## ECE3040 - Assignment 9

1. The figures show inverting amplifier circuits.
(a) For the circuit of Fig. (a), specify $R_{1}, R_{F}$, and $R_{O}$ for a voltage gain of -50 , an input resistance of $2 \mathrm{k} \Omega$, and an output resistance of $1 \mathrm{k} \Omega$. If the op amp clips at a peak output voltage of 12 V , specify the maximum peak input voltage if the op amp is not to be driven into clipping. Answers: $R_{1}=2 \mathrm{k} \Omega, R_{F}=100 \mathrm{k} \Omega, R_{O}=1 \mathrm{k} \Omega$, and $\left|v_{I(p e a k)}\right|=0.24 \mathrm{~V}$.
(b) Fig. (b) shows the circuit of Fig. (a) with a load resistor connected to the output. Calculate the new voltage gain if $R_{L}=1 \mathrm{k} \Omega$. What is the maximum peak output voltage if the op amp is not to clip? Answers: $v_{O} / v_{I}=-25$ and $\left|v_{O(\text { peak })}\right|=6 \mathrm{~V}$.
(c) Repeat part (b) if the circuit is modified as shown in Fig. (c). Answers: $v_{O} / v_{I}=-50$ and $\left|v_{O(\text { peak })}\right|=5.97 \mathrm{~V}$. Hint, use voltage division to solve for $\left|v_{O(\text { peak })}\right|$ in terms of $\left|v_{O(\text { peak })}^{\prime}\right|$.

(a)
(b)

2. Fig. (a) shows an inverting amplifier with a $T$ feedback network. Fig. (b) shows the amplifier with a Thévenin equivalent made looking into the feedback network from the input. The amplifier is to be designed for an input resistance of $1 \mathrm{k} \Omega$ and a voltage gain of -1000 . If $R_{2}=R_{4}$ and $R_{3}=100 \Omega$, specify the value of $R_{2}$ and $R_{4}$. Answers: $R_{1}=1 \mathrm{k} \Omega$ and $R_{2}=R_{4}=9.9005 \mathrm{k} \Omega$.

3. The figure shows a current to voltage converter. The circuit is to be designed to convert an input current of $-50 \mu \mathrm{~A}$ into an output voltage of +4 V .

(a) Calculate the required value of $R_{F}$. Answer: $R_{F}=80 \mathrm{k} \Omega$.
(b) If the op amp clips at a peak output voltage of 12 V , calculate the maximum peak input current. Answer: $i_{1(\text { peak })}=150 \mu \mathrm{~A}$.
(c) The circuit is driven from an amplifier which can be modeled by a voltage-controlled voltage source with an open-circuit voltage gain of 10 and an output resistance of $2 \mathrm{k} \Omega$. Calculate the overall voltage gain of the two circuits in combination. Answer: $v_{O} / v_{I}=$ -400 .
4. The figure shows a non-inverting amplifier. The circuit is to be designed for an input resistance of $10 \mathrm{k} \Omega$, and output resistance of $100 \Omega$, and an open-circuit voltage gain of 20 . When the peak output voltage is 10 V , the current through $R_{F}$ and $R_{1}$ is to be 0.2 mA . Specify the resistors in the circuit. Answers: $R_{i}=10 \mathrm{k} \Omega, R_{O}=100 \Omega, R_{1}=2.5 \mathrm{k} \Omega$, and $R_{F}=47.5 \mathrm{k} \Omega$.

5. The figure shows a 4 input inverting summer. The circuit is to be designed for an output voltage given by $v_{O}=-\left(2 v_{I 1}+4 v_{I 2}+6 v_{I 3}+8 v_{I 4}\right)$. When the peak output voltage is 10 V , the current through $R_{F}$ is to be 0.5 mA . Specify the resistors in the circuit. Answers: $R_{F}=20 \mathrm{k} \Omega, R_{1}=10 \mathrm{k} \Omega, R_{2}=5 \mathrm{k} \Omega, R_{3}=3.33 \mathrm{k} \Omega$, and $R_{4}=2.5 \mathrm{k} \Omega$.

6. The figure shows a non-inverting summer. The gain $v_{O} / v_{+}$is specified to be 50. If $R_{3}=$ $R_{4}=1 \mathrm{k} \Omega$, specify $R_{1}, R_{2}$, and $R_{F}$ for an output voltage given by $v_{O}=5 v_{I 1}+2 v_{I 2}$. Answers: $R_{F}=49 \mathrm{k} \Omega, R_{1}=8.6 \mathrm{k} \Omega$, and $R_{2}=21.5 \mathrm{k} \Omega$.

