are controlled by a LabView interface. The data acquired from the accelerometers is then fed to a MATLAB program that determines the next optimal position for acquiring more data. The source and receiver setup for the real-time experiment is shown in Fig. 7.

A. Single Target Case

In this case a single anti-tank (AT) mine, VS-1.6, is buried at a depth of 5 cm. The AT mine has a diameter of 20 cm. The data collected across each 10-element line of the seismic array is processed by the Prony-based velocity spectrum analysis, and the direct and the scattered waves are separated using the spectrum analysis. The reflected waves are resynthesized at three out of the ten sensors in each line and retained to form a $3 \times 3$ array for imaging.

There are two phases involved in the system’s maneuvering for target detection. First is the probe phase, where two fixed array positions with respect to the source are used to form an initial target position estimate via the ML imaging algorithm Fig. 8(a). Then the system enters its search phase, where the array position is calculated from the FIM, and the target position estimate is refined according to (14) and (18). Using the optimal maneuver strategy, the target is found with three search phase array measurements, Fig. 8(b), which requires measurements at only a of 150 individual sensor points. This is approximately a 24-fold reduction in measurements required to scan the entire 2-D region and make an image which requires approximately 3600 individual sensor points.

In this experiment, the optimal array movement is constrained by the circle constraint, using a radius of 25 cm to control the step size. The initial and final location estimates, shown in Fig. 8(a) and (b), show the improvement in the target estimate. With the circle constraint, we observe that the array moves toward the target while increasing the effective aperture, and thereby reduces the size of uncertainty ellipse around target position estimate.

B. Multi-target case: Experiment M-1

In the first multi-target experiment, we assume that the number of targets is known. Two VS-1.6 anti-tank (AT) mines are buried at a depth of 5 cm. During the probe phase, three fixed array positions with respect to the source are used to find the starting locations for the search phase. The three fixed array positions are shown only as their centers (+ signs), but the shape and size of the array is the same $3 \times 3$ array used before. The ML estimates of the two target positions are shown in Fig. 9. Since two initial ML estimates have been obtained, the method for maneuvering the array optimally with the FIM must be modified. The FIM is now of size $4 \times 4$, so we partition the matrix into four $2 \times 2$ submatrices related to the individual targets and their cross terms.

Since we want to minimize the determinant of the FIM as in (18), there are various options available. One is to use the full $4 \times 4$ FIM matrix in (18), and the other would be to devise a partitioned approach with the two smaller $2 \times 2$ FIMs, one for each target. The second approach is inherently more complicated and might involve multi-objective optimization to satisfy both measures.

Thus we use the $4 \times 4$ approach and determine the next array position by using (18) with a circle constraint. A circle
of radius 25 cm is used, and the array center at position-2 is used as the center of the first circle. The surface obtained by using the $4 \times 4$ FIM is shown in Fig. 10(a) and the values on the circle from $-90^\circ$ to $90^\circ$ are shown in Fig. 10(b). There are two well defined peaks with one direction favored more than the other. The higher peak in this plot is picked to generate the next optimal array position.

The succeeding array positions are obtained optimally and the surface plot at step 4 and also at the last step are shown in Fig. 11(a) and (b). From this figure, we can make a couple of interesting observations: once the optimal maneuvers pick one target the algorithm continues to move toward that target. At the same time as the array moves toward target-1, the signature of target-2 becomes weaker. At the last step (Fig. 11(b)) target-1 has been localized completely with appreciable reduction in the size of its uncertainty ellipse. However, the weak signature of target-2 is still somewhat present, and with a very accurate position estimate.

Once we have completely localized the first target, we would return to the original starting positions and remove the effects of the already localized target from the array data. The remaining targets can then be localized. The removal technique used will be based on the CLEAN algorithm originally developed for radio astronomy [25], [31]. Suppose that there are $M$ targets, and out of these we have estimated all the positions except for the $m^{th}$ target. Then the “cleaned” array data at a frequency $\omega_l$ which can be used for this target is given by:

$$y_m(\omega_l) = y(\omega_l) - \sum_{j=1, j \neq m}^{M} g(p_j, \omega_l) s_j(\omega_l)$$  \hspace{1cm} (20)

where $g$ is the steering vector whose elements are given by the known 2-D Green’s function, $p_j$ is the $j^{th}$ target position estimate, and $s_j(\omega_l)$ is the signal reflected from the $j^{th}$ target which can be estimated using (13). Once we remove the first target using (20), the FIM will be reduced to a $2 \times 2$ matrix. The probe phase for the second target uses the previous starting position as shown in Fig. 12(a), the only difference being that the effect of target-1 has now been completely removed from the array data. The next few optimal moves to locate target-2 are shown in Fig. 12(b). Now this second target has been completely localized in addition to considerable reduction in the size of its uncertainty ellipse.

