Noise Analysis of Transformer-Coupled Preamplifiers*

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Noise analyses of input stages of typical transformer-coupled preamplifiers are presented. Conditions are derived for which the transformer improves the signal-to-noise ratio of the circuits. Examples are presented which are typical for microphone preamplifiers.

0 INTRODUCTION

A transformer connected between a transducer such as a microphone and its preamplifier is described in the literature as a noise-matching device [1]–[3]. If the source resistance, the preamplifier equivalent noise input voltage, and the preamplifier equivalent noise input current are known, the optimum turns ratio of the transformer which minimizes the noise can be calculated. Fig. 1(a) shows a source with an open-circuit output voltage v_s and an output resistance R_s coupled by a transformer with a turns ratio of $1:N$ to a preamplifier. The thermal noise voltage generated by R_s is denoted by $e_\tau(R_s)$. The preamplifier is modeled as having an input resistance R_i, a voltage gain A_v, an output resistance R_o, an equivalent noise input voltage e_n, and an equivalent noise input current j_n. Fig. 1(b) shows the equivalent circuit of the combined source, transformer, and noise sources, where it is assumed that the transformer is ideal.

The open-circuit, or Thévenin, output voltage from the circuit in Fig. 1(b) is denoted by v_{ti}. This voltage is given by

$$v_{ti} = Nv_s + Ne_\tau(R_s) + e_n + j_n N^2 R_s$$

$$= N\left[v_s + e_\tau(R_s) + \frac{e_n}{N} + j_n N R_s \right] . \qquad (1)$$

Let e_{ni} denote the equivalent noise input voltage in series with v_s which generates the same noise at the preamplifier output as the noise sources $e_\tau(R_s)$, e_n, and

j_n in combination. It follows from Eq. (1) that the mean-squared value of e_{ni} is given by

$$\langle e_{ni}^2 \rangle = \langle e_\tau^2(R_s) \rangle + \frac{1}{N^2} \langle e_n^2 \rangle + \langle j_n^2 \rangle N^2 R_s^2 . \qquad (2)$$

This is minimized when the transformer turns ratio satisfies the relation

$$N = \sqrt[4]{\frac{\langle e_n^2 \rangle}{\langle j_n^2 \rangle R_s^2}} . \qquad (3)$$

Eq. (3) is commonly quoted for the transformer turns ratio which miminizes the noise output voltage of a transformer-coupled preamplifier. The formula results in the condition for minimum noise only if it is assumed that the transformer is noiseless and that the preamplifier
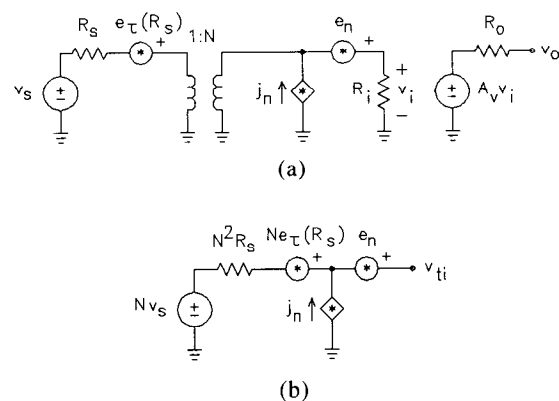

(a)

(b)

Fig. 1. (a) Source coupled by transformer to preamplifier. (b) Equivalent circuit seen looking out of preamplifier input terminals.

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equivalent input noise sources e_n and j_n are fixed, that is, the rms values of these sources are fixed. In practice the condition for minimum noise is more complicated to determine for several reasons. First, the source resistance seen by the preamplifier is a function of the transformer turns ratio N. Second, the optimum bias current in the preamplifier input stage which minimizes the noise is a function of the source resistance. Third, the equivalent noise sources e_n and j_n are functions of the bias current. And fourth, the transformer is not noiseless, that is, its primary and secondary windings exhibit series resistance which generates thermal noise. It follows that a more in-depth analysis is required to determine the condition for minimum noise. This paper presents such an analysis for bipolar junction transistor (BJT) and junction field-effect transistor (JFET) input stage devices.

1 SINGLE BIPOLAR JUNCTION TRANSISTOR INPUT STAGE

The noise performance of a preamplifier is determined predominantly by the input stage. Except for the case of a very low source output resistance [3], [4], the fewer the number of active devices in this stage, the lower the noise. It follows, therefore, that a single BJT input stage is optimum. However, a two-BJT differential amplifier is often used as a compromise because of other design considerations. In this section, the noise analysis of a single BJT input stage is presented. In the following section, the two-BJT differential-amplifier input stage is analyzed. Feedback is assumed to be series-shunt feedback from the output node back to the input stage. The analysis also applies to the case where no feedback is used.

Fig. 2 shows the midband signal circuit of a BJT common-emitter input stage with a signal source coupled to its base input through a transformer. The primary and secondary transformer winding resistances are denoted by R_1 and R_2, respectively. It is assumed that the BJT is biased at the collector current I_C and that the bias circuit, which is not shown, is noiseless. The Thévenin equivalent circuit seen looking out of the emitter consists of a source with an open-circuit voltage $v_{tf} = bv_o$ in series with the resistance R_{tf}, where v_o is the preamplifier output voltage and b is the feedback ratio. The source v_{tf} represents the series-shunt feedback applied from the preamplifier output node (which is
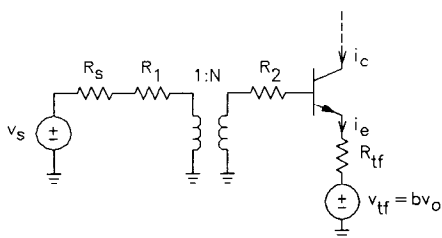
not shown) back to the emitter. In the case of no feedback, v_{tf} is set equal to zero.