7. Fig. (a) shows a differential amplifier. Fig. (b) shows the equivalent circuit for the special case $R_{1}=R_{3}$ and $R_{2}=R_{F}$. It is desired to design the circuit so that $v_{O}=10\left(v_{I 1}-v_{I 2}\right)$. In addition, the input resistance seen between the two input nodes is to be $10 \mathrm{k} \Omega$.

(a) Specify the resistors in the circuit. Answers: $R_{1}=R_{3}=5 \mathrm{k} \Omega, R_{2}=R_{F}=50 \mathrm{k} \Omega$.
(b) For $v_{I 2}=0$, solve for the resistance seen looking into the $v_{I 1}$ input. Answer: $55 \mathrm{k} \Omega$.
(c) For $v_{I 1}=0$, solve for the resistance seen looking into the $v_{I 2}$ input. Answer: $5 \mathrm{k} \Omega$.
8. The figure shows a differential amplifier with a source connected between its two inputs. The circuit elements values are the same as those found in problem 7 . Solve for the voltage gain $v_{O} / v_{D}, v_{I 1}, v_{I 2}$, the voltage at each op amp input, and the common-mode input voltage $v_{I C M}$. Answers: $v_{O} / v_{D}=10, v_{I 1}=5.5 v_{D}, v_{I 2}=4.5 v_{D}, v_{+}=v_{-}=5 v_{D}, v_{I C M}=v_{O} / 2=5 v_{D}$.

9. The figure shows a two op amp diff amp. Design the circuit for an output voltage given by $v_{O}=50\left(v_{I 1}-v_{I 2}\right)$. The input resistance to each input is to be $5 \mathrm{k} \Omega$. Answers: $R_{1}=R_{2}=$ $R_{F 1}=R_{3}=5 \mathrm{k} \Omega, R_{F 2}=250 \mathrm{k} \Omega$.

10. The figure shows a three op amp instrumentation amplifier. (a) Design the circuit such that $v_{O 1}-v_{O 2}=10\left(v_{I 1}-v_{I 2}\right)$ and $v_{O}=10\left(v_{O 1}-v_{O 2}\right)$. Answers: $1+2 R_{F 1} / R_{1}=10$, choose $R_{1}=2 \mathrm{k} \Omega$ and $R_{F 1}=9 \mathrm{k} \Omega, R_{F 2} / R_{2}=10$, choose $R_{F 2}=10 \mathrm{k} \Omega$, and $R_{2}=1 \mathrm{k} \Omega$. (b) For $v_{I 1}=0.03 \mathrm{~V}$ and $v_{I 2}=0.01 \mathrm{~V}$, calculate $v_{O 1}, v_{O 2}$, and $v_{O}$. Answers: $v_{O 1}=0.12 \mathrm{~V}$, $v_{O 2}=-0.08 \mathrm{~V}$, and $v_{O}=2 \mathrm{~V}$.

11. The figure shows a balanced output amplifier. Design the circuit so that $v_{O 1}=-v_{O 2}=6 v_{I}$. When $v_{O 1}= \pm 12 \mathrm{~V}$ and $v_{O 2}=\mp 12 \mathrm{~V}$, the current through $R_{F 1}$ and $R_{F 2}$ is not to exceed 1 mA . Answers: for $1 \mathrm{~mA}, R_{1}=2 \mathrm{k} \Omega, R_{F 1}=10 \mathrm{k} \Omega$, and $R_{F 2}=12 \mathrm{k} \Omega$.

12. For the circuit shown, use superposition of $v_{1}$ and $v_{2}$ to show that $v_{O}$ is given by

$$
v_{O}=-v_{1}\left[\left(1+\frac{R_{4}}{R_{3} \| R_{5}}\right) \frac{R_{2}}{R_{1}}+\frac{R_{2}}{R_{5}}\right]+v_{2}\left(1+\frac{R_{2}}{R_{1} \| R_{5}}+\frac{R_{4}}{R_{5}} \frac{R_{2}}{R_{1}}\right)
$$


13. Solve for the transfer function for $V_{o} / V_{i}$ for the circuit below. Sketch the Bode plot, label the break frequencies, and label the gain on the zero-slope asymptotes. Answer:

$$
\frac{V_{o}}{V_{i}}=-\frac{Z_{F}}{R_{1}}=-\frac{R_{2}}{R_{1}} \frac{1+R_{3} C s}{1+\left(R_{2}+R_{3}\right) C s}
$$



The circuit is to be designed as a lag-lead compensator for a motor control system. The specifications are low-frequency asymptotic gain: -2 , input resistance: $10 \mathrm{k} \Omega$, pole frequency: 1 Hz , zero frequency: 10 Hz . Specify the element values. Answers: $R_{1}=10 \mathrm{k} \Omega, R_{2}=20 \mathrm{k} \Omega$, $R_{3}=2222.2 \Omega$, and $C=7.1620 \mu \mathrm{~F}$.
14. Solve for $V_{o} / V_{i}$ for the circuits below. Sketch and label the Bode magnitude plots.


