MOSFET Transistors and Basic Circuits

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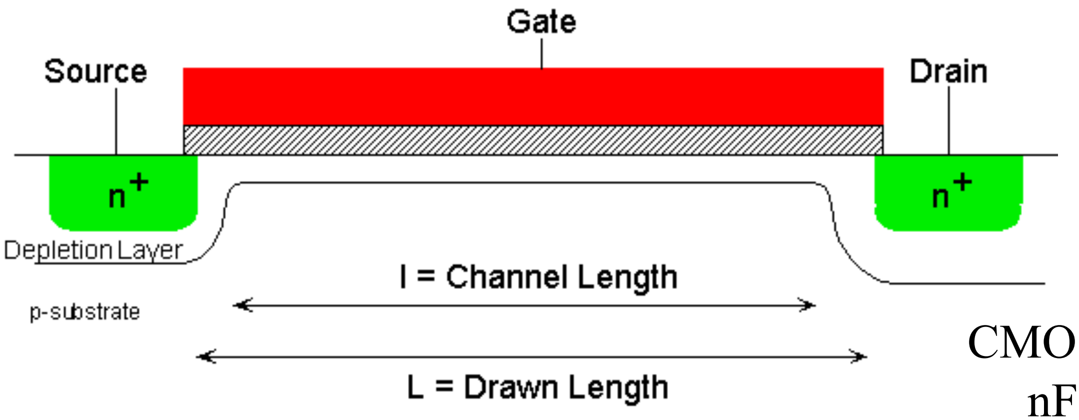
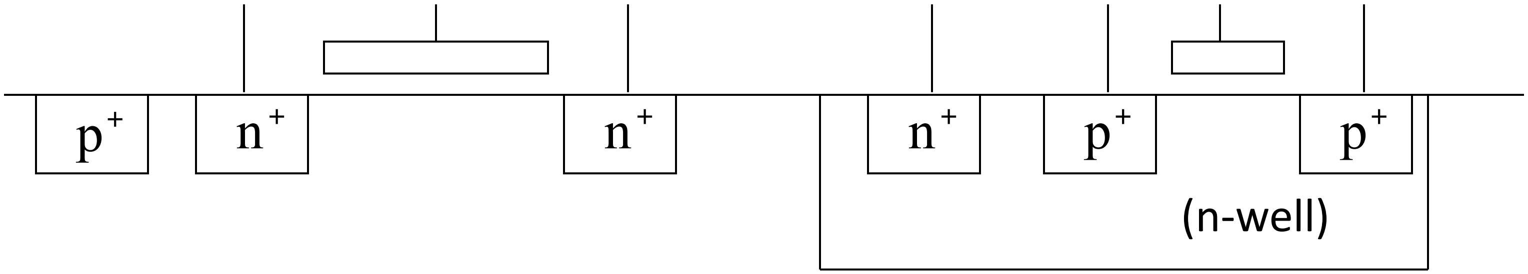
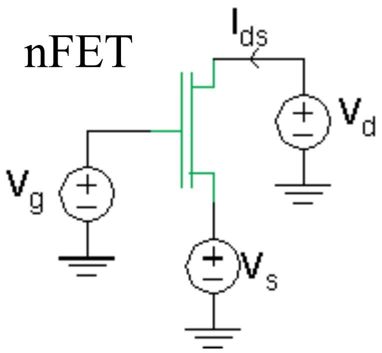
CMOS Process Cross Section

