Hybrid-π Model with Body Effect

Let the drain current and each voltage be written as the sum of a dc component and a small-signal ac component as follows:

$$i_D = I_D + i_d \tag{1}$$

$$v_{GS} = V_{GS} + v_{gs} \tag{2}$$

$$v_{BS} = V_{BS} + v_{bs} \tag{3}$$

$$v_{DS} = V_{DS} + v_{ds} \tag{4}$$

If the ac components are sufficiently small, we can write

$$i_d = \frac{\partial I_D}{\partial V_{GS}} v_{gs} + \frac{\partial I_D}{\partial V_{BS}} v_{bs} + \frac{\partial I_D}{\partial V_{DS}} v_{ds} \tag{5}$$

where the derivatives are evaluated at the dc bias values. Let us define

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = K \left(V_{GS} - V_{TH}\right) = 2\sqrt{K I_D} \tag{6}$$

$$g_{mb} = \frac{\partial I_D}{\partial V_{BS}} = \frac{\gamma \sqrt{K I_D}}{\sqrt{\phi - V_{BS}}} = \chi g_m \tag{7}$$

$$\chi = \frac{\gamma}{2\sqrt{\phi - V_{BS}}} \tag{8}$$

$$r_0 = \left[\frac{\partial I_D}{\partial V_{DS}}\right]^{-1} = \left[\frac{k'}{2} \frac{W}{L} \lambda \left(V_{GS} - V_{TH}\right)^2\right]^{-1} = \frac{V_{DS} + 1/\lambda}{I_D} \tag{9}$$

The small-signal drain current can thus be written

$$i_d = i_{dg} + i_{db} + \frac{v_{ds}}{r_0} \tag{10}$$

where

$$i_{dg} = g_m v_{gs} \tag{11}$$

$$i_{db} = g_{mb} v_{bs} \tag{12}$$

The small-signal circuit which models these equations is given in Fig. 1. This is called the hybrid-π model.

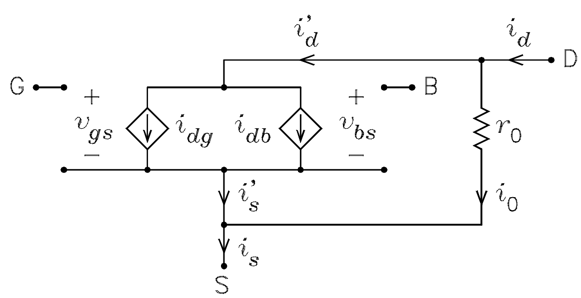

Figure 1: Hybrid-π model of the MOSFET.