Solution. The impedance function is given by

$$Z_{VC}(s) = 7 + 0.0018s + 70\frac{(1/3.3)(s/138.2)}{(s/138.2)^2 + (1/3.3)(s/138.2) + 1}$$

Example 10 *Solve for the element values in the equivalent circuit of the voice coil for the driver of Example 1.*

Solution. $R_{ES} = (Q_{MS}/Q_{ES}) R_E = 70\,\Omega$, $L_{CES} = B^2\ell^2 C_{MS} = 0.153\,\mathrm{H}$, $C_{MES} = M_{MS}/B^2\ell^2 = 341\,\mu\mathrm{F}$.

6.17 The Lossy Voice-Coil Inductance

If L_e is a lossless inductor, its impedance is given by $Z_e(\omega) = j\omega L_e$. To a good approximation, the impedance of a lossy inductor can be written

$$Z_e(\omega) = (j\omega)^n L_e = \frac{\omega^n L_e}{\cos(n\pi/2) - j\sin(n\pi/2)} \tag{6.50}$$

where the units of L_e are no longer henries and $0 < m < 1$. If we use this model for the voice-coil inductance, it follows that

$$\frac{1}{Z_e(\omega)} = \frac{\cos(n\pi/2)}{\omega^n L_e} - j\frac{\sin(n\pi/2)}{\omega^n L_e} = \frac{1}{R'_E(\omega)} - j\frac{1}{\omega L_E(\omega)} \tag{6.51}$$

Thus $R'_E(\omega)$ and $L_E(\omega)$ are given by

$$R'_E(\omega) = \left[\frac{L_e}{\cos(n\pi/2)}\right]\omega^n \qquad L_E(\omega) = \left[\frac{L_e}{\sin(n\pi/2)}\right]\omega^{n-1} \tag{6.52}$$

For a given driver, the constant n can be determined from a log-log plot of the magnitude of the high-frequency voice-coil impedance versus frequency. Well above the fundamental resonance frequency, such a plot should be a straight line with a slope n dec/dec. If accurate impedance data for Z_{VC} are known, n should be determined from the high-frequency plot of $|Z_{VC} - R_E|$ rather than the plot of $|Z_{VC}|$. This requires knowledge of both the real and imaginary parts of Z_{VC}. A method for doing this is described in Section 12.7. For many loudspeaker drivers, a typical value for n is in the range of 0.6 to 0.7.

For example, let $|Z_{e1}|$ be the value of $|Z_{VC} - R_E|$ at f_1 and $|Z_{e2}|$ be the value at f_2. If the log-log plot of $|Z_{VC} - R_E|$ versus f exhibits a straight line between f_1 and f_2, it follows that n and L_e are given by

$$n = \frac{\log(|Z_{e2}|/|Z_{e1}|)}{\log(f_2/f_1)} \qquad L_e = \frac{|Z_{e1}|}{(2\pi f_1)^n} = \frac{|Z_{e2}|}{(2\pi f_2)^n} \tag{6.53}$$

If $|Z_e|$ is measured at more than one frequency, a linear regression analysis can be used to determine L_e and n. A method for doing this is described in Section 12.7.6.

Example 11 *Impedance measurements on a particular loudspeaker driver yield $Z_{e1} = 7.54 + j14.6$ Ω at $f_1 = 2242$ Hz and $Z_{e2} = 36.6 + j66.6\ \Omega$ at $f_2 = 20$ kHz. Calculate n and L_e in Eq. (6.50).*

Solution. $|Z_{e1}| = 16.4$, $|Z_{e2}| = 76.0$. Thus $n = \log(76.0/16.4)/\log(20000/2242) = 0.700$ and $L_e = 16.4/(2\pi 2242)^{0.7} = 0.0205$.

Example 12 *Calculate the phase of Z_{e1} and Z_{e2} for the data in Example 11 and the error in the phase of the approximating function for Z_e at f_1 and f_2.*

Solution. $\arg Z_{e1} = \tan^{-1}(14.6/7.54) = 62.7^{\mathrm{o}}$, $\arg Z_{e2} = \tan^{-1}(66.6/36.6) = 61.2^{\mathrm{o}}$. The phase of the approximating function is $\arg Z_e = 0.7 \times 90^{\mathrm{o}} = 63^{\mathrm{o}}$. Thus the phase error at f_1 is 0.3^{o}. At f_2, it is 1.8^{o}.

