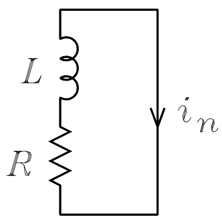
ECE 6416 Quiz 1
October 6, 2008

Professor Leach Name______________________________

Instructions. Print your name in the space above and at the top of all other pages in your quiz. Place a box around each answer. Express each numerical answer as a decimal number. Numerical values are $4kT_0 = 1.6 \times 10^{-20}$ J and $q = 1.6 \times 10^{-19}$ C. **Honor Code:** *I have neither given nor received help on this quiz.* Initials __________

1. The circuit shows a series resistor R and ideal inductor L. You are given that $\int_0^\infty dx/\left(1+x^2\right) = \pi/2$



(a) Using the Thévenin form for the thermal noise generated by R, solve for the mean-square value of the short-circuit noise current i_n^2 over the frequency band from $f=0$ to $f=\infty$.

$$
\begin{aligned}
i_n^2 &= \int_0^\infty \frac{v_t^2}{|R+j2\pi fL|^2} = \int_0^\infty \frac{4kTRdf}{R^2+(2\pi fL)^2} = \frac{4kT}{R}\int_0^\infty \frac{df}{1+\left(\frac{2\pi fL}{R}\right)^2} \\
&= \frac{2kT}{\pi L}\int_0^\infty \frac{dx}{1+x^2} = \frac{kT}{L}
\end{aligned}
$$

(b) Repeat part (a) using the generalized admittance Nyquist theorem. Show that the same result is obtained.

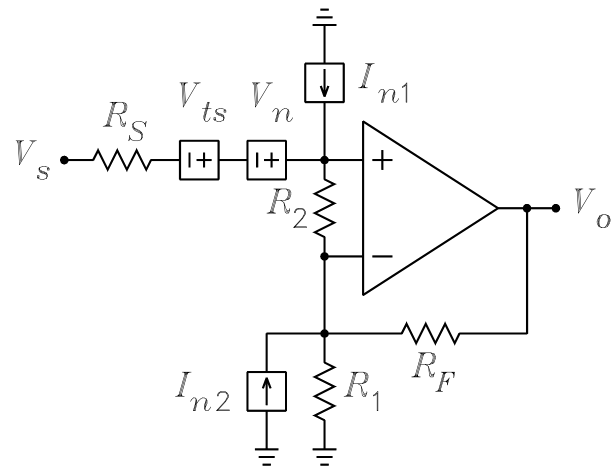
$$
\begin{aligned}
i_n^2 &= \int_0^\infty 4kT\,\mathrm{Re}\left(\frac{1}{R+j2\pi fL}\right) df = 4kT\int_0^\infty \frac{R}{R^2+(2\pi fL)^2}df \\
&= \frac{4kT}{R}\int_0^\infty \frac{df}{1+\left(\frac{2\pi L}{R}f\right)^2} = \frac{kT}{L}
\end{aligned}
$$

2. The op amp shown is ideal. It is given that $\Delta f = 10\,\text{kHz}$, $R_S = 2\,\text{k}\Omega$, $R_1 = 1\,\text{k}\Omega$, $R_2 = 5.1\,\text{k}\Omega$, $R_F = 47\,\text{k}\Omega$, $v_n = 2\,\text{nV}/\sqrt{\text{Hz}}$, and $i_{n1} = i_{n2} = 1.5\,\text{pA}/\sqrt{\text{Hz}}$. The op-amp noise sources are shown. The thermal noise sources of the resistors must be added. Assume that all noise sources are uncorrelated.

(a) Using the Norton noise model for R_2, add the thermal noise sources and solve for the equation for V_o as a function of V_s and all noise sources. Note that R_2 does not affect the gain.

$$V_s + V_{ts} + V_n + (I_{n1}+I_{t2})R_S = \frac{R_1}{R_F+R_1}V_o + V_{teq} + (I_{n2}-I_{t2})R_1\|R_F$$

$$V_o = \left(1+\frac{R_F}{R_1}\right)\left[V_o + V_{ts} + V_n + I_{n1}R_S - I_{n2}R_1\|R_F - V_{teq} + I_{t2}\left(R_S + R_1\|R_F\right)\right]$$



(b) Put the equation obtained in part (a) into the form $V_o = K(V_s + V_{ni})$ and give the symbolic equations for K and V_{ni}.

$$K = 1 + \frac{R_F}{R_1} \qquad V_{ni} = V_{ts} + V_n + I_{n1}R_S - I_{n2}R_1\|R_F - V_{teq} + I_{t2}(R_S + R_1\|R_F)$$

(c) Over the band $\Delta f = 10\,\text{kHz}$, solve for the numerical values of v_{ni}, F, and NF.

$$\begin{aligned} v_{ni} &= \sqrt{\left[4kTR_S + v_n^2 + i_{n1}^2R_S^2 + i_{n2}^2(R_1\|R_F)^2 + 4kTR_1\|R_F + \frac{4kT}{R_2}(R_S + R_1\|R_F)^2\right]\Delta f} \\ &= 9.522 \times 10^{-7}\,\text{V} \end{aligned}$$

$$F = \frac{v_{ni}^2}{4kTR_S\Delta f} = 2.833 \qquad NF = 10\log(F) = 4.523\,\text{dB}$$

(d) If V_{ni} is written $V_{ni} = V_{ts} + V_n' + I_n'R_S$, solve for the symbolic equations for V_n', I_n', and $\gamma' = \overline{V_n'I_n'^*}/v_n'i_n'$.

$$V_n' = V_n - I_{n2}R_1\|R_F - V_{teq} + I_{t2}R_1\|R_F$$

$$I_n' = I_{n1} + I_{t2} \qquad \gamma = \frac{1}{v_n'i_n'}i_{t2}^2R_1\|R_F$$

(e) Over the band $\Delta f = 10\,\text{kHz}$, solve for the numerical values of v_n', i_n', and γ'.

$$v_n' = \sqrt{\left[v_n^2 + 4kT(R_1\|R_F) + \left(\frac{4kT}{R_2} + i_{n2}^2\right)(R_1\|R_F)^2\right]\Delta f} = 4.983 \times 10^{-7}\,\text{V}$$

$$i_n' = \sqrt{\left(i_{n1}^2 + \frac{4kT}{R_2}\right)\Delta f} = 2.2321 \times 10^{-10}\,\text{A} \qquad \gamma = \frac{1}{v_n'i_n'}\frac{4kT\Delta f}{R_2}(R_1\|R_F) = 0.266$$