## ECE 6416 Assignment 5

1. The figure shows a CMOS amplifier consisting of a p-channel input transistor $M_{1}$ and an n-channel load transistor $M_{2}$ biased by a fixed gate voltage $V_{B}$.

(a) Show that the small-signal voltage gain is given by

$$
\frac{v_{o}}{v_{i}}=-g_{m 1}\left(r_{d s 1} \| r_{d s 2}\right)
$$

(b) Show that the small-signal short-circuit output current is given by

$$
i_{o(s c)}=-g_{m 1}\left(v_{i}+v_{n 1}\right)-g_{m 2} v_{n 2}
$$

(c) If only flicker noise is modeled, show that the mean-square equivalent noise input voltage is given by

$$
v_{n i}^{2}=\frac{K_{f 1} \Delta f}{2 \mu_{p} L_{1} W_{1} C_{o x}^{2} f}\left[1+\frac{K_{f 2}}{K_{f 1}}\left(\frac{L_{1}}{L_{2}}\right)^{2}\right]
$$

How should the $W$ and $L$ for each device be chosen to minimize the noise? ( $L_{2}$ and $W_{1}$ should be large and $L_{1}$ and $W_{2}$ should be small)
2. The following MOSFET data are given

|  | n-Channel $\left(M_{2}\right)$ | p-Channel $\left(M_{1}\right)$ |
| :---: | :---: | :---: |
| $\frac{\mu_{0} C_{o x}}{2}$ | $7 \mu \mathrm{~A} / \mathrm{V}^{2}$ | $3 \mu \mathrm{~A} / \mathrm{V}^{2}$ |
| $\frac{K_{f}}{2 \mu_{0} C_{o x}^{2}} \int_{20}^{20 k} \frac{d f}{f}$ | $380 \times 10^{3}(\mu \mathrm{~V} \times \mu \mathrm{m})^{2}$ | $48 \times 10^{3}(\mu \mathrm{~V} \times \mu \mathrm{m})^{2}$ |

If the value of $C_{o x}$ is the same for both MOSFETs in the circuit of Problem 1, calculate $v_{n i}$ for the following values of $W$ and $L$ :

|  | $W_{1}$ | $L_{1}$ | $W_{2}$ | $L_{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| Case 1 | $1000 \mu \mathrm{~m}$ | $5 \mu \mathrm{~m}$ | $400 \mu \mathrm{~m}$ | $4 \mu \mathrm{~m}$ |
| Case 2 | $1000 \mu \mathrm{~m}$ | $5 \mu \mathrm{~m}$ | $200 \mu \mathrm{~m}$ | $8 \mu \mathrm{~m}$ |
| Case 3 | $500 \mu \mathrm{~m}$ | $10 \mu \mathrm{~m}$ | $400 \mu \mathrm{~m}$ | $4 \mu \mathrm{~m}$ |

$(16.9 \mu \mathrm{~V}, 8.88 \mu \mathrm{~V}$, and $33.4 \mu \mathrm{~V})$
3. The figure shows an n-channel NMOS enhancement-mode common-source amplifier with an active n-channel NMOS enhancement-mode load. The two transistors are biased at the same drain current $I_{D}$ and have the same value for $C_{o x}$.

(a) Show that the small-signal short-circuit output current is given by

$$
i_{o(s c)}=-g_{m 1}\left(v_{i}+v_{n 1}\right)+g_{m 2} v_{n 2}
$$

(b) Show that the small-signal output resistance is given by

$$
r_{o u t}=r_{d s 1}\left\|r_{d s 2}\right\|\left(\frac{1}{g_{m 2}\left(1+\chi_{2}\right)}\right)
$$

(c) Show that the open-circuit output voltage is given by

$$
v_{o(o c)}=\left(-g_{m 1}\left(v_{i}+v_{n 1}\right)+g_{m 2} v_{n 2}\right) \times r_{d s 1}\left\|r_{d s 2}\right\|\left(\frac{1}{g_{m 2}\left(1+\chi_{2}\right)}\right)
$$

(d) If only flicker noise is modeled, show that the mean-square equivalent noise input voltage is given by

$$
v_{n i}^{2}=\frac{K_{f 1} \Delta f}{2 \mu_{n} C_{o x}^{2} L_{1} W_{1} f}\left[1+\left(\frac{L_{1}}{L_{2}}\right)^{2}\right]
$$

It is obvious that $W_{1}$ should be large to minimize the noise. What should $L_{1}$ be to minimize the noise? $\left(L_{1}=L_{2}\right)$
(e) If only thermal noise is modeled, show that the mean-square equivalent noise input voltage is given by

$$
v_{n i}^{2}=\frac{4 k T \Delta f}{3 \sqrt{K_{1} I_{D}}}\left[1+\sqrt{\frac{L_{1} W_{2}}{L_{2} W_{1}}}\right]
$$

How should the $W$ and $L$ for each device be chosen to minimize the noise? ( $L_{2}$ and $W_{1}$ should be large and $L_{1}$ and $W_{2}$ should be small)
4. Repeat problem 2 for part (d) of problem 3. $(14.0 \mu \mathrm{~V}, 10.3 \mu \mathrm{~V}$, and $23.5 \mu \mathrm{~V})$