Answers: (a) The transfer function is a low-pass shelving function with a dc gain of

$$
K_{d c}=\frac{R_{2}+R_{3}}{R_{1}+R_{2}+R_{3}}
$$

and a high-frequency gain of

$$
K_{\infty}=\frac{R_{2}+R_{3} \| R_{4}}{R_{1}+R_{2}+R_{3} \| R_{4}}
$$

The transfer function is

$$
\frac{V_{o}}{V_{i}}=\frac{R_{2}+R_{3}}{R_{1}+R_{2}+R_{3}} \frac{1+\left(R_{2} \| R_{3}+R_{4}\right) C s}{1+\left[\left(R_{1}+R_{2}\right) \| R_{3}+R_{4}\right] C s}
$$

(b) The transfer function is a high-pass shelving function. The zero-frequency gain is

$$
K_{d c}=1+\frac{R_{1}+R_{2}}{R_{3}}
$$

The high-frequency gain is

$$
K_{\infty}=1+\frac{R_{2}}{R_{3}}
$$

The transfer function is given by

$$
\frac{V_{o}}{V_{i}}=\left(\frac{V_{f}}{V_{o}}\right)^{-1}=\left(1+\frac{R_{1}+R_{2}}{R_{3}}\right) \frac{1+\left[\left(R_{2}+R_{3}\right) \| R_{1}\right] C s}{1+R_{1} C s}
$$

15. Solve for the voltage-gain transfer function for the circuit below Sketch and label the Bode magnitude plot.


Answer:

$$
\frac{V_{o}}{V_{i}}=-\frac{R_{3}}{Z_{1}}=-\frac{R_{3}}{R_{1}+R_{2}} \frac{1+R_{2} C s}{1+\left(R_{1} \| R_{2}\right) C s}
$$

16. Using a single $100 \mu \mathrm{~F}$ capacitor, design a single op amp circuit which has the voltage-gain transfer function

$$
\frac{V_{o}}{V_{i}}=10 \frac{1+s / 10}{1+s / 100}
$$

Sketch and label the Bode magnitude plot. One possible answer is the circuit below.

where

$$
\frac{V_{o}}{V_{i}}=\left(\frac{V_{f}}{V_{o}}\right)^{-1}=\left(1+\frac{R_{3}}{R_{1}}\right) \frac{1+\left(R_{1} \| R_{3}+R_{2}\right) C s}{1+R_{2} C s}
$$

The element values are $R_{1}=1 \mathrm{k} \Omega, R_{2}=100 \Omega, R_{3}=9 \mathrm{k} \Omega$, and $C=15.92 \mu \mathrm{~F}$.
17. For the circuit shown, show that

$$
V_{o}=\frac{R_{2}}{R_{1}} \frac{V_{i 1}-V_{i 2}}{1+R_{2} C s}
$$


18. For the circuit shown, show that

$$
\frac{V_{o}}{V_{i}}=\frac{1+\left(R_{1}+R_{2}\right) C s}{1+R_{1} C s}
$$


19. For the potentiometer circuit shown, let the resistance below the wiper be $x R_{p}$ and the resistance above the wiper be $(1-x) R_{p}$. Show that

$$
\frac{V_{o}}{V_{i}}=\frac{x}{1+x(1-x) R C s}
$$

Show that the circuit has a minimum bandwidth when $x=0.5$ and that the corresponding pole frequency is given by

$$
f_{\text {pole }}=\frac{1}{\pi R C}
$$


20. For the circuit shown, show that

$$
\frac{V_{o}}{V_{i}}=\left(1+\frac{R_{F}}{R_{1}+R_{2}}\right) \frac{1+\left[R_{1} \|\left(R_{2}+R_{F}\right)\right] C s}{1+\left(R_{1} \| R_{2}\right) C s}
$$


21. The figure shows a Schmidt trigger. It is given that $V_{\mathrm{SAT}}=12 \mathrm{~V}$ and $R_{F}=10 \mathrm{k} \Omega$. Solve for $V_{\text {REF }}$ and $R_{1}$ for $V_{A}=-4 \mathrm{~V}$ and $V_{B}=+2 \mathrm{~V}$. Answers: $R_{1}=3.33 \mathrm{k} \Omega, V_{R E F}=-1.33 \mathrm{~V}$.