The phase of $Z_e(\omega)$ given by Eq. (6.50) is $n\pi/2$ rad, which is a constant. To account for deviations from this value, an alternate model for $Z_e(\omega)$ can be used. Let

$$Y_e(\omega) = G_e(\omega) + jB_e(\omega) = \frac{1}{Z_e(\omega)} = \frac{1}{R'_E(\omega)} - j\frac{1}{X_E(\omega)} \tag{6.54}$$

where $X_E(\omega) = \omega L_E(\omega)$. In the alternate model, separate functions are used to represent $R'_E(\omega)$ and $X_E(\omega)$. The representations are as follows:

$$R'_E(\omega) = \frac{1}{G_e(\omega)} = R_e\omega^{n_r} \qquad X_E(\omega) = \frac{-1}{B_e(\omega)} = X_e\omega^{n_x} \tag{6.55}$$

Let the real and imaginary parts of Z_{VC} be known at two frequencies $f_1$and f_2 that are sufficiently high so that the motional impedance component of Z_{VC} is negligible. $G_e(\omega)$ and $B_e(\omega)$ are given by $G_e = \mathrm{Re}\left[(Z_{VC} - R_E)^{-1}\right]$ and $B_e = \mathrm{Im}\left[(Z_{VC} - R_E)^{-1}\right]$. The constants R_e, n_r, X_e, and n_x are calculated as follows:

$$\begin{aligned} n_r &= \frac{\log\left[G_e(f_1)/G_e(f_2)\right]}{\log(f_2/f_1)} \qquad R_e = \frac{1}{(2\pi f_1)^{n_r} G_e(f_1)} = \frac{1}{(2\pi f_2)^{n_r} G_e(f_2)} \\ n_x &= \frac{\log\left[B_e(f_1)/B_e(f_2)\right]}{\log(f_b/f_a)} \qquad X_e = \frac{-1}{(2\pi f_1)^{n_x} B_e(f_1)} = \frac{-1}{(2\pi f_2)^{n_x} B_e(f_2)} \end{aligned} \tag{6.56}$$

Example 13 *Calculate R_e, n_r, X_e, and n_x for the voice-coil data given in Example 11.*

Solution. $Y_{e1} = 1/Z_{e1} = 0.0279 - j0.0541$, $Y_{e2} = 1/Z_{e2} = 0.00634 - j0.0115$.

$$\begin{aligned} n_r &= \frac{\log(0.0279/0.00634)}{\log(20000/2242)} = 0.678 \qquad R_e = \frac{1}{(2\pi 2242)^{0.678} \times 0.0279} = 0.0553 \\ n_x &= \frac{\log(0.0541/0.0115)}{\log(20000/2242)} = 0.706 \qquad X_e = \frac{1}{(2\pi 2242)^{0.706} \times 0.0541} = 0.0218 \end{aligned}$$

Fig. 6.15 shows the plots of R'_E and $X_E = \omega L_E$ versus frequency for a 10-inch driver having the small-signal parameters $R_E = 5.08\ \Omega$, $f_S = 35.2$ Hz, $Q_{ES} = 0.443$, and $Q_{MS} = 2.80$. The measured values are plotted as circles. Only those values above 1 kHz are plotted. The solid curves represent the approximations calculated with the values $R_e = 0.0728$, $n_r = 0.653$, $X_e = 0.0243$, and $n_x = 0.697$. These values were derived from experimental data on a driver using a linear regression technique described in Section 12.7.6.

