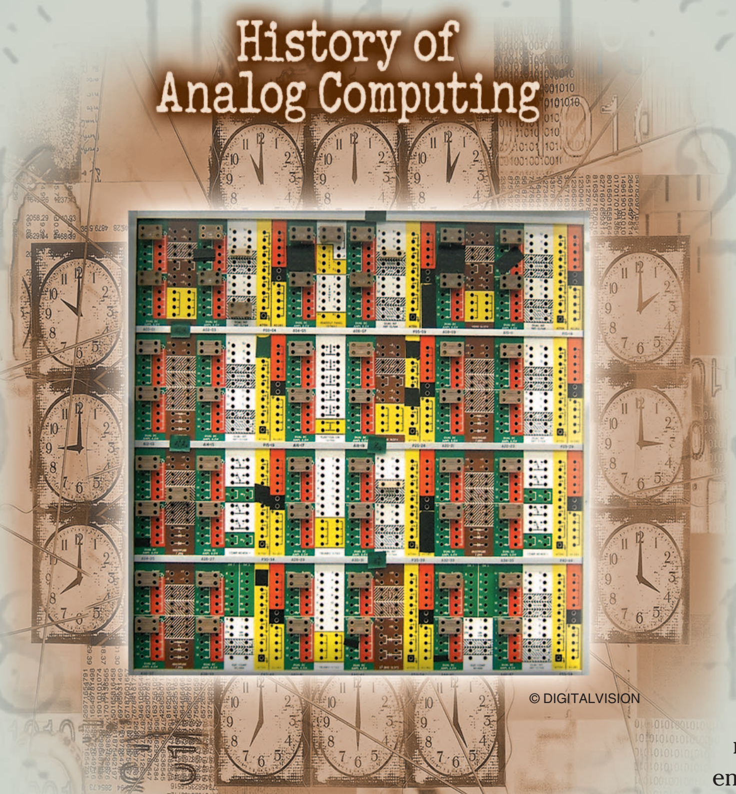
FEATURE

Analog Computers in Academia and Industry

A history of analog computing at the University of Michigan and the founding of Applied Dynamics International

By Robert M. Howe



At the end of World War II, the U.S. Air Force recognized that none of its officers had any academic training in the emerging field of guided missiles. To remedy this situation, the Air Force established the Guided Missiles Training Program at the University of Michigan. The program consisted of two years of graduate studies in the Department of Aeronautical Engineering for qualified junior and senior officers, with an emphasis on new courses related to guided missile technology. A similar program was sponsored at MIT. At the same time, ownership of Willow Run airport, a facility 12 miles east of Ann Arbor that was built during World War II as part of the Ford plant to mass produce B-24 Liberator bombers, was transferred from the U.S. Government to the University of Michigan. This facility enabled the University of Michigan to create the Michigan Aeronautical Research Center as an organization for conducting large government-funded projects. The initial Willow Run program was Project Wizard, sponsored by the U.S. Air Force, which involved the design of a surface-to-air guided missile to destroy enemy ballistic missiles in flight.

With these post-World-War-II developments, the Department of Aeronautical Engineering at the University of Michigan began to add faculty members with backgrounds in physics, electrical engineering, and applied mathematics to the existing faculty with expertise in the traditional areas of aeronautical engineering (aerodynamics, propulsion, structures, and aircraft design). The new faculty members were charged with creating and teaching graduate courses in guided missiles and control systems technology, as well as conducting research associated with the Aeronautical Research Center at Willow Run. In 1947, under the auspices of

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Project Wizard, the author (then a graduate research assistant), the author's father (C.E. Howe, a physics professor from Oberlin College who spent his summers doing research at the University of Michigan), and D.W. Hagelbarger (a new faculty member in the Department of Aeronautical Engineering) initiated a study of the utility of electronic analog computers for solving engineering problems [1]. This study led directly to the development and use of analog computers for simulation in the laboratory courses associated with the USAF Guided Missiles Training Program. It also spurred a number of follow-on government-sponsored research efforts and the founding in 1957 of the company Applied Dynamics to manufacture and market analog computer systems.