Fig. 3 is the circuit of Fig. 2 with the noise sources shown. The BJT noise sources are shown as external generators. The BJT base spreading resistance r_x is shown as an external resistance in series with the base lead. The thermal noise voltages generated by the resistors in the circuit are denoted by $e_\tau(R_s)$, $e_\tau(R_1)$, $e_\tau(R_2)$, $e_\tau(r_x)$, and $e_\tau(R_{tf})$. The shot noise currents associated with the collector bias current I_C and the base bias current I_B are denoted by $j_\sigma(I_C)$ and $j_\sigma(I_B)$, where $I_B = I_C/\beta$. The small-signal collector current in the circuit can be written

$$i_c = \alpha i_e' + j_\sigma(I_C) \tag{4}$$

where α is the emitter-to-collector current gain.

To solve for i_e', the transistor is replaced by the small-signal Thévenin equivalent circuit seen looking into its emitter. This consists of a voltage source v_{tb} in series with a resistor r_{ie}, where v_{tb} is the open-circuit, or Thévenin, voltage seen looking out of the base. The resistor r_{ie} is given by

$$r_{ie} = \frac{\alpha V_T}{I_C} + \frac{\alpha}{\beta} R_{tb} \tag{5}$$

where R_{tb} is the resistance of the Thévenin equivalent circuit seen looking out of the base. It follows from Fig. 3 that v_{tb} and R_{tb} are given by

$$v_{tb} = N[v_s + e_\tau(R_s) + e_\tau(R_1)] + e_\tau(R_2) + e_\tau(r_x)$$
$$+ j_\sigma(I_B)[N^2(R_s + R_1) + R_2 + r_x] \tag{6}$$

$$R_{tb} = N^2(R_s + R_1) + R_2 + r_x . \tag{7}$$

The calculation for i_e' is simplified by representing the circuit seen looking out of the emitter by a Thévenin equivalent circuit consisting of a voltage v_{te} in series with a resistor R_{te}. These are given by

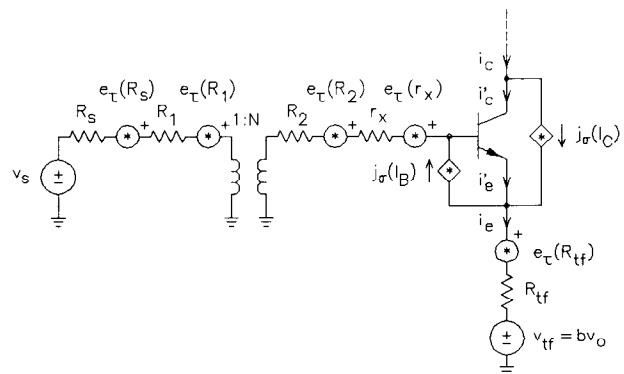$$v_{te} = bv_o + e_\tau(R_{tf}) + [j_\sigma(I_C) - j_\sigma(I_B)]R_{tf} \tag{8}$$

Fig. 2. Midband signal circuit of source coupled to BJT common-emitter stage by transformer.

Fig. 3. Circuit of Fig. 2 with noise sources shown.

$$R_{te} = R_{tf} . \tag{9}$$

Fig. 4 gives the final BJT emitter equivalent circuit. This circuit and Eq. (4) can be used to solve for the small-signal collector output current as follows:

$$i_C = \alpha \frac{v_{tb} - v_{te}}{r_{ie} + R_{te}} + j_\sigma(I_C)$$

$$= N \frac{\alpha}{r_{ie} + R_{te}} [v_s + e_{ni}] - \frac{\alpha}{r_{ie} + R_{te}} bv_o \tag{10}$$

where e_{ni} is the equivalent input noise voltage in series with v_s which generates the same noise current in i_c as all noise sources combined. It is given by

$$e_{ni} = e_\tau(R_s) + e_\tau(R_1) + \frac{1}{N} [e_\tau(R_2) + e_\tau(r_x) - e_\tau(R_{tf})]$$

$$+ \frac{1}{N} j_\sigma(I_B) [N^2(R_s + R_1) + R_2 + r_x + R_{tf}]$$

$$+ \frac{1}{N} j_\sigma(I_C) \left[\frac{V_T}{I_C} + \frac{N^2(R_s + R_1) + R_2 + r_x + R_{tf}}{\beta} \right] \tag{11}$$

where the relation $\beta = \alpha/(1 - \alpha)$ has been used in the simplification of the latter term.

The mean-squared thermal noise voltage produced by a resistor in the frequency band Δf is $\langle e_\tau^2(R) \rangle = 4kT\Delta f R$, where k is Boltzmann's constant and T is the absolute temperature. The mean-squared shot noise current associated with the dc current I in the frequency band Δf is given by $\langle j_\sigma^2(I) \rangle = 2qI\Delta f$, where q is the electronic charge. If it is assumed that the terms in Eq. (11) are uncorrelated, it follows that the mean-squared value of e_{ni} in the band Δf is given by

$$\langle e_{ni}^2 \rangle = 4kT\Delta f \left[R_s + R_1 + \frac{R_2 + r_x + R_{tf}}{N^2} \right]$$

$$+ \frac{1}{N^2} 2q \frac{I_C}{\beta} \Delta f [N^2(R_s + R_1) + R_2 + r_x + R_{tf}]^2$$

$$+ \frac{1}{N^2} 2qI_C\Delta f \left[\frac{V_T}{I_C} \right.$$

$$\left. + \frac{N^2(R_s + R_1) + R_2 + r_x + R_{tf}}{\beta} \right]^2 . \tag{12}$$