6.18 On-Axis Pressure Sensitivity

The reference on-axis pressure sensitivity of a driver is defined as the magnitude of the midband on-axis pressure at $r = 1$ m for a voice coil voltage $e_g = 1$ V rms. We denote the pressure sensitivity by p_{sens}^{1V}. It is obtained from Eq. (6.29) by setting $e_g = 1$ and $|G(j\omega)\,T_{u1}(j\omega)| = 1$ and is given by

$$p_{\mathrm{sens}}^{1V} = \frac{\rho_0}{2\pi}\frac{B\ell}{S_D R_E M_{AS}} = \frac{\sqrt{2\pi\rho_0}}{c} f_S^{3/2}\left[\frac{V_{AS}}{R_E Q_{ES}}\right]^{1/2} \tag{6.57}$$

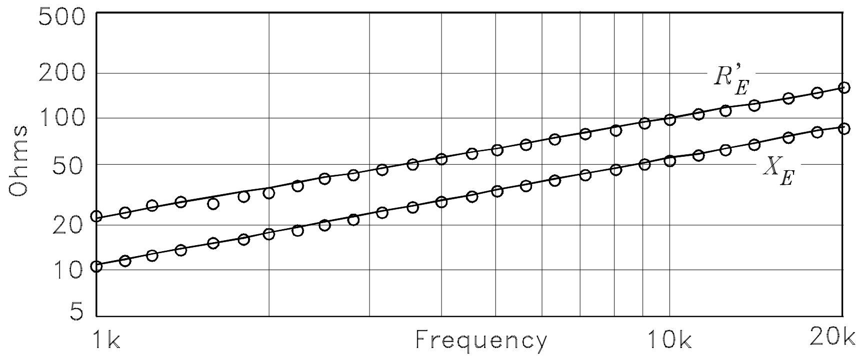

Figure 6.15: R'_E and X_E versus frequency for a typical driver.

The on-axis sensitivity is often specified as the SPL at $r = 1$ m either for a voice-coil voltage of $e_g = 1$ V rms or $e_g = \sqrt{R_E}$ V rms. The latter is the rms voltage required for a power of 1 W into a resistor of value R_E, i.e. the voice-coil resistance. We denote the two SPL sensitivities by SPL_{sens}^{1V} and SPL_{sens}^{1W}, respectively. They are given by

$$SPL_{\text{sens}}^{1V} = 20 \log \left[\frac{p_{\text{sens}}^{1V}}{p_{\text{ref}}} \right] \text{ dB} \qquad SPL_{\text{sens}}^{1W} = 20 \log \left[\frac{p_{\text{sens}}^{1V} \sqrt{R_E}}{p_{\text{ref}}} \right] \text{ dB} \tag{6.58}$$

where $p_{\text{ref}} = 2 \times 10^{-5}$ Pa.

Example 14 *For the driver of Example 1, calculate the on-axis pressure sensitivity p_{sens}^{1V} and the SPL_{sens}^{1V} without and with the resistor of Example 7.*

Solution. Without the resistor, the pressure sensitivity is evaluated in Example 3. It is $p_{\text{sens}}^{1V} = 0.312$ Pa. The corresponding SPL is $SPL_{\text{sens}}^{1V} = 20 \log \left[0.312/2 \times 10^{-5}\right] = 83.9$ dB. From Example 7, the added resistor causes the pressure sensitivity to drop by the factor 7/19.1 to the value $p_{\text{sens}}^{1V} = 0.114$ Pa. The corresponding SPL is $SPL_{\text{sens}}^{1V} = 20 \log \left[0.114/2 \times 10^{-5}\right] = 75.1$ dB. Thus the resistor attenuates the output by 8.8 dB.

Example 15 *If the power amplifier puts out 1 W of average sine-wave power (sometimes erroneously called 1 W rms), calculate the 1-meter on-axis SPL_{sens}^{1W} for the driver of Example 1 with and without the resistor of Example 7.*

Solution. Without the resistor, $SPL_{\text{sens}}^{1W} = 20 \log \left[0.312\sqrt{7}/2 \times 10^{-5}\right] = 92.3$ dB. With the resistor, $SPL_{\text{sens}}^{1W} = 20 \log \left[0.114\sqrt{19.1}/2 \times 10^{-5}\right] = 87.9$ dB. Although the output is less by 4.4 dB with the resistor, the generator voltage for the two cases is different. The voltage output of the power amplifier must be increased to keep its output power at 1 W. Therefore, the dB change does not correspond to the attenuation introduced by the resistor.