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The Study of the Utility of Electronic Analog Computers at the University of Michigan

The development and application of electronic analog computers in the Aeronautical Engineering Department at the University of Michigan, initiated in 1947, employed operational amplifiers based on a high-gain dc amplifier circuit published at that time in an article by Ragazzini et al. [2]. The amplifier circuit utilized two vacuum tubes and exhibited an open-loop gain of approximately 50,000, with an output voltage range that exceeded the ±100-V dc reference. Each operational amplifier was housed in its own chassis, which included sockets for input and feedback resistors mounted on twin banana-jack plugs when the amplifier was used as a summer, and a feedback capacitor when the amplifier was used as an integrator. Carbon film resistors with 1% accuracy were used as input and feedback resistors, and a Western Electric 1-μF polystyrene capacitor accurate to 1% was used as an integrator feedback capacitor. The polystyrene dielectric was utilized because of its low dielectric absorption. Input and feedback impedances were matched to 0.1% to improve the overall accuracy of analog solutions. Figure 1 shows two summer and two integrator operational amplifiers (as constructed in the University of Michigan Aeronautical Engineering Laboratories) connected to solve a second-order linear differential equation.

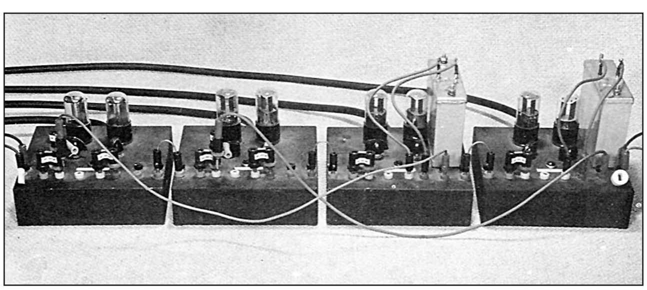

Figure 1. *Four of the original University of Michigan operational amplifiers connected to solve a second-order linear differential equation. Input and feedback resistors are mounted on the twin banana-jack plugs in the front of each amplifier chassis. The polystyrene integrating capacitors can be seen next to the vacuum tubes on the right hand pair of amplifier chassis.*

Thanks to the success of the 1947–1948 study of the utility of analog computers in solving engineering problems, the analog computers constructed for the study were introduced into the laboratories of two graduate courses created to serve the needs of the Guided Missiles Training Program at the University of Michigan. Specifically, the analog computer was used to simulate dynamic systems, such as seismic instruments and feedback control systems, in courses on engineering measurements and design of control systems [3].

Follow-On Analog Computer Developments in the Department of Aeronautical Engineering

In 1950, the author returned to become a faculty member in the University of Michigan Department of Aeronautical Engineering following a two-year absence to earn his doctorate in physics from MIT. At the same time L.L. Rauch, who joined the departmental faculty in 1949 from Princeton, initiated a program to construct new and improved operational amplifiers based on a circuit developed by the Rand Corporation. One of the problems associated with the dc operational amplifiers used in the original 1947–1948 study was the drift over time in the amplifier output voltage. Partial elimination of the solution errors caused by this drift could be achieved by frequently rebalancing the amplifiers. An ingenious method for practically eliminating this drift was worked out by RCA and Leeds and Northrup. The scheme involved passing the input to the dc amplifier through a low-pass filter. The input was then converted to an ac signal by means of a 60-Hz Leeds and Northrup chopper, passed through an ac amplifier, reconverted to a dc signal, and then added back to the dc amplifier input through a second input terminal. Because the ac amplifier is drift free, the dc operational amplifier voltage offset referred to the amplifier input is now practically eliminated, being reduced to less than one part in 10^6 of full scale (±100 V). Operational amplifiers utilizing this feature are called drift-stabilized amplifiers.