It can be seen from this equation that $\langle e_{ni}^2 \rangle$ increases if I_C is very small or very large. It follows that there is

a value of I_C which minimizes the noise. It is straightforward to show that the optimum collector current which minimizes the noise is given by

$$I_{C(opt)} = \frac{\sqrt{\beta}\ V_T}{\sqrt{1 + 1/\beta}\ [N^2(R_s + R_1) + R_2 + r_x + R_{tf}]} . \tag{13}$$

The corresponding minimum mean-squared noise voltage is given by

$$\langle e_{ni}^2 \rangle_{min} = 4kT\Delta f \left[1 + \frac{1 + \sqrt{1 + \beta}}{\beta} \right]$$

$$\times R_s + R_1 + \frac{R_2 + r_x + R_{tf}}{N^2} \tag{14}$$

where $V_T = kT/q$ has been used in the simplification. By setting $N = 1$ and $R_1 = R_2 = 0$ in this equation, the minimum mean-squared noise voltage is obtained for the case without the transformer. It follows that the transformer results in less noise only if the following condition is satisfied:

$$\frac{N^2 R_1 + R_2}{N^2 - 1} < r_x + R_{tf} . \tag{15}$$

For an example calculation, a commercial transformer has the measured parameters $N = 12$, $R_1 = 72\Omega$, and $R_2 = 4200\ \Omega$. It follows that this transformer results in less noise if $r_x + R_{tf} > 102\ \Omega$. Low-noise BJTs are commonly available with r_x less than about 10 Ω. If it is assumed that $r_x = 10\ \Omega$, it follows that the transformer results in more noise for $R_{tf} < 92\ \Omega$. If no feedback is used, it is possible to design the input stage with $R_{tf} = 0$. With feedback, a value of $R_{tf} < 92\ \Omega$ is easily realized. Therefore it can be concluded that the circuit noise is lower without the transformer.

For a second example calculation, the optimum bias current in the BJT is calculated with and without the example transformer. In this case it is assumed that $R_s = 200\ \Omega$ and $r_x + R_{tf} = 102\ \Omega$, so that the mean-squared noise voltages with and without the transformer are equal. In addition, it is assumed that the current gain of the BJT is $\beta = 150$. The temperature is taken to be $T = 300$ K. It follows from Eq. (13) that the
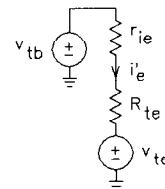

Fig. 4. Emitter equivalent circuit used to calculate i'_e.

optimum collector bias current is $I_{C(opt)} = 7.27$ μA with the transformer and $I_{C(opt)} = 1.05$ mA without the transformer. Because the slew rate of a preamplifier is proportional to the bias current in the input stage, it follows that the optimally biased input stage has a slew rate that is potentially 144 times greater without the transformer. (The actual increase in slew rate depends on the frequency compensation of the circuit.) The superiority of the circuit without the transformer is evident.

For a third example calculation, the signal-to-noise ratio of the circuit is calculated. The bandwidth is taken to be $\Delta f = 20$ kHz and an absolute temperature of $T = 300$ K is used. It is assumed that v_s is 55 dB below 1 V rms so that the signal-to-noise ratio is given by SNR $= -(55 + 10 \log \langle e_{ni}^2 \rangle)$. Because of the value used for $r_x + R_{tf}$, the mean-squared noise voltage given by Eq. (14) is the same with and without the transformer and has the value $\langle e_{ni}^2 \rangle = 1.09 \times 10^{-13}$ V$_2$. The signal-to-noise ratio is SNR $= 74.6$ dB with and without the transformer.

For a fourth example calculation, the signal-to-noise ratio is calculated for the following two cases: 1) the BJT without the transformer but optimally biased for the transformer and 2) the BJT with the transformer but optimally biased for no transformer. In each case the mean-squared noise voltage is calculated from Eq. (12). For case 1, $\langle e_{ni}^2 \rangle = 6.91 \times 10^{-13}$ V^2 and SNR $= 66.6$ dB. For case 2, $\langle e_{ni}^2 \rangle = 6.93 \times 10^{-13}$ V^2 and SNR $= 66.6$ dB. Thus the signal-to-noise ratio is 8 dB less in each case compared to the optimally biased cases.

2 BJT DIFFERENTIAL-AMPLIFIER INPUT STAGE

Fig. 5 shows the midband signal circuit of a BJT differential-amplifier input stage with a signal source coupled to the base input of Q_1 through a transformer. The dc tail current is labeled I_T. The two emitter resistors labeled R_E have been included for completeness. However, these resistors are not normally used in low-noise designs because they generate undesired thermal noise. It is assumed that the transistors have equal current gains α and β and equal base spreading resistances r_x. The Thévenin equivalent circuit seen looking out of the base of Q_2 consists of a source with an open-circuit voltage $v_{tf} = bv_o$ in series with the resistance R_{tf}, where v_o is the preamplifier output voltage and b is the feedback ratio. In the case of no feedback, v_{tf} is set equal to zero.

Fig. 6 is the circuit of Fig. 5 with the noise sources shown. The BJT noise sources are shown as external generators and the base spreading resistances as external resistances in series with the base leads. The noise current of the tail bias network is denoted by j_t. It is assumed that the small-signal resistance seen looking into the tail supply is large compared to the other resistances in the circuit so that it can be considered to be an open circuit. This is equivalent to the assumption that the differential amplifier has a high common-mode rejection ratio. The thermal noise voltages generated

by the resistors in the circuit are denoted by $e_\tau(R_s)$, $e_\tau(R_1)$, $e_\tau(R_2)$, $e_{\tau 1}(r_x)$, $e_{\tau 1}(R_E)$, $e_{\tau 2}(R_E)$, $e_{\tau 2}(r_x)$, and $e_\tau(R_{tf})$. The shot noise currents associated with the collector bias currents and the base bias currents are denoted by $j_\sigma(I_{C1})$, $j_\sigma(I_{C2})$, $j_\sigma(I_{B1})$, and $j_\sigma(I_{B2})$, where $I_{B1} = I_{C1}/\beta$, $I_{B2} = I_{C2}/\beta$, and $I_{C1} + I_{C2} = \alpha I_T$.

