ECE3040 – Assignment 7

1. This problem illustrates several different methods for solving the single-stage amplifier circuits. Some are more difficult than others, but all give the same numerical answers when numbers are used.



- (a) Figure (a) shows a common-emitter amplifier.
 - 1. If $R_{te} = 0$, use the pi model with $i'_c = g_m v_{be}$, voltage division, and Ohm's Law to show that

$$v_o = g_m \times \frac{v_{tb} r_\pi}{R_{tb} + r_\pi} \times \left[-\left(r_0 \| R_{tc}\right) \right]$$

Show that $r_{in} = R_{tb} + r_{\pi}$ and $r_{out} = r_0 || R_{tc}$.

2. If $R_{te} = 0$, use the pi model with $i'_c = \beta i_b$ and Ohm's Law to show that

$$v_o = \beta \times \frac{v_{tb}}{R_{tb} + r_{\pi}} \times \left[- \left(r_0 \| R_{tc} \right) \right]$$

- 3. Show that the two above answers are equivalent. To show this, you must use $\beta = g_m r_{\pi}$.
- 4. If $R_{te} = 0$, use the simplified T model with $i'_c = \alpha i'_e$ and Ohm's Law to show that

$$v_o = \alpha \times \frac{v_{tb}}{r'_e} \times \left[-\left(r_0 \| R_{tc}\right) \right] \qquad r'_e = \frac{R_{tb}}{1+\beta} + r_e$$

5. If $R_{te} \neq 0$ and $r_0 = \infty$, use the simplified T model to show that v_o is given by

$$v_o = -v_{tb} \frac{\alpha R_{tc}}{r'_e + R_{te}} \qquad r'_e = \frac{R_{tb}}{1+\beta} + r_e$$

- (b) Figure (b) shows a common-collector amplifier.
 - 1. Use superposition, voltage division, and Ohm's Law with the pi model to show that

$$v_{be} = v_{tb} \frac{r_{\pi}}{R_{tb} + r_{\pi}} - v_o \frac{r_{\pi}}{R_{tb} + r_{\pi}}$$
$$v_o = v_{tb} \frac{r_0 \| R_{te}}{R_{tb} + r_{\pi} + r_0 \| R_{te}} + i'_c [r_0 \| R_{te} \| (r_{\pi} + R_{tb})]$$

Use the equation $i'_c = g_m v_{be}$ and the above two equations to show that

$$v_o = v_{tb} \frac{\frac{r_0 \| R_{te}}{R_{tb} + r_\pi + r_0 \| R_{te}} + g_m \frac{r_\pi}{R_{tb} + r_\pi} \left[r_0 \| R_{te} \| \left(r_\pi + R_{tb} \right) \right]}{1 + g_m \frac{r_\pi}{R_{tb} + r_\pi} \left[r_0 \| R_{te} \| \left(r_\pi + R_{tb} \right) \right]}$$

Show that $r_{in} = R_{tb} + r_{ib}$, where $r_{ib} = r_{\pi} + (1 + \beta) (r_0 || R_{te})$.

2. Use voltage division with the simplified T model to show that

$$v_o = v_{tb} \frac{r_0 \| R_{te}}{r'_e + r_0 \| R_{te}}$$
 $r'_e = \frac{R_{tb}}{1 + \beta} + r_e$

Which is the simpler solution, this one or the one above? Show that $r_{out} = r'_e ||r_0||R_{te}$.

- 3. Can you show that the above two answers for v_o are the same? You must use $\beta = g_m r_{\pi}$ and $r_e = r_{\pi}/(1+\beta)$ to do this.
- (c) Figure (c) shows a common-base amplifier.
 - 1. For $R_{tb} = 0$ and $r_0 = \infty$, use the pi model with $i'_c = g_m v_{be}$, superposition, voltage division, and Ohm's Law to show that

$$v_{be} = -v_{te} \frac{r_{\pi}}{R_{te} + r_{\pi}} - i'_c \left(R_{te} \| r_{\pi} \right) = -v_{te} \frac{r_{\pi}}{R_{te} + r_{\pi}} - g_m v_{be} \left(R_{te} \| r_{\pi} \right)$$

Solve this equation to obtain

$$v_{be} = -v_{te} \frac{\frac{r_{\pi}}{R_{te} + r_{\pi}}}{1 + g_m \left(R_{te} \| r_{\pi}\right)}$$

Show that v_o is given by

$$v_o = +v_{te} \frac{\frac{g_m r_\pi}{R_{te} + r_\pi} R_{tc}}{1 + g_m \left(R_{te} \| r_\pi \right)}$$

2. For $R_{tb} = 0$ and $r_0 = \infty$, use the T model and Ohm's Law to show that

$$i'_e = \frac{-v_{te}}{R_{te} + r_e}$$

Show that v_o is given by

$$v_o = + v_{te} \frac{\alpha R_{tc}}{R_{te} + r_e}$$

Show that $r_{in} = R_{te} + r_e$ and $r_{out} = R_C$.