In 1951, the department acquired a Series 100 REAC analog computer manufactured by the Reeves Instrument Corp. This computer consisted of 20 operational amplifiers, four servomultipliers, and four resolvers. The machine also utilized a removable patch panel to program and store the connections between analog components. With the arrival of the REAC computer, the department's capabilities were expanded to include the solution of nonlinear differential equations involving multiplication and coordinate conversion. Because the multipliers and resolvers utilized servo-driven potentiometers, the useful range of problem frequencies available for accurate computation was restricted to values below 1 Hz, in contrast with the linear operational amplifier accurate performance for problem frequencies up to 50 Hz.

Also in 1951, the department was awarded an Office of Naval Research (ONR) contract to utilize the analog computer for the study of wave-equation solutions for underwater sound propagation in a bilinear velocity gradient [4]. This application was a direct outgrowth of our earlier experience in solving boundary-value problems in the original 1947–1948 study, including the use of the stepping-relay scheme to approximate time-varying coefficients. The contract also included the design and delivery to ONR of an analog computer capable of solving the underwater-sound wave equation with a bilinear velocity gradient [5]. The computer was comprised of ten drift-stabilized operational amplifiers, including six integrators, as well as a 17-digit, 25-step variable-coefficient generator utilizing stepping relays. The front of the three relay racks making up the computer is shown in Figure 2, and the dc operational amplifier chassis with plug-in drift stabilizer is shown in Figure 3. This machine represented the first analog computer designed from the ground up by the University of Michigan Aeronautical Engineering Department.

The patch panels associated with three groups of operational amplifiers can be seen in the center relay rack in Figure 2. In between these patch panels are two panels, each of which contains four 4 × 4 arrays of toggle switches. Each array can be used to generate computing resistors up to 16 MΩ in 0.001-MΩ steps. The right relay rack in Figure 2 contains an array of 17 × 25 toggle switches. Each of the 25 toggle-switch rows corresponds to a fixed time instant in the variable-coefficient generator that utilizes a 25-position stepping relay. Each of the 17 columns of toggle switches is used to open or close at each time step a relay that shorts or opens a plug-in binary computing resistor. At each of the 25 time steps, any desired variable-coefficient resistance is obtained as the sum of the individual binary resistors. Each binary relay is a double-pole relay and can be used to control two plug-in binary resistors. Thus, two separate but identical variable-coefficient resistances can be generated. The time Δt between time steps was normally set at 1 s resulting in a total solution time of 25 s. In a sense, this stepping-relay scheme for simulating variable coefficients

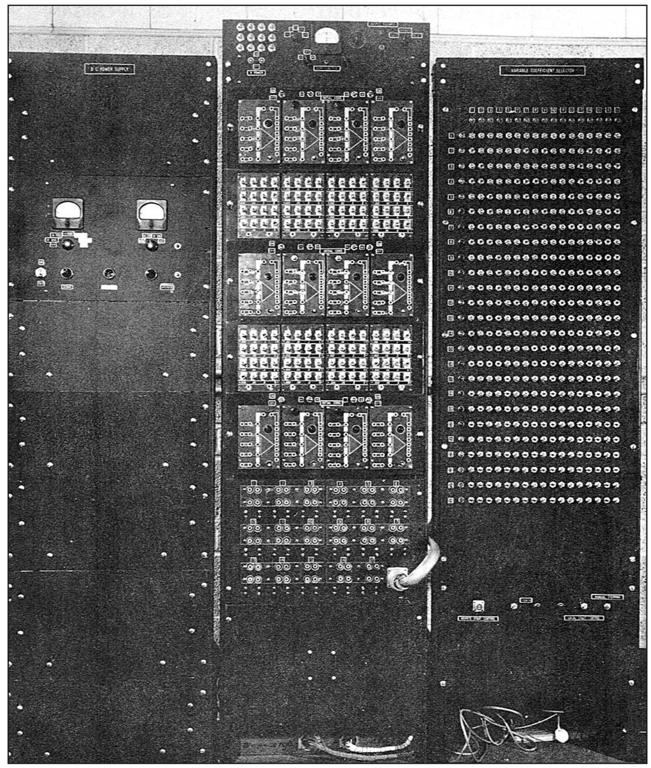

Figure 2. *The analog computer designed and delivered to the Office of Naval Research in 1953. The system included ten drift-stabilized operational amplifiers and a 17-digit, 25-step variable-coefficient generator.*