Fig. 7 shows the emitter equivalent circuit for the differential amplifier. In this circuit v_{tb1}, r_{ie1}, v_{tb2}, and r_{ie2} are given by

$$v_{tb1} = N[v_s + e_\tau(R_s) + e_\tau(R_1)] + e_\tau(R_2) + e_{\tau 1}(r_x)$$
$$+ j_\sigma(I_{B1}) [N^2(R_s + R_1) + R_2 + r_x] \qquad (16)$$

$$r_{ie1} = \frac{\alpha V_T}{I_{C1}} + \frac{\alpha}{\beta} [N^2(R_s + R_1) + R_2 + r_x] \qquad (17)$$

$$v_{tb2} = bv_o + e_\tau(R_{tf}) + e_{\tau 2}(r_x) + j_o(I_{B2}) [R_{tf} + r_x] \qquad (18)$$

$$r_{ie2} = \frac{\alpha V_T}{I_{C2}} + \frac{\alpha}{\beta} (R_{tf} + r_x) . \qquad (19)$$

It follows from this circuit that the small-signal emitter current i'_{e1} is given by

$$i'_{e1} = \frac{1}{2R_E + r_{ie1} + r_{ie2}} [v_{tb1} - e_{\tau 1}(R_E)$$
$$+ e_{\tau 2}(R_E) - v_{tb2}]$$
$$- \frac{2R_E + r_{ie2}}{2R_E + r_{ie1} + r_{ie2}} [j_\sigma(I_{C1}) - j_\sigma(I_{B1})]$$
$$- \frac{r_{ie2}}{2R_E + r_{ie1} + r_{ie2}} [j_\sigma(I_{C2}) - j_\sigma(I_{B2})]$$
$$- \frac{R_E + r_{ie2}}{2R_E + r_{ie1} + r_{ie2}} j_t . \qquad (20)$$

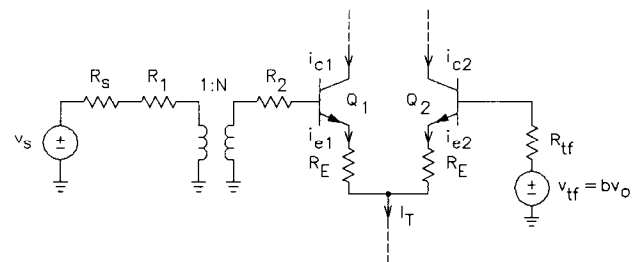The emitter current i'_{e2} is obtained by interchanging subscripts 1 and 2 in Eq. (20).

Fig. 5. Midband signal circuit of source coupled to BJT differential-amplifier stage by transformer.

For minimum noise output from a differential-amplifier input stage, the output signal should be a differential signal, that is, the output voltage should be proportional to the difference current $i_{c1} - i_{c2}$. The collector current for each BJT is given by Eq. (4). To solve for the collector difference current, the emitter difference current is required, which is given by

$$i'_{e1} - i'_{e2} = \frac{2}{2R_E + r_{ie1} + r_{ie2}} [v_{tb1} - e_{\tau1}(R_E)$$

$$+ e_{\tau2}(R_E) - v_{tb2}]$$

$$- \frac{2R_E + r_{ie2} - r_{ie1}}{2R_E + r_{ie1} + r_{ie2}} [j_\sigma(I_{C1}) - j_\sigma(I_{B1})]$$

$$+ \frac{2R_E + r_{ie1} - r_{ie2}}{2R_E + r_{ie1} + r_{ie2}} [j_\sigma(I_{C2}) - j_\sigma(I_{B2})]$$

$$- \frac{r_{ie2} - r_{ie1}}{2R_E + r_{ie1} + r_{ie2}} j_t . \qquad (21)$$

This expression shows that the noise output caused by the tail noise current j_t is not canceled unless $r_{ie1} = r_{ie2}$. Eqs. (17) and (19) show that $r_{ie1} = r_{ie2}$ if $I_{C1} = I_{C2}$ and $N^2(R_s + R_1) + R_2 = R_{tf}$. These conditions will be assumed to hold in the following.

It follows that the collector difference output current can be written

$$i_{c1} - i_{c2} = \alpha(i'_{e1} - i'_{e2}) + j_\sigma(I_{C1}) - j_\sigma(I_{C2})$$

$$= N \frac{\alpha}{r_{ie} + R_E} (v_s + e_{ni}) - \frac{\alpha}{r_{ie} + R_E} bv_o$$

$$\qquad (22)$$

where $r_{ie} = r_{ie1} = r_{ie2}$ has been used and e_{ni} is the equivalent input noise voltage in series with v_s, which generates the same noise current in $i_{c1} - i_{c2}$ as all noise sources combined. It is given by

$$e_{ni} = e_\tau(R_s) + e_\tau(R_1) + \frac{1}{N} [e_\tau(R_2) + e_{\tau1}(r_x)$$

$$- e_{\tau1}(R_E) + e_{\tau2}(R_E) - e_{\tau2}(r_x) - e_\tau(R_{tf})]$$

$$+ \frac{1}{N} [j_\sigma(I_{B1}) - \frac{1}{N} j_\sigma(I_{B2})] [N^2(R_s + R_1)$$

$$+ R_2 + r_x + R_E]$$

$$+ \frac{1}{N} [j_\sigma(I_{C1}) - j_\sigma(I_{C2})] \left(\frac{r_{ie} + R_E}{\alpha} - R_E \right) .$$