C. Multi-target case: Experiment M-2

In most realistic situations, there is no a priori knowledge of the number of targets. For these situations, a different strategy must be developed used. At each iteration, we could assume that there is only one strong target, and then locate this target using optimal moves. Once it is localized, we could then remove the contribution of this target from the array data by using the CLEAN algorithm, and proceed to find the next strongest target, and so on. We would repeat these operations until all the possible targets were localized. An important question to answer for this strategy is how to be sure that there is nothing left to be located. To answer this question, a power distribution function is proposed based on the metric...
in [31]. It uses the array data at the probe phase only.

\[
P(p, \omega_l) = \sum_{i=1}^{N} \frac{|g^H(p, \omega_l)y(\omega_l)|^2}{\|g(p, \omega_l)\|^2}
\]

This power distribution is calculated over the area of interest as a function of position \( p \), using the Green’s function vector (steering vector) \( g(p, \omega_l) \) and \( y(\omega_l) \) is the array data at frequency \( \omega_l \).

For a stopping criterion, a scalar metric is calculated based on matrix norm of \( P(p, \omega_l) \). The norm of a matrix is a scalar that gives some measure of the magnitude of the elements of the matrix. The norm used is the Euclidean norm (\( L_f \)). When there are strong targets present in a uniform background, we get a distribution with higher values for \( L_f \). However, once we locate the targets, and remove their contributions from the array data, the power distribution decreases along with the values of the norm. Hence, one way to decide when to stop is to calculate this distribution along with the matrix norm, after localizing each target. If the values in the metric become very small as compared to the starting value, then this indicates that no stronger targets remain.

To simulate this scenario, an AT mine (VS-2.2) and a rock of nearly same size and shape are buried at a depth of 6 cm. Both of these targets are nearly at the same distance from the source, with the rock being slightly closer. Assuming that there is only one target, we let the array maneuver optimally. After three iterations, the rock, which is stronger in this case is localized as shown in Fig. 13(a). After this, the same steps are repeated, with the contributions of the already localized target removed from the array. This time the array moves toward the second target (the AT mine) and localizes it in three optimal moves as shown in Fig. 13(b).

After each probe phase we calculate the power distribution and its norm. At first there are two strong targets. The histogram of this power distribution is shown in Fig. 14(a), with calculated value of a norm of 31.58 (Table I). Then once this target is localized, we remove its contribution from the array at the same probe position. The second power histogram is shown in Fig. 14(b) with a norm value of 12.45 (Table I). The third power histogram when both targets have been removed is shown in Fig. 14(c), and the calculated norm value is 7.24 (Table I). This final norm value is the same as for an empty sandbox and can be used to define a stopping criterion.

For the stopping criterion we will use a threshold that is within \( \pm 10\% \) of the empty sandbox value (\( L_f = 7.5 \)). For example, consider the data set with a single TS-50 AP mine, which is Case 4 in Table II and Fig. 15. Its value with the contribution of the mine removed from the array is \( L_f = 7.41 \) which is within \( 10\% \) of 7.5. In other realistic situations we would have to calibrate the array by using an area without any targets to calculate the benchmark metric values for the stopping criterion. The power distribution will depend upon several factors including the propagation properties of the medium, the dynamic range of the seismic source, as well as the target types, sizes and burial depths.
E. Targets in Clutter (rocks)

To test the behavior of the algorithm in clutter, rocks are introduced in addition to land mines. The fact that land mines resonate, and hence have more reflected energy, can be used to discriminate them from clutter such as rocks. However, rocks also scatter seismic waves, just like land mines, so discriminating the rocks from land mines with the method of optimal maneuvers depends upon their relative positions with respect to the source, and their burial depths. In these experiments rocks are introduced to confuse the system in locating mines.

1) Experiment C-1: The first experiment is with a small TS-50 AP mine buried at a depth of 1 cm surrounded by four rocks which are nearly the same size as the mine. The locations and burial depths of the rocks and the mine are shown in Fig. 16(a). The location estimate using optimal maneuvers is shown in Fig. 16(b), and the array is able to pick out the target even in the presence of these rocks.