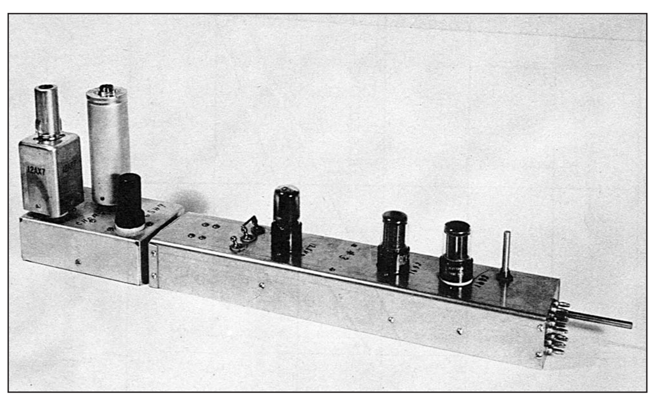

Figure 3. *A dc amplifier and drift stabilizer. This circuitry was used in the Office of Naval Research computer.*

(or its earlier counterpart described in [1]) probably represents the first implementation of hybrid computing, with the periodically changed binary toggle-switch array representing the digital subsystem and the remaining integrating and summing operational amplifiers representing the analog subsystem.

Use of Nonlinear Components

Before 1954, the only true nonlinear analog capability in the department resided in the four servomultipliers and resolvers included with the REAC 100 computer, acquired in 1951 as noted earlier. In 1953, the department received a contract from the Air Force to study the computer section of flight simulators [6]. At the same time, the design and construction of a sufficient number of operational amplifiers and high-performance servomultipliers was initiated to provide the capability of running a full real-time analog solution of the six-degree-of-freedom nonlinear aircraft flight equations. The trigonometric resolution needed for coordinate conversion was accomplished with multipliers, thus eliminating the need for servo-driven sine-cosine potentiometers [7]. The design of the servomultipliers gave the department valuable insight into practical considerations associated with the design of electromechanical servos, experience that was later utilized in both the lecture and laboratory courses. The analog computer designed and constructed by the department primarily for simulation of the complete nonlinear six-degree-of-freedom flight equations is shown in Figure 4.

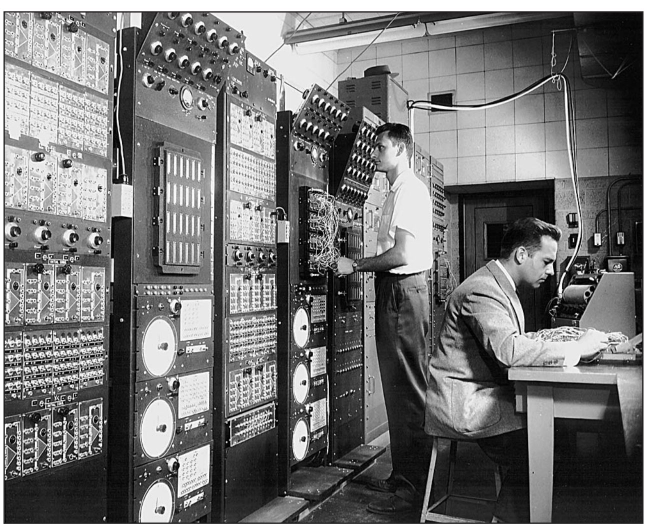

Figure 4. *Analog computer developed in the Department of Aeronautical Engineering at the University of Michigan for simulating the six-degree-of-freedom aircraft flight equations. The circular readout dials of the servomultipliers can be seen in the bottom half of the second and fourth relay racks from the left.*