$$\qquad (23)$$
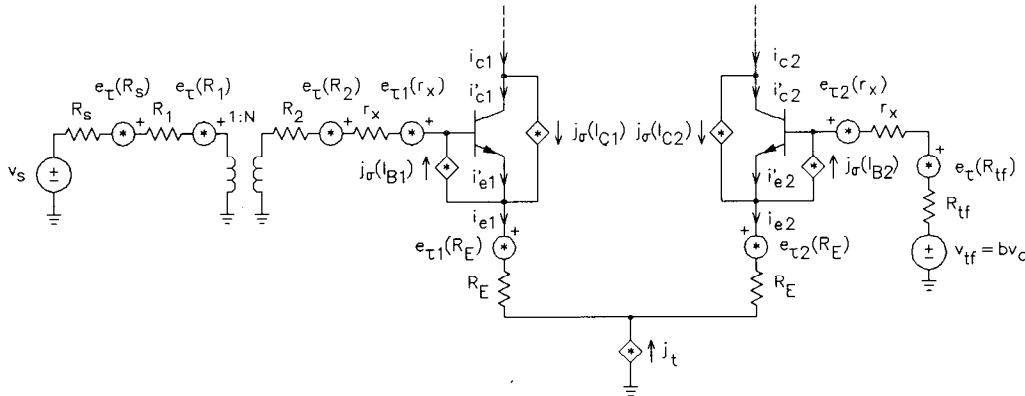
This can be converted to a mean-squared voltage in
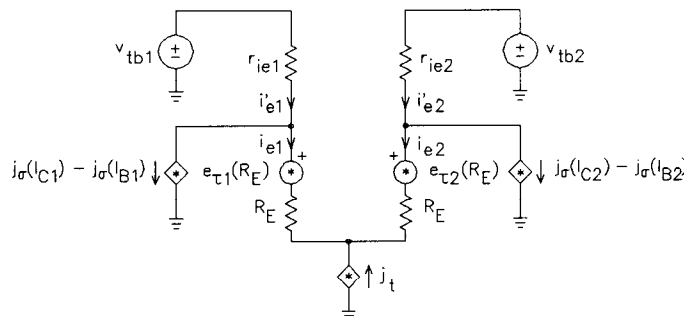

Fig. 6. Circuit of Fig. 5 with noise sources shown.

Fig. 7. Emitter equivalent circuit used to calculate i'_{e1} and i'_{e2}.

the frequency band Δf to obtain

$$
\begin{aligned}
\langle e_{\mathrm{ni}}^2 \rangle = 8kT\Delta f & \left[R_{\mathrm{s}} + R_1 + \frac{R_2 + r_{\mathrm{x}} + R_{\mathrm{E}}}{N^2} \right] \\
& + \frac{2}{N^2} 2q \frac{I_{\mathrm{C}}}{\beta} \Delta f \left[N^2(R_{\mathrm{s}} + R_1) + R_2 + r_{\mathrm{x}} + R_{\mathrm{E}} \right]^2 \\
& + \frac{2}{N^2} 2q I_{\mathrm{C}} \Delta f \left[\frac{V_{\mathrm{T}}}{I_{\mathrm{C}}} \right. \\
& \left. + \frac{N^2(R_{\mathrm{s}} + R_1) + R_2 + r_{\mathrm{x}} + R_{\mathrm{E}}}{\beta} \right]^2 \Bigg]
\end{aligned}
$$

(24)

where $I_{\mathrm{C}} = I_{\mathrm{C}1} = I_{\mathrm{C}2} = \alpha I_{\mathrm{T}}/2$.

Following Eqs. (12) and (13) it follows that the noise output is minimized if the collector current in each BJT is given by

$$
\begin{aligned}
& I_{\mathrm{C(opt)}} \\
& = \frac{\sqrt{\beta}\, V_{\mathrm{T}}}{\sqrt{1 + 1/\beta}\, [N^2(R_{\mathrm{s}} + R_1) + R_2 + r_{\mathrm{x}} + R_{\mathrm{E}}]} .
\end{aligned}
$$

(25)

The corresponding value for $\langle e_{\mathrm{ni}}^2 \rangle$ is

$$
\begin{aligned}
\langle e_{\mathrm{ni}}^2 \rangle_{\mathrm{min}} = 8kT\Delta f & \left(1 + \frac{1 + \sqrt{1 + \beta}}{\beta} \right) \\
& \times R_{\mathrm{s}} + R_1 + \frac{R_2 + r_{\mathrm{x}} + R_{\mathrm{E}}}{N^2} .
\end{aligned}
$$

(26)

By setting $N = 1$ and $R_1 = R_2 = 0$ in this equation, the minimum mean-squared noise voltage is obtained for the case without the transformer. It follows that the transformer results in less noise only if the following condition is satisfied:

$$
\frac{N^2 R_1 + R_2}{N^2 - 1} < r_{\mathrm{x}} + R_{\mathrm{E}} .
$$

(27)

For an example calculation, consider the transformer with the parameters given in the numerical examples of the preceding section. This transformer results in less noise if $r_{\mathrm{x}} + R_{\mathrm{E}} > 102\ \Omega$. If it is assumed that $r_{\mathrm{x}} = 10\ \Omega$, the transformer would result in more noise for $R_{\mathrm{E}} < 92\ \Omega$. Because $R_{\mathrm{E}} = 0$ gives the least noise, it follows that the transformer results in more noise.

