ECE 6416 Quiz 1

September 28, 2005

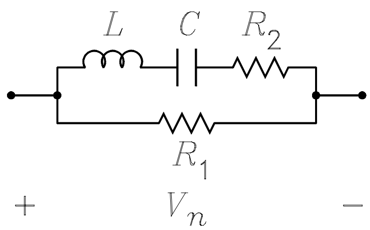
Professor Leach Name________________________

Instructions. Print your name in the space above and at the top of all other pages in your quiz. Place a box around each answer. Express each numerical answer as a decimal number. Staple your formula sheet to the back of the quiz. Numerical values are $4kT_0 = 1.6 \times 10^{-20}$ J and $q = 1.6 \times 10^{-19}$ C. **Honor Code:** *I have neither given nor received help on this quiz.* Initials ____________

Formulas:

$$\begin{aligned} v_n^2 &= 4kT\,\mathrm{Re}\,(Z)\,\Delta f \qquad i_n^2 = 4kT\,\mathrm{Re}\,(Y)\,\Delta f \qquad \omega = 2\pi f \qquad j = \sqrt{-1} \\ Z_C &= \frac{1}{j\omega C} \qquad Z_L = j\omega L \qquad \overline{V_n V_n^*} = v_n^2 \qquad \overline{I_n I_n^*} = i_n^2 \qquad \overline{V_n I_n^*} = \gamma v_n i_n \end{aligned}$$

1. (a) Use the Thévenin noise model for each resistor to solve for the phasor noise voltage V_n across the circuit shown. Express V_n as a function of V_{t1} and V_{t2}.



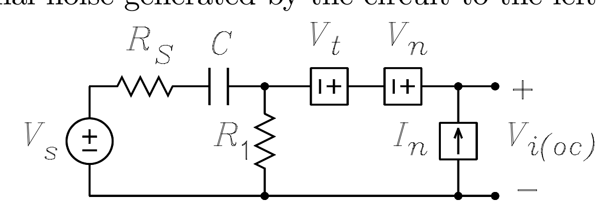
$$V_n = V_{t1}\frac{R_2 + j\omega L + 1/j\omega C}{R_1 + R_2 + j\omega L + 1/j\omega C} + V_{t2}\frac{R_1}{R_1 + R_2 + j\omega L + 1/j\omega C}$$

(b) Use the expression for V_n to obtain the mean-square noise voltage v_n^2 in the band Δf.

$$v_n^2 = 4kT\Delta f \frac{R_1\left[R_2^2 + (\omega L - 1/\omega C)^2\right] + R_1^2 R_2}{(R_1 + R_2)^2 + (\omega L - 1/\omega C)^2}$$

(c) For this problem, do you think it is simpler to use the Thévenin noise model of each resistor or the generalized Nyquist formula to solve for v_n^2? Explain your logic. It is more difficult to determine $\mathrm{Re}\,(Z)$ that $|Z|^2$ for this problem.

2. The figure shows the circuit for calculating the open-circuit input voltage to a particular amplifier. The sources V_n and I_n, respectively, are the amplifier input noise voltage and noise current. The complex correlation coefficient between these is γ. The source V_t represents the thermal noise generated by the circuit to the left of V_t.



(a) Solve for $V_{i(oc)}$ as a function of V_s, V_t, V_n, and I_n.

$$V_{i(oc)} = V_s \frac{R_1}{R_S + R_1 + 1/j\omega C} + V_t + V_n + I_n R_1 \| \left(R_S + \frac{1}{j\omega C} \right)$$

(b) Write the equation for $V_{i(oc)}$ in the form $V_{i(oc)} = A_v (V_s + V_{ni})$ and obtain the phasor equation for the noise equivalent input voltage V_{ni} in series with V_s that generates the same noise as all noise sources in the circuit.

$$V_{i(oc)} = A_v \left[V_s + (V_t + V_n) \left(1 + \frac{R_S}{R_1} + \frac{1}{j\omega R_1 C} \right) + I_n \left(R_S + \frac{1}{j\omega C} \right) \right]$$

$$V_{ni} = (V_t + V_n) \left(1 + \frac{R_S}{R_1} + \frac{1}{j\omega R_1 C} \right) + I_n \left(R_S + \frac{1}{j\omega C} \right)$$

(c) Solve for the expression for the mean-square value of V_t.

$$Z_{eq} = R_1 \| \left(R_S + \frac{1}{j\omega C} \right) = R_1 \frac{1 + j\omega R_S C}{1 + j\omega (R_S + R_1) C}$$

$$v_t^2 = 4kT \, \mathrm{Re}(Z_{eq}) \, \Delta f = 4kT R_1 \frac{1 + \omega^2 R_S (R_S + R_1) C^2}{1 + [\omega (R_S + R_1) C]^2} \Delta f$$

(d) Solve for the expression for the mean-square value of V_{ni}.

$$\begin{aligned} v_{ni}^2 &= \left(v_t^2 + v_n^2 \right) \left[\left(1 + \frac{R_S}{R_1} \right)^2 + \frac{1}{(\omega R_1 C)^2} \right] + i_n^2 \left(R_S^2 + \frac{1}{(\omega C)^2} \right) \\ &\quad + 2 v_n i_n \, \mathrm{Re} \left[\gamma \left(1 + \frac{R_S}{R_1} + \frac{1}{j\omega R_1 C} \right) \left(R_S - \frac{1}{j\omega C} \right) \right] \end{aligned}$$