The Use of Analog Computers in Laboratory Courses at the University of Michigan

As a direct result of the development and use of analog computers in the Department of Aeronautical Engineering at the University of Michigan in the 1950s, analog computers played a major role in several senior and graduate-level courses in the department. In reviewing the 1956–1957 College of Engineering catalog, the author identified a total of nine courses in which the analog computer was used either for lecture demonstrations or for simulation experiments in laboratories associated with courses. For example, in the introductory course on automatic control, the analog computer was utilized not only to simulate various control system designs but also functioned as the controller-circuit subsystem used in a laboratory servo. In the laboratory associated with the course on engineering measurements and instrumentation, analog computers were used to simulate various physical systems over a wide range of parameters. The course on control and guidance of aircraft and missiles also utilized analog computers for simulation experiments in the laboratory, as did the advanced course on feedback control, where a number of different nonlinear control systems and sampled-data systems were simulated in the laboratory. Courses on theory of oscillation of nonlinear systems and the response of nonlinear systems used the analog computer for lecture demonstrations. A course in the design of electronic analog computers utilized analog computers for laboratory experiments, as did beginning and advanced courses on applications of the electronic differential analyzer.

It should be noted that the role played by the analog computer in dynamic system simulation in the above courses has in recent years been taken over by digital-computing laboratories. Yet there still appears to be a place for the analog computer, a true continuous dynamic system, in simulating the continuous portion of dynamic systems controlled by digital processors. In particular, the problems associated with analog-to-digital and digital-to-analog converters in microprocessor control of continuous systems can be demonstrated with the analog computer emulating the continuous subsystem.

The University of Michigan Simulation Center

In 1968, the College of Engineering, with support from the National Science Foundation (NSF), established the University of Michigan Simulation Center. Under the directorship of Laurence E. Fogarty, professor of Aerospace Engineering, the center was created to serve the needs of other units of the college and the university. The center acquired both an Applied Dynamics AD-4 and a PDP-9 digital computer to constitute a state-of-the-art analog/hybrid system. With Dr. Roy B. Hollstien as facilities manager, the center was involved in the teaching of courses conducted by faculty members of both the departments of Aerospace Engineering and Electrical Engineering and Computer Science. The center was used for extensive research activity in the application of analog/hybrid computers to the optimized design of systems. One of the center's most significant achievements was the development of an autopatch system for the AD-4. In this system, components of the analog subsystem were prewired on an AD-4 patchboard to a matrix of switches under program control of the PDP-9. Users who had only a limited knowledge of analog computation were able to create programs by "compiling" a set of simulation-language statements that described problems in standard mathematical terms [8]. Each problem programmed on the autopatch system could be operated from up to five remote terminals. The autopatch system was utilized in laboratories associated with a number of both engineering and nonengineering courses.

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Other Early Uses of Analog Computers at the University of Michigan

Other units at the University of Michigan utilized analog computers for teaching and research during the three decades following World War II. In particular, the Willow Run Research Laboratories performed missile simulations with a Series 100 REAC computer similar to, but much larger than, the REAC computer in the Department of Aeronautical Engineering (now known as the Department of Aerospace Engineering). The Electrical Engineering Department used a PACE 16-31R computer for research involving the calculation of electron trajectories, and the Mechanical Engineering Department used analog computers for simulating dynamic systems in a laboratory course.