For a second example calculation, the optimum BJT bias current is calculated with and without the example transformer. It is assumed that $R_{\mathrm{s}} = 200\ \Omega$, $r_{\mathrm{x}} = 10$

Ω, $R_{\mathrm{E}} = 0$, $\beta = 150$, and $T = 300$ K. It follows from Eq. (25) that the optimum collector bias current is $I_{\mathrm{C(opt)}} = 7.29\ \mu\mathrm{A}$ with the transformer and $I_{\mathrm{C(opt)}} = 1.51$ mA without the transformer. Thus the optimally biased input stage has a slew rate that is potentially 173 times greater without the transformer. Again, the superiority of the circuit without the transformer is evident.

For a third example calculation, the signal-to-noise ratio of the differential amplifier is calculated. It is assumed that $\Delta f = 20$ kHz, $T = 300$ K, and v_{s} is 55 dB below 1 V rms. The mean-squared noise voltage given by Eq. (26) has the value $\langle e_{\mathrm{ni}}^2 \rangle = 2.17 \times 10^{-13}$ V^2 with the transformer and $\langle e_{\mathrm{ni}}^2 \rangle = 1.51 \times 10^{-13}\ \mathrm{V}^2$ without the transformer. The signal-to-noise ratio is 71.6 dB with the transformer and 73.2 dB without the transformer.

The minimum mean-squared equivalent noise input voltage $\langle e_{\mathrm{ni}}^2 \rangle_{\mathrm{min}}$ that has been derived here for the BJT differential amplifier is strictly valid only if $R_{\mathrm{tf}} = N^2 (R_{\mathrm{s}} + R_1) + R_2$. The noise can be minimized further if $R_{\mathrm{tf}} < N^2(R_{\mathrm{s}} + R_1) + R_2$.

3 SINGLE JUNCTION FIELD-EFFECT TRANSISTOR INPUT STAGE

Fig. 8 shows the midband signal circuit of a JFET common-source input stage with a signal source coupled to its gate input through a transformer. It is assumed that the JFET is biased at the drain current I_{D} and that the bias circuit, which is not shown, is noiseless. The Thévenin equivalent circuit seen looking out of the source consists of a source with an open-circuit voltage $v_{\mathrm{tf}} = bv_0$ in series with the resistance R_{tf}, where v_0 is the preamplifier output voltage and b is the feedback ratio. In the case of no feedback, v_{tf} is set equal to zero.

Fig. 9 is the circuit of Fig. 8 with the noise sources shown. The JFET noise source is shown as an external generator. The thermal noise voltages generated by the resistors in the circuit are denoted by $e_\tau(R_{\mathrm{s}})$, $e_\tau(R_1)$, $e_\tau(R_2)$, and $e_\tau(R_{\mathrm{tf}})$. The thermal noise current generated in the JFET channel is denoted by $j_\tau(3/2g_{\mathrm{m}})$, where g_{m} is the JFET transconductance. The equivalent noise input voltage e_{ni} in series with v_{s} which generates the same noise current in i_{d} as all noise sources combined can be obtained from Eq. (11) for the common-emitter BJT input stage with the following modifications: $e_\tau(r_{\mathrm{x}})$ is replaced by zero, $j_\sigma(I_{\mathrm{B}})$ is replaced by zero, $j_\sigma(I_{\mathrm{C}})$
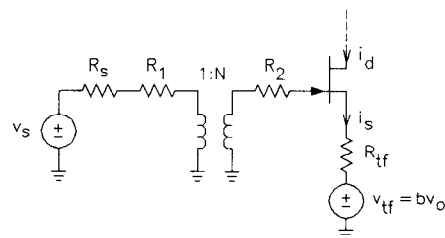

Fig. 8. Midband signal circuit of source coupled to JFET common-source stage by transformer.

is replaced by $j_\tau(3/2g_m)$, V_T/I_C is replaced by $1/g_m$, and $\beta \to \infty$. The result is

$$e_{ni} = e_\tau(R_s) + e_\tau(R_1) + \frac{1}{N}[e_\tau(R_2)$$

$$- e_\tau(R_{tf})] + \frac{1}{N}j_\tau\left(\frac{3}{2g_m}\right)\frac{1}{g_m} . \quad (28)$$

With $\langle j_\tau^2(3/2g_m)\rangle = 4kT\Delta f 2g_m/3$, it follows that the mean-squared value of e_{ni} is given by

$$\langle e_{ni}^2 \rangle = 4kT\Delta f \left[R_s + R_1 + \frac{R_2 + R_{tf}}{N^2}\right]$$

$$+ \frac{1}{N^2}4kT\Delta f \frac{2}{3g_m} . \quad (29)$$

In this expression, the JFET transconductance g_m is given by

$$g_m = 2\sqrt{\beta I_D} \quad (30)$$

where I_D is the dc drain current and β is the transconductance parameter. (The JFET transconductance parameter is not to be confused with the BJT current gain. It is related to the drain-to-source saturation current I_{DSS} and the pinch-off or threshold voltage V_{TO} by $\beta = I_{DSS}/V_{TO}^2$.) It follows that the noise is minimized if the drain current is chosen as large as possible. In the active mode, the JFET drain current must be in the range $0 \leq I_D \leq I_{DSS}$, where I_{DSS} is the drain-to-source saturation current. For a maximum symmetrical current swing, it is common to take $I_D = I_{DSS}/2$. This gives a noise voltage that is within 3 dB of the lowest value, which occurs when $I_D = I_{DSS}$.