- 3. Show that the two above answers for v_o are equivalent. To do this, you must use $\beta = g_m r_{\pi}$ and $\alpha = \beta / (1 + \beta)$
- 4. For $R_{tb} \neq 0$ and $r_0 = \infty$, use the simplified T model to show that

$$i'_e = -\frac{v_{te}}{R_{te} + r'_e} \qquad r'_e = \frac{R_{tb}}{1+\beta} + r_e$$

Show that v_o is given by

$$v_o = +v_{te} \frac{\alpha R_{tc}}{R_{te} + r'_e}$$

For $R_{tb} = 0$, show that this reduces to the answer obtained with the T model. Show that $r_{in} = R_{te} + r'_{e}$.

2. The figure shows a CE amplifier. For the dc analysis, all capacitors are open circuits. For the ac signal analysis, C_1 , C_2 , and C_3 are to be considered to be short circuits. C_L represents the load capacitance which is usually negligible in the frequency band of interest. It is to be considered to be an open circuit for the ac signal analysis. It is given that $R_1 = 430 \text{ k}\Omega$, $R_2 = 30 \text{ k}\Omega$, $R_3 = 0$, $R_C = 12 \text{ k}\Omega$, $R_E = 1 \text{ k}\Omega$, $R_s = 1.2 \text{ k}\Omega$, $R_L = 20 \text{ k}\Omega$, $\beta = 99$, $r_0 = 50 \text{ k}\Omega$, $V_{BE} = 0.65 \text{ V}$, $V_T = 25 \text{ mV}$, $V^+ = 24 \text{ V}$, and $V^- = -24 \text{ V}$.



- (a) Show that $V_{BB} = -20.87 \text{ V}$, $R_{BB} = 28.04 \text{ k}\Omega$, $I_E = 1.937 \text{ mA}$, $V_C = 0.986 \text{ V}$, $V_B = -21.41 \text{ V}$, and $V_{CB} = 22.4 \text{ V}$.
- (b) Show that $v_{tb} = 0.959v_s$, $R_{tb} = 1.151 \text{ k}\Omega$, $R_{te} = 0$, and $R_{tc} = 7.5 \text{ k}\Omega$.
- (c) Use all four expressions from problem 1a to show that $v_o = -253.6v_s$.
- (d) If $R_s = 0$, show that $v_o = -500.3v_s$. What is the main reason the gain changes so much for this case?
- (e) Show that $r_{in} = 1.069 \,\mathrm{k}\Omega$ and $r_{out} = 9.677 \,\mathrm{k}\Omega$.
- 3. The figure shows a CC amplifier. For the dc analysis, all capacitors are open circuits. For the ac signal analysis, C_1 and C_2 are to be considered to be short circuits. It is given that $R_1 = 183.5 \text{ k}\Omega$, $R_2 = 593 \text{ k}\Omega$, $R_E = 5.8 \text{ k}\Omega$, $R_s = 1.2 \text{ k}\Omega$, $R_L = 20 \text{ k}\Omega$, $\beta = 99$, $r_0 = 50 \text{ k}\Omega$, $V_{BE} = 0.65 \text{ V}$, $V_T = 25 \text{ mV}$, $V^+ = 24 \text{ V}$, and $V^- = -24 \text{ V}$.



- (a) Show that $V_{BB} = 12.66 \text{ V}$, $R_{BB} = 140.1 \text{ k}\Omega$, $I_E = 5 \text{ mA}$, $V_C = 24 \text{ V}$, $V_B = 5.65 \text{ V}$, and $V_{CB} = 18.35 \text{ V}$.
- (b) Show that $v_{tb} = 0.992v_s$, $R_{tb} = 1.19 \,\mathrm{k}\Omega$ and $R_{te} = 4.496 \,\mathrm{k}\Omega$.
- (c) Use all three expressions from problem 1b to show that $v_o = 0.988 v_s$.
- (d) If $R_s = 0$, show that $v_o = 0.996v_s$.
- (e) Show that $r_{in} = 106.9 \,\mathrm{k}\Omega$ and $r_{out} = 16.83 \,\Omega$.
- 4. The figure shows a CE amplifier. For the dc analysis, all capacitors are open circuits. For the ac signal analysis, C_1 , C_2 , and C_3 are to be considered to be short circuits. It is given that $R_1 = 430 \text{ k}\Omega$, $R_2 = 30 \text{ k}\Omega$, $R_3 = 0$, $R_C = 12 \text{ k}\Omega$, $R_E = 1 \text{ k}\Omega$, $R_s = 50 \Omega$, $R_L = 20 \text{ k}\Omega$, $\beta = 99$, $r_0 = \infty$, $V_{BE} = 0.65 \text{ V}$, $V_T = 25 \text{ mV}$, $V^+ = 24 \text{ V}$, and $V^- = -24 \text{ V}$.



- (a) Show that $V_{BB} = -20.87 \text{ V}$, $R_{BB} = 28.04 \text{ k}\Omega$, $I_E = 1.937 \text{ mA}$, $V_C = 0.986 \text{ V}$, $V_B = -21.41 \text{ V}$, and $V_{CB} = 22.4 \text{ V}$.
- (b) Show that $v_{te} = 0.952v_s$, $R_{te} = 47.62 \Omega$, $R_{tb} = 0$, and $R_{tc} = 7.5 \text{ k}\Omega$.
- (c) Use all three expressions from problem 1c to show that $v_o = 116.8v_s$.
- (d) If $R_s = 0$, show that $v_o = -575.3v_s$. What is the main reason the gain changes so much for this case?
- (e) Show that $r_{in} = 12.74 \Omega$ and $r_{out} = 12 \text{ k}\Omega$.