The Founding of Applied Dynamics International

In 1957, Applied Dynamics International (ADI) was founded by four faculty members of the Department of Aeronautical Engineering at the University of Michigan: the author, professors (and twin brothers) Edward Gilbert and Elmer Gilbert, and Jay King, a design engineer. The initial ADI product was the LM-10, a ten-amplifier tabletop analog computer used to simulate up to sixth-order linear and certain nonlinear differential equations (see Figure 5). The first computer developed exclusively by ADI was the ADI-16, a modular analog computer expandable to 16

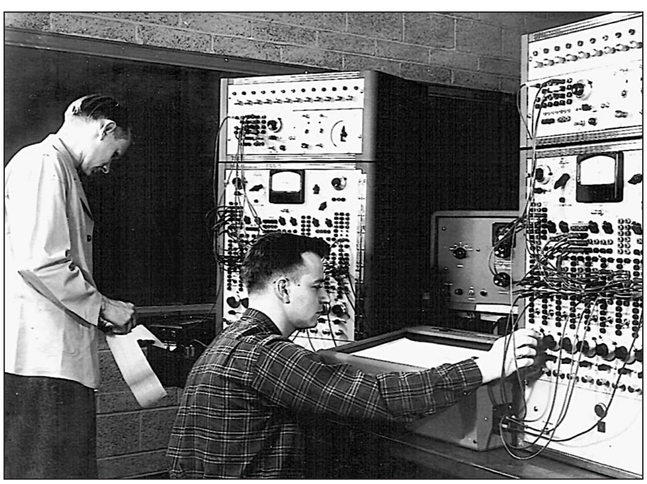

Figure 5. *The LM-10, the first ADI product. This ten-amplifier computer, together with the servo-multiplier in the cabinet sitting on top of each LM-10 cabinet, was developed in the University of Michigan Aeronautical Engineering Department. Shown in the figure is Prof. Edward O. Gilbert (standing) and Richard French, the technician involved in constructing the computers.*

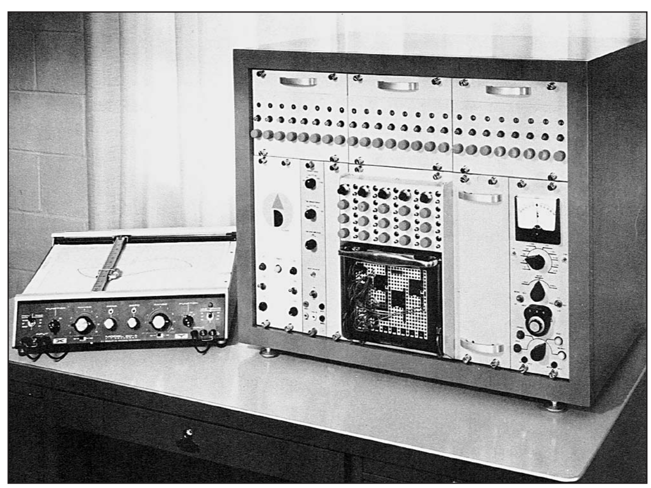

Figure 6. *The ADI AD2-24PB analog computer. This 24-amplifier tabletop analog computer with removable patchboard for problem storage, when fully expanded, contained eight integrators, 16 coefficient pots, five quarter-square multipliers and diode function generators, and five passive diode networks.*

amplifiers. Shortly thereafter, ADI marketed the ADI-64PB, a 64-amplifier tabletop computer, which used a removable patchboard for problem storage. By 1959, ADI had also developed the AD2-24PB (see Figure 6), AD2-32PB, and AD2-80PB analog computers. The console-type AD2-80PB began to compete for the first time with the products of other major U.S. analog computer manufacturers.

The analog computer was used to simulate dynamic systems, such as seismic instruments and feedback control systems, in courses on engineering measurements and design of control systems.

During the 1960s, the use of combined analog-digital or hybrid computers for real-time simulation emerged. The digital subsystems consisted of patchable logic components, along with a general-purpose digital computer, used principally to implement multivariable function generation in what were otherwise all-analog simulations. In 1963, to compete in the hybrid computer marketplace, ADI initiated development of the AD-256. This large, high-performance system incorporated a number of new features, including bipolar operational amplifiers, electronic mode control of integrators, a sizeable complement of asynchronous patchable logic, and a large complement of nonlinear analog components. During this period, ADI developed a family of high-accuracy, all-passive quarter-square multipliers and sine-cosine function generators based on the circuit shown in the author's article, "Analog Computer Fundamentals" [9]. These ADI nonlinear analog components were not only incorporated into ADI computers but were also purchased by competitors for use in their units.