By setting $N = 1$ and $R_1 = R_2 = 0$ in Eq. (29), the minimum mean-squared noise voltage is obtained for the case without the transformer. It follows that the transformer results in less noise only if the following condition is satisfied:

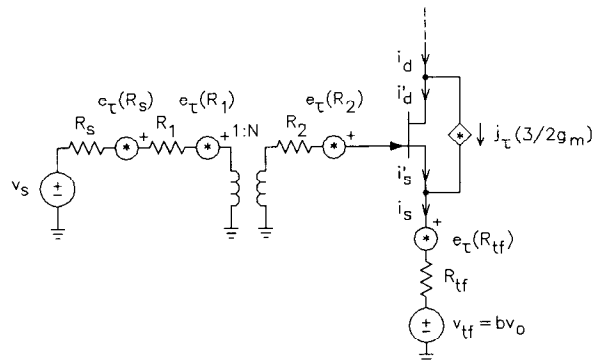$$\frac{N^2R_1 + R_2}{N^2 - 1} < R_{tf} . \quad (31)$$

For an example calculation, consider the source and transformer with the parameters given in the numerical examples of Sec. 1. The JFET parameters are taken to be $\beta = 3.33 \times 10^{-4}$ A/V^2 and $I_{DSS} = 3$ mA. In addition, it is assumed that $T = 300$ K, $\Delta f = 20$ kHz, $R_{tf} = 92$ Ω, and $I_D = 1.5$ mA. The mean-squared noise voltage given by Eq. (29) is $\langle e_{ni}^2 \rangle = 1.01 \times 10^{-13}$ V^2 with the transformer and $\langle e_{ni}^2 \rangle = 2.22 \times 10^{-13}$ V^2 without the transformer. The signal-to-noise ratio is 75.0 dB with the transformer and 71.5 dB without the transformer. In contrast to the BJT examples, the transformer results in less noise for the example common-source JFET input stage.

4 JFET DIFFERENTIAL-AMPLIFIER INPUT STAGE

Fig. 10 shows the midband signal circuit of a JFET differential-amplifier input stage with a signal source coupled to the gate input of Q_1 through a transformer. The dc tail current is labeled I_T. The two source resistors labeled R_S have been included for completeness. However, these resistors are not normally used in low-noise designs because they generate undesired thermal noise. The Thévenin equivalent circuit seen looking out of the gate of Q_2 consists of a source with an open-circuit voltage $v_{tf} = bv_o$ in series with the resistance R_{tf}, where v_o is the preamplifier output voltage and b is the feedback ratio. In the case of no feedback, v_{tf} is set equal to zero.

Fig. 11 is the circuit of Fig. 10 with the noise sources shown. The JFET noise sources are shown as external generators. The noise current of the tail bias network is denoted by j_t. It is assumed that the small-signal resistance seen looking into the tail supply is large compared to the other resistances in the circuit so that it can be considered to be an open circuit. This is equivalent to the assumption that the differential amplifier has a high common-mode rejection ratio. The thermal noise voltages generated by the resistors in the circuit are denoted by $e_\tau(R_s)$, $e_\tau(R_1)$, $e_\tau(R_2)$, $e_{\tau 1}(R_S)$, $e_{\tau 2}(R_S)$, and $e_\tau(R_{tf})$. The thermal noise currents generated in the JFET channels are denoted by $j_\tau(3/2g_{m1})$ and $j_\tau(3/2g_{m2})$.

For minimum noise output from a differential-amplifier input stage, the output signal should be a differential signal, that is, the output voltage should be

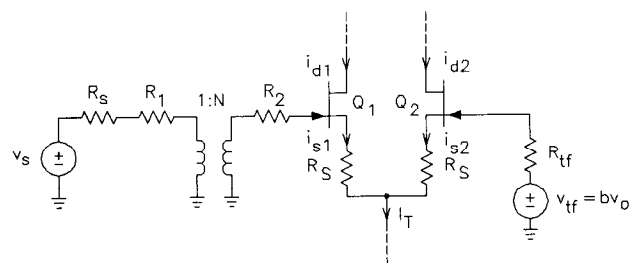Fig. 9. Circuit of Fig. 8 with noise sources shown.

Fig. 10. Midband signal circuit of source coupled to JFET differential-amplifier stage by transformer.

proportional to the difference current $i_{d1} - i_{d2}$. In addition, the noise output caused by the tail noise current j_t is not canceled unless the small-signal resistances seen looking into the sources of the two JFETs are equal. The small-signal source input resistance is given by $r_{is} = 1/g_m$, where g_m is the transconductance given by Eq. (30). Because this is independent of the Thévenin resistance seen looking out of the gate, it is not necessary to assume that $N^2(R_s + R_1) + R_2 = R_{tf}$ as for the BJT differential amplifier. In the following it will be assumed that $I_{D1} = I_{D2} = I_T/2$ and $g_{m1} = g_{m2}$.

The equivalent noise input voltage e_{ni} in series with v_s which generates the same noise current in $i_{d1} - i_{d2}$ as all noise sources combined can be obtained from Eq. (23) for the BJT differential-amplifier input stage with the following modifications: $e_\tau(r_{x1})$ and $e_\tau(r_{x2})$ are replaced by zero, $j_\sigma(I_{B1})$ and $j_\sigma(I_{B2})$ are replaced by zero, $j_\sigma(I_{C1})$ and $j_\sigma(I_{C2})$ are replaced by $j_\tau(3/2g_{m1})$ and $j_\tau(3/2g_{m2})$, α is replaced by 1, R_E is replaced by R_S, and r_{ie} is replaced by $1/g_m$. The result is

$$e_{ni} = e_\tau(R_s) + e_\tau(R_1) + \frac{1}{N} [e_\tau(R_2)$$
$$- e_{\tau1}(R_S) + e_{\tau2}(R_S) - e_\tau(R_{tf})]$$
$$+ \frac{1}{N} \left[j_\tau \left(\frac{3}{2g_{m1}} \right) - j_\tau \left(\frac{3}{2g_{m2}} \right) \right] \frac{1}{g_m} . \quad (32)$$

This can be converted to a mean-squared voltage in the frequency band Δf to obtain

$$\langle e_{ni}^2 \rangle = 4kT\Delta f \left[R_s + R_1 + \frac{R_2 + 2R_S + R_{tf}}{N^2} \right]$$
$$+ \frac{2}{N^2} 4kT\Delta f \frac{2}{3g_m} . \quad (33)$$

If $R_S = 0$ and $R_{tf} = N^2(R_s + R_1) + R_2$, this equation predicts 3 dB more noise than Eq. (29) for the single JFET.