Example 16 *Calculate the 1-meter on-axis pressure sensitivity p_{sens}^{1V} and the SPL_{sens}^{1V} for the driver of Example 1 with the new value of $B\ell$ obtained in Example 6.*

Solution. The original pressure sensitivity is evaluated in Example 3. It is $p_{\text{sens}}^{1V} = 0.312$ Pa. Eq. (6.57) shows that p_{sens}^{1V} is directly proportional to $B\ell$. Thus $p_{\text{sens}}^{1V} = 0.312 \times 6.9/11.4 = 0.189$ Pa and $SPL_{\text{sens}}^{1V} = 20 \log \left[0.189/2 \times 10^{-5}\right] = 79.5$ dB.

Example 17 *If the power amplifier puts out 1 W of average sine-wave power, calculate the 1-meter on-axis SPL_{sens}^{1W} for the driver of Example 1 with the new value of $B\ell$ obtained in Example 6.*

Solution. $SPL_{\text{sens}}^{1W} = 20 \log \left[0.189\sqrt{7}/2 \times 10^{-5}\right] = 88$ dB.

By taking the product of f_C and Q_{ECT} given by the equations above, we have

$$f_C Q_{ECT} = \frac{1}{2\pi R_{AE} C_{AT}} = \frac{\rho_0 c^2}{2\pi R_{AE} V_{AS}} \left(1 + \frac{V_{AS}}{V_T}\right) \tag{12.134}$$

Similarly, we have for the product of f_S and Q_{ES}

$$f_S Q_{ES} = \frac{1}{2\pi R_{AE} C_{AS}} = \frac{\rho_0 c^2}{2\pi R_{AE} V_{AS}} \tag{12.135}$$

The equation for V_{AS} is obtained by taking the ratio of these two equations. It is given by

$$V_{AS} = V_T \left[\frac{f_C}{f_S} \frac{Q_{ECT}}{Q_{ES}} - 1\right] \tag{12.136}$$

12.7.5 Conversion to Infinite-Baffle Parameters

The above procedures give the parameters of the driver in free air, i.e. not in an infinite baffle. Because only the air-load mass is different for the two cases, the parameters can be converted to the infinite baffle parameters by dividing f_S by the mass correction factor $k_M = \left[M_{AS(\text{ib})}/M_{AS(\text{fa})}\right]^{1/2}$ and by multiplying Q_{MS}, Q_{ES}, and Q_{TS} by the same factor, where $M_{AS(\text{fa})}$ is the acoustic mass of the diaphragm plus air load in free air and $M_{AS(\text{ib})}$ is the acoustic mass of the diaphragm plus air load in an infinite baffle. These are related as follows:

$$M_{AS(\text{fa})} - 0.2705\frac{\rho_0}{a} = M_{AS(\text{ib})} - 2 \times \frac{8\rho_0}{3\pi^2 a} \tag{12.137}$$

where a is the equivalent piston radius of the diaphragm. Because

$$M_{AS(\text{fa})} = \frac{\rho_0 c^2}{\left[2\pi f_{S(\text{fa})}\right]^2 V_{AS}} \tag{12.138}$$

it follows that the correction factor is given by

$$k_M = \left[\frac{M_{AS(\text{ib})}}{M_{AS(\text{fa})}}\right]^{1/2} = \left[1 + 0.2699 \left(2\pi f_{S(\text{fa})}\right)^2 \frac{V_{AS}}{c^2 a}\right]^{1/2} \tag{12.139}$$

After k_M is calculated, the low-frequency parameters measured in free air can be converted to the infinite baffle parameters as described above. The correction has a minor effect on the parameters in most cases.

12.7.6 Measuring the Voice-Coil Inductance

The impedance of a lossy inductor can be written

$$Z_e(\omega) = (j\omega)^n L_e \tag{12.140}$$

where L_e and n are constants. It follows that $|Z_e(\omega)| = \omega^n L_e$ and $\arg[Z_e(\omega)] = n\pi/2$. This simple model for the lossy inductance often gives excellent results when applied to loudspeaker voice coils.

To experimentally determine the constants L_e and n.for a particular driver, the measurement test set shown in Fig. 12.18 can be used with the exception that a capacitor is connected in series with one lead of the driver. Because this capacitor must not be too large or too small, its value may have to be determined experimentally. Before connecting the capacitor, the first frequency above the resonance frequency f_S at which the Lissajous figure collapses to a straight line should be determined. This frequency is typically 4 to 5 times f_S. Above this frequency, the inductance causes the voice-coil impedance to increase with frequency. A series capacitor should be experimentally chosen to resonate with the voice-coil inductance well above this frequency. For example, a driver with a resonance frequency of 40 Hz may show a Lissajous figure that collapses to a straight line at 200 Hz. The capacitor might be chosen to resonate with the voice-coil impedance at 2 kHz.