In 1966, ADI developed the all-solid-state AD-4 analog/hybrid computer as the successor to the AD-256. Over the next decade, more than 100 AD-4 systems were delivered, including a number incorporating the digital coefficient unit (DCU), an all-solid-state replacement for servo-set coefficient potentiometers. In addition to the 100-V AD-4 system, ADI also developed the lower-cost AD-5 10-V analog/hybrid system.

Starting in 1975, ADI developed the all-digital AD-10, a special-architecture multiprocessor computer using 16-bit fixed-point words to represent problem variables. Using all solid-state memory as well as ECL (emitter coupled logic) processors, the initial AD-10 was designed to rapidly perform the table lookup and linear interpolation operations involved in multivariable function generation [10]. With the addition of a 48-bit numerical-integration processor, the AD-10 could perform all the required calculations in real-time simulation of high-bandwidth dynamic systems, which had previously only been able to be run on analog/hybrid computer systems. For this reason, the AD-10 was successful in replacing analog/hybrid computers in major simulation laboratories worldwide.

The experience gained with the AD-10 enabled ADI in 1985 to develop a follow-on all-digital, real-time simulation system, the AD-100. Once again consisting of a special architecture designed for optimal simulation performance, the AD-100 utilized ECL multiprocessors with a 64-bit floating-point word. With its floating-point design, the AD-100 matched the speed of analog computers in simulating complex dynamic systems without the heavy burden of scaling all the variables, as required with analog computers as well as the fixed-point AD-10. To accompany the hardware design, ADI introduced the user-friendly simulation language ADSIM to program the AD-100. At the time of its introduction, the AD-100 proved to be the fastest computer in existence for simulating dynamic systems described by scalar-type differential equations. Over the decade from 1985–1995, the AD-100 represented the computer of choice for real-time simulation of very fast dynamic systems.

It should be noted that the University of Michigan faculty members who founded ADI were responsible for many of the technical innovations incorporated in the analog/hybrid ADI computers. Particular mention should be made of Dr. Edward O. Gilbert, who served as a full-time consultant to ADI from the mid-1960s until his death in 1996. Dr. Gilbert was responsible for the development of the highly successful all-digital AD-10 and AD-100 computers. The faculty members of the University of Michigan Aerospace Engineering Department also made many contributions to the transition from all-analog to hybrid and, finally, to all-digital real-time simulation [11]–[14].

In the early to mid-1990s, single-chip microprocessors began to approach the speed of the AD-100. To take advantage of this development, ADI introduced the Real Time Station (RTS), a VME-based system consisting of multiple microprocessor chips hosted by a workstation, along with significant new software for hardware-in-the-loop (HIL) simulations. The ADI RTS, utilizing microprocessor chips that are several times faster than the AD-100, continues to be in demand for rugged, reliable, high-speed real-time simulation in aerospace and defense applications. Recent advances in the speed and I/O capability of PC-based systems have allowed ADI to balance its HIL product offerings with the rtX, a PC-based version of the RTS aimed at lower-cost applications that still require the computing power, versatility, and openness of ADI software. This new rtX product is especially tuned to the automotive market for real-time simulations involving both open- and closed-loop testing. An experienced staff of application engineers for customer support and special projects rounds out the modern day profile of ADI.

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Beholder's Eye

There are moments in our lives, when in a figurative sense we strike pay dirt. This moment occurred when the model differential analyzer produced for the first time a series of graphs of unsurpassed beauty: wave functions of hydrogen atoms, and then a few weeks later, chromium atoms.

—Arthur Porter, "Building the Manchester differential analyzers: A personal reflection," *IEEE Annals of the History of Computing*, vol. 25, no. 2, p. 88, April-June 2003.