2) Experiment C-2: In the second experiment, shown in Fig. 17(a), a VS-1.6 AT mine is surrounded by nine rocks. The optimal maneuvers to locate the mine are shown in Fig. 17(b).

Since the VS-1.6 is a big mine, its signature is very strong as compared to the rocks. Hence, it can be picked out of rocks very easily.

F. Drunken Waves Case

Drunken waves are generated whenever there is a drastic change in the propagation properties of the intervening medium. With such a change, waves can take a curved route instead of propagating on a straight path. The main pulse might not strike the mine, and hence it will be difficult to locate them. Therefore, it will be interesting to see how the optimal maneuvering algorithm handles the drunken wave case. In this example, the main pulse of the wave bends and the mine is not placed in the path of main pulse. These drunken waves were generated in a sandbox by changing the soil properties of some portions of the sandbox. These properties can be affected by water content and the degree of cohesion between sand particles. A TS-50 AP mine, buried at a depth of 1 cm is used for this experiment. A single large shaker is used as a seismic source. The surface displacement plots for this case are shown in Fig. 18, which shows seismic waves at four different time instants. In the ideal case, waves should pass straight through the center of scan region, but in this case the waves bend and turn to right. It is very difficult to pick the mine from this raw data, and the only indication is in Figs. 18(b,c), where the position of the mine is indicated by an arrow. A weak indication of resonance can be seen at the mine location. The results of applying optimal maneuvering to this case are shown in Fig. 19. For this drunken wave, it is surprising that the algorithm is able to pick up the mine with exact location estimates.

---

**TABLE II**

Estimated norm values for power distribution for different scenarios as plotted in Fig. 15.

<table>
<thead>
<tr>
<th>Case</th>
<th>Description</th>
<th>$L_f$ (Initial)</th>
<th>$L_f$ (Rock removed)</th>
<th>$L_f$ (Rock and AT mine removed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1:</td>
<td>Single AT mine and a rock</td>
<td>31.58</td>
<td>12.45</td>
<td>7.24</td>
</tr>
<tr>
<td>Case 2:</td>
<td>Single AP mine and 4 rocks</td>
<td>19.39</td>
<td>11.6</td>
<td>7.98</td>
</tr>
<tr>
<td>Case 3:</td>
<td>Single AT mine and 9 rocks</td>
<td>22.077</td>
<td>12.762</td>
<td>8.19</td>
</tr>
<tr>
<td>Case 4:</td>
<td>Single AP mine</td>
<td>13.07</td>
<td></td>
<td>7.41</td>
</tr>
</tbody>
</table>

Fig. 15. $L_f$ norm of the power versus iteration number for four different cases (mines and rocks). At each iteration we calculate the strongest target along with the metric; then, we remove this target, and repeat the process. The values should converge to that for an empty sandbox, when all strong targets are located and removed.

Fig. 16. TS-50 surrounded by four rocks (a) Experimental setup showing rocks and the mine. (b) Final location estimate.

Fig. 17. VS-1.6 surrounded by nine rocks (a) Experimental setup showing rocks and the mine. (b) Final location estimate.
Control of an array to find buried targets. A seismic surface wave induces a considerable resonance in the buried land mines. Howeve r, confirmation is required. An investigation of acoustic-to-seismic coupling to detect buried antitank landmines is presented.

**Figures:**
- Drunken wave case. Surface displacement showing the interaction of waves with a TS-50 mine buried at a depth of 1 cm, at four instants. Location of mine is shown by an arrow (40 dB scale), (a) wavefronts at the start of the scan region (b) at the middle. Waves start to bend, and the main pulse goes to the right (c) at another instant, a weak resonance can be seen at the mine location (d) main pulse has bent toward the right and out of the scan region.
- Drunken wave case for a TS-50 mine buried at a depth of 1 cm. Location estimate after final move.

**IV. Conclusions**

The algorithm presented in this paper shows that it is possible to control a maneuvering array to find buried targets like landmines. A complete mine finding system would require one more step to distinguish a land mine from clutter. Since the maneuver algorithm can obtain very accurate estimate of the target location, the array would be positioned to exploit the induced resonance of buried land mines to make the confirmation. It is known that a seismic surface wave induces a considerable resonance in the buried land mines. However, to isolate and identify the resonance, a different imaging algorithm must be employed and the measurement has to be done on top of the mine.

**References**