By setting $N = 1$ and $R_1 = R_2 = 0$ in Eq. (33), the

minimum mean-squared noise voltage is obtained for the case without the transformer. It follows that the transformer results in less noise only if the following condition is satisfied:

$$\frac{N^2 R_1 + R_2}{N^2 - 1} < 2R_S + R_{tf} . \quad (34)$$

For an example calculation, let the JFET common-source amplifier in the example of the preceding section be replaced by a JFET differential amplifier. Let each JFET in the differential amplifier have the same parameters and the same drain current as in that example. If $R_S = 0$, it follows that the mean-squared noise voltage given by Eq. (33) is $\langle e_{ni}^2 \rangle = 2.02 \times 10^{-13}$ V^2 with the transformer and $\langle e_{ni}^2 \rangle = 4.44 \times 10^{-13}$ V^2 without the transformer. The signal-to-noise ratio is 72.0 dB with the transformer and 68.5 dB without the transformer.

5 CONCLUSIONS

The noise equivalent input voltages have been calculated for the input stage of a transformer-coupled feedback preamplifier for a BJT common-emitter input stage, a BJT differential-amplifier input stage, a JFET common-source input stage, and a JFET differential-amplifier input stage. Conditions for less noise with a transformer having a specified turns ratio have been derived. In the case of a BJT input stage, there is an optimum bias current which minimizes the noise for a given transformer turns ratio. It is shown by example that the optimally biased BJT input stage can have less noise when a transformer is not used. In the case of a JFET input stage, the examples predict better noise performance with the transformer, but the improvement is not large.

The analysis has neglected excess noise or flicker noise in the BJT and JFET devices. Although this is normally an acceptable approximation for audio band calculations for low-noise BJTs, the JFET can exhibit larger amounts of flicker noice which cannot be neglected. For low-noise circuits the JFET should be chosen which has the lowest possible flicker noise coefficient.
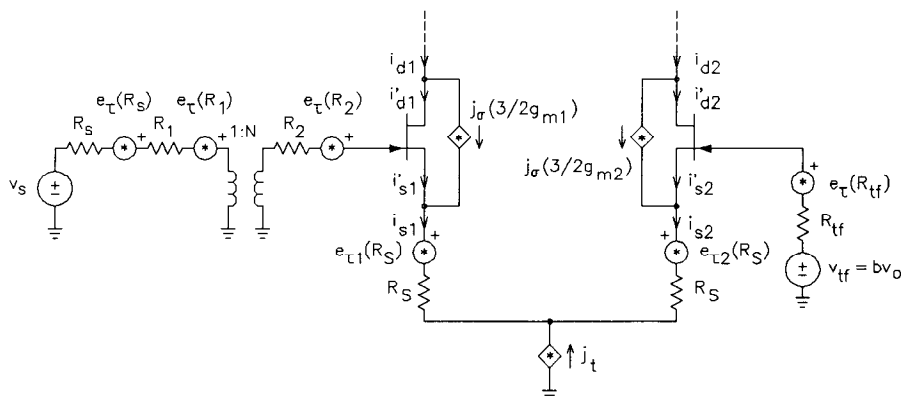

Fig. 11. Circuit of Fig. 10 with noise sources shown.

6 GLOSSARY OF SYMBOLS

α = BJT emitter-to-collector current gain

b = feedback ratio or feedback factor

β = BJT base-to-collector current gain, JFET transconductance parameter

Δf = noise bandwidth, Hz

e = noise voltage or noise voltage source

e_{ni} = equivalent noise voltage in series with amplifier input signal source which generates the same noise at the amplifier output as all other noise sources in the circuit combined

$e_\tau(R)$ = thermal noise voltage generated by resistor R, $\langle e_\tau^2(R) \rangle = 4kT\Delta fR$

g_m = transconductance

i = small-signal ac current or ac current source

I = dc current or dc current source

I_{DSS} = JFET drain-to-source saturation current

j = noise current or noise current source

$j_\sigma(I)$ = shot noise current associated with dc current I; $\langle j_\sigma^2(I) \rangle = 2qI\Delta f$

$j_\tau(R)$ = thermal noise current generated by resistor R; $\langle j_\tau^2(R) \rangle = 4kT\Delta f/R$

k = Boltzmann's constant; $= 1.38 \times 10^{-23}$ J/K

N = transformer turns ratio

q = electronic charge; $= 1.60 \times 10^{-19}$ C

r_{ie} = small-signal resistance seen looking into BJT emitter

r_{is} = small-signal resistance seen looking into JFET source

R_{tb} = Thévenin resistance seen looking out of BJT base

R_{te} = Thévenin resistance seen looking out of BJT emitter

r_x = small-signal BJT base spreading resistance

T = absolute temperature or centigrade temperature

v = small-signal ac voltage or ac voltage source

V_T = thermal voltage; $= kT/q = 0.0259$ V at $T = 27°C$

v_{tb} = Thévenin voltage seen looking out of BJT base

v_{te} = Thévenin voltage seen looking out of BJT emitter

V_{TO} = JFET threshold or pinch-off voltage

$\langle x \rangle$ = mean or average value of variable x

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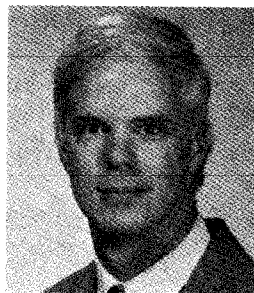
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