If we assume that the frequency is high enough so that the back emf due to the voice coil velocity is negligible, the input impedance to the capacitor in series with the voice coil can be written

$$Z(\omega) = \frac{1}{j\omega C} + R_E + Z_e(\omega) = \frac{1}{j\omega C} + R_E + R_e(\omega) + jX_e(\omega) \tag{12.141}$$

where $Z_e(\omega)$ is the impedance of the lossy voice-coil inductance, $R_e(\omega) = \mathrm{Re}[Z_e(\omega)]$, and $X_e(\omega) = \mathrm{Im}[Z_e(\omega)]$. $Z(\omega)$ is real when $\omega C = 1/X_e(\omega)$. At this frequency, the Lissajous figure collapses to a straight line and $Z(\omega) = R_E + R_e(\omega)$.

Let capacitor C_1, determined by the method described above, be connected in series with the voice coil. Let the frequency $f_1 = \omega_1/2\pi$ be the frequency at which $Z(\omega)$ is real. It is straightforward to show that $R_e(f_1)$ and $X_e(f_1)$ are given by

$$R_e(f_1) = Z(f_1) - R_E = R_S \times \frac{V_b}{V_a} - R_E \qquad X_e(f_1) = \frac{1}{2\pi f_1 C_1} \tag{12.142}$$

Next, replace C_1 with capacitor C_2 which is chosen so that the frequency $f_2 = \omega_2/2\pi$ at which $Z(\omega)$ is real is in the upper audio frequency range. For example, the capacitor might be chosen to resonate with the voice-coil impedance at 20 kHz. $R_e(f_2)$ and $X_e(f_2)$ are given by

$$R_e(f_2) = Z(f_2) - R_E = R_S \times \frac{V_b}{V_a} - R_E \qquad X_e(f_2) = \frac{1}{2\pi f_2 C_2} \tag{12.143}$$

The measured values of R_e and X_e are related to the constants L_e and n by the equations

$$(2\pi f_1)^n L_e = |Z_e(f_1)| \qquad (2\pi f_2)^n L_e = |Z_e(f_2)| \tag{12.144}$$

where

$$|Z_e(f_1)| = \sqrt{R_e^2(f_1) + X_e^2(f_1)} \qquad |Z_e(f_2)| = \sqrt{R_e^2(f_2) + X_e^2(f_2)} \tag{12.145}$$

These equations can be solved for L_e and n to obtain

$$n = \frac{\log[|Z_e(f_2)| / |Z_e(f_1)|]}{\log(f_2/f_1)} \qquad L_e = \frac{|Z_e(f_1)|}{(2\pi f_1)^n} = \frac{|Z_e(f_2)|}{(2\pi f_2)^n} \tag{12.146}$$

The method described above is based on measurements at only two frequencies. A more accurate procedure is to make measurements at a series of frequencies and then plot the values of $|Z_e(f)|$ as a function of frequency on log-log scales. The plot should be approximately a straight line. A linear regression analysis of the data can be used to obtain the constants L_e and n from the data and to test the accuracy of the resulting approximations.

For a linear regression analysis, let $|Z_e(f)| = |Z_E(\omega) - R_E|$ be measured at N frequencies well above the fundamental resonance frequency where the impedance is dominated by the voice-coil inductance. Let k_1 through k_4 be defined by

$$k_1 = \sum \log f_i \qquad k_2 = \sum (\log f_i)^2 \qquad k_3 = \sum f_i \qquad k_4 = \sum f_i^2 \tag{12.147}$$

The linear regression solutions for constants L_e and n are

$$n = \frac{1}{Nk_2 - k_1^2}\left[N \sum \log f_i \, \log|Z_e(f_i)| - k_1 \sum \log|Z_e(f_i)|\right] \tag{12.148}$$

$$\log L_e = \frac{1}{Nk_2 - k_1^2}\left[k_2 \sum \log|Z_e(f_i)| - k_1 \sum \log f_i \, \log|Z_e(f_i)|\right] - n\log(2\pi) \tag{12.149}$$

where all summations have the limits $1 \leq i \leq N$.

