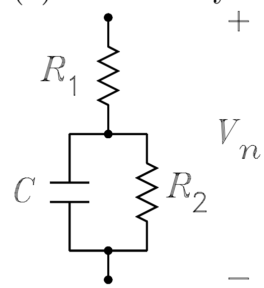
ECE 6416 Quiz 1

October 3, 2007

Professor Leach Name__________________________________

Instructions. Print your name in the space above and at the top of all other pages in your quiz. Place a box around each answer. Express each numerical answer as a decimal number. Numerical values are $4kT_0 = 1.6 \times 10^{-20}$ J and $q = 1.6 \times 10^{-19}$ C. **Honor Code:** *I have neither given nor received help on this quiz.* Initials ____________

1. The figure shows an RC network. [Normalize all expressions containing ω so that they are of the form $a\,(1 + b\omega^n)$.]
(a) Replace R_1 with its Thévenin noise model and R_2 with its Norton noise model. Use phasor analysis to write the equation for V_n. Convert the equation into a mean-square voltage. The noise sources are not correlated.
(b) Show that you get the same result with the generalized Nyquist formula.



$$V_n = V_{t1} + I_{t2}\left(R_2 \| \frac{1}{j\omega C}\right) = V_{t1} + I_{t2}\frac{R_2}{1 + j\omega R_2 C}$$

$$\begin{aligned} v_n^2 &= v_{t1}^2 + i_{t2}^2 \frac{R_2^2}{1 + (\omega R_2 C)^2} \\ &= 4kTR_1\Delta f + \frac{4kT\Delta f}{R_2}\frac{R_2^2}{1 + (\omega R_2 C)^2} \\ &= 4kT\Delta f\left(R_1 + \frac{R_2}{1 + (\omega R_2 C)^2}\right) \end{aligned}$$

$$Z = R_1 + R_2 \| \frac{1}{j\omega C} = R_1 + \frac{R_2}{1 + j\omega R_2 C} = R_1 + R_2\frac{1 - j\omega R_2 C}{1 + (\omega R_2 C)^2}$$

$$v_n^2 = 4kT\,\mathrm{Re}\left(R_1 + R_2\frac{1 - j\omega R_2 C}{1 + (\omega R_2 C)^2}\right)\Delta f = 4kT\,\mathrm{Re}\left(R_1 + \frac{R_2}{1 + (\omega R_2 C)^2}\right)\Delta f$$

2. The op amp is ideal and noiseless. Redraw the circuit with the resistors replaced with their noise equivalent models (either Thévenin or Norton).(a) Solve for V_+ and V_-, set $V_+ = V_-$, and solve for the equation for V_o as a function of V_s and the thermal noise sources in the circuit.

$$V_+ = V_{t1}$$

$$V_- = \left(\frac{V_s + V_{ts}}{R_S} + \frac{V_o + V_{t2}}{R_2}\right)(R_S \| R_2)$$

$$V_o = R_2\left(\frac{V_{t1}}{R_S\|R_2} - \frac{V_s + V_{ts}}{R_S}\right) - V_{t2}$$

(b) Put the equation obtained above into the form $V_o = K\left(V_s + V_{ni}\right)$ and give the equations for K and V_{ni}.

$$\begin{aligned} V_o &= -\frac{R_2}{R_S}\left(V_s + V_{ts} - \frac{V_{t1}R_S}{R_S\|R_2} + V_{t2}\frac{R_S}{R_2}\right) \\ &= -\frac{R_2}{R_S}\left[V_s + V_{ts} - V_{t1} - R_S\left(\frac{V_{t1}}{R_2} - \frac{V_{t2}}{R_2}\right)\right] \end{aligned}$$

$$K = -\frac{R_2}{R_S} \qquad V_{ni} = V_{ts} - V_{t1} - R_S\left(\frac{V_{t1}}{R_2} - \frac{V_{t2}}{R_2}\right)$$

(c) Use the V_{ni} equation to solve for the V_n and I_n of the amplifier.

$$V_n = -V_{t1} \qquad I_n = -\frac{V_{t1}}{R_2} + \frac{V_{t2}}{R_2}$$

(d) Solve for v_n^2, i_n^2, and γ.

$$v_n^2 = 4kTR_1\Delta f \qquad i_n^2 = 4kT\Delta f\left(\frac{R_1 + R_2}{R_2^2}\right)$$

$$\begin{aligned} \gamma &= \frac{1}{\sqrt{4kTR_1\Delta f \times 4kT\Delta f\left(\frac{R_1+R_2}{R_2^2}\right)}}\frac{4kTR_1\Delta f}{R_2} \\ &= \sqrt{\frac{R_1}{R_1 + R_2}} \end{aligned}$$