It is straightforward to model the lossy inductor in SPICE with the analog behavioral modeling of *PSpice*. For example, let the inductor connect between nodes 2 and 3 in the circuit. The SPICE deck line for Z_e is as follows:

GZE 2 3 LAPLACE {V(2,3)}={1/(L_e*PWR(S,n))}

where numerical values for L_e and n must be used.

A more elaborate model for $Z_e(\omega)$ is to model $R'_E(\omega)$ and $X_E(\omega)$ with separate functions. Let $Y_e(\omega) = 1/Z_e(\omega) = G_e(\omega) + jB_e(\omega)$, where

$$G_e(\omega) = \mathrm{Re}\left(\frac{1}{R_e(\omega) + jX_e(\omega)}\right) \qquad B_e(\omega) = \mathrm{Im}\left(\frac{1}{R_e(\omega) + jX_e(\omega)}\right) \tag{12.150}$$

The approximations are as follows:

$$R'_E(\omega) = \frac{1}{G_e(\omega)} = R_e\omega^{n_r} \qquad X_E(\omega) = \frac{-1}{B_e(\omega)} = X_e\omega^{n_x} \tag{12.151}$$

If $Y_e(\omega)$ is determined at two frequencies f_1 and f_2 as described above, the constants R_e, n_r, X_e, and n_x are given by

$$n_r = \frac{\log\left[G_e(f_1)/G_e(f_2)\right]}{\log(f_2/f_1)} \qquad R_e = \frac{1}{(2\pi f_1)^{n_r} G_e(f_1)} = \frac{1}{(2\pi f_2)^{n_r} G_e(f_2)} \tag{12.152}$$

$$n_x = \frac{\log\left[B_e(f_1)/B_e(f_2)\right]}{\log(f_2/f_1)} \qquad X_e = \frac{-1}{(2\pi f_1)^{n_x} B_e(f_1)} = \frac{-1}{(2\pi f_2)^{n_x} B_e(f_2)} \tag{12.153}$$

If a series of N measurements are made, the linear regression solutions for the constants are

$$n_r = \frac{1}{Nk_2 - k_1^2}\left[N\sum \log f_i \, \log\left[\frac{1}{G_e(f_i)}\right] - k_1\sum \log\left[\frac{1}{G_e(f_i)}\right]\right] \tag{12.154}$$

$$\begin{aligned} \log R_e &= \frac{1}{Nk_2 - k_1^2}\left[k_2\sum \log\left[\frac{1}{G_e(f_i)}\right] - k_1\sum \log f_i \, \log\left[\frac{1}{G_e(f_i)}\right]\right] \\ &\quad -n_r \log(2\pi) \end{aligned} \tag{12.155}$$

$$n_x = \frac{1}{Nk_2 - k_1^2}\left[N\sum \log f_i \, \log\left[\frac{-1}{B_e(f_i)}\right] - k_1\sum \log\left[\frac{-1}{B_e(f_i)}\right]\right] \tag{12.156}$$

$$\begin{aligned} \log X_e &= \frac{1}{Nk_2 - k_1^2}\left[k_2\sum \log\left[\frac{-1}{B_e(f_i)}\right] - k_1\sum \log f_i \, \log\left[\frac{-1}{B_e(f_i)}\right]\right] \\ &\quad -n_x \log(2\pi) \end{aligned} \tag{12.157}$$

Both R'_E and jX_E can be modeled with the analog behavioral modeling of *PSpice*. Let the lossy inductor connect between nodes 2 and 3 in the circuit. The SPICE deck lines for R'_E and jX_E are as follows:

GREP 2 3 LAPLACE {V(2,3)}={1/(R_e*PWR(ABS(S),n_r))}
GXE 2 3 LAPLACE {V(2,3)}={1/(S*X_e*PWR(ABS(S),$n_x - 1$))}

where numerical values for R_e, n_r, X_e, and $n_x - 1$ must be used. Note that jX_E is implemented as $j\omega X_e\omega^{n_x-1}$.