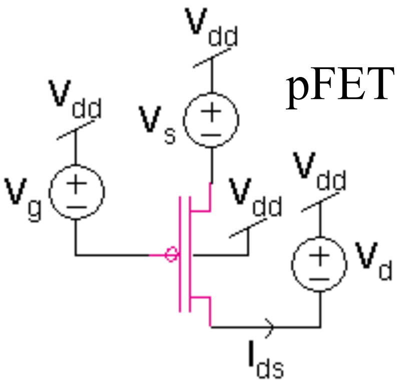


all p-n junction must
be reversed bias

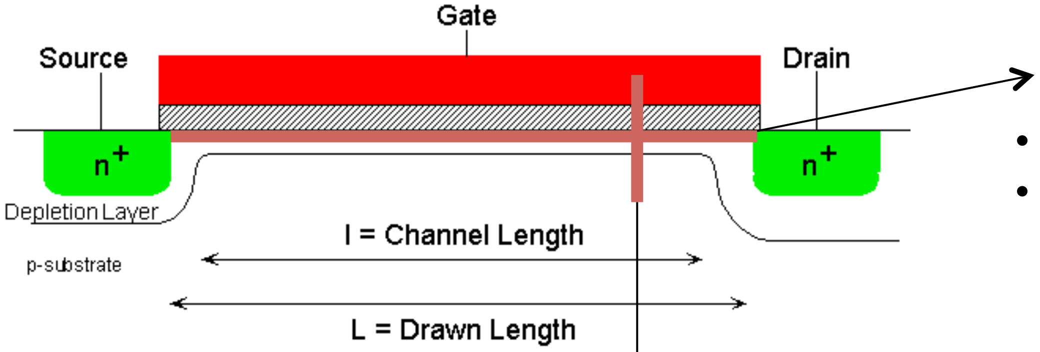
CMOS Process =
nFETs and pFETs are available



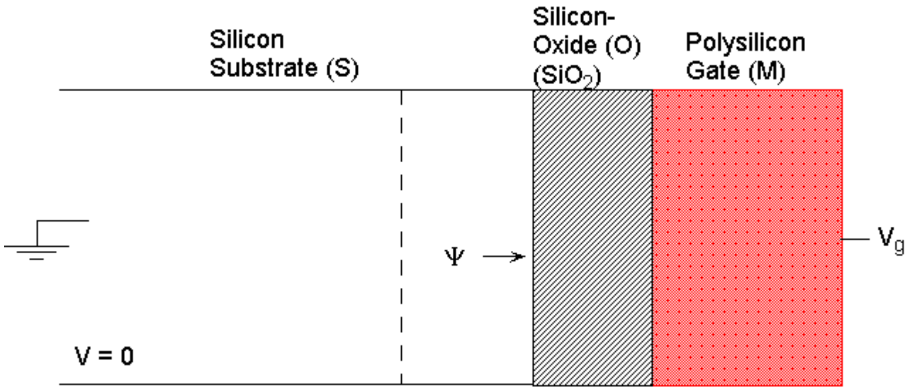


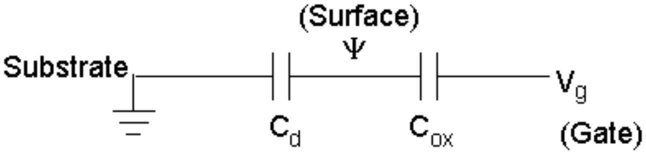


MOSFET Device Physics



MOS Capacitor Picture



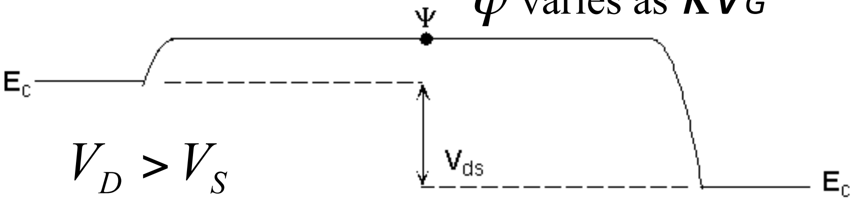


$$\kappa = \frac{\Psi}{V_g} = \frac{C_{ox}}{C_{ox} + C_d}$$

MOS Channel Behavior

- Sub-VT: $I < I_{th}$ → Channel Potential (Ψ) is flat
- Sub-VT operation simplifies this 2D problem to 2, 1D problems

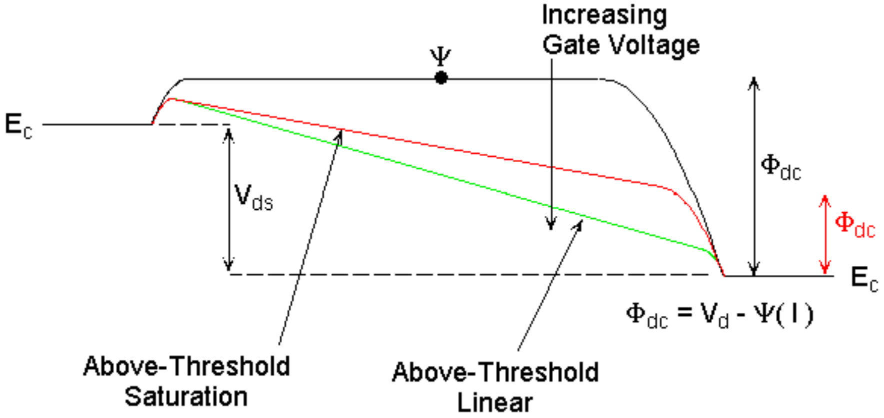
ψ varies as κV_G



$V_D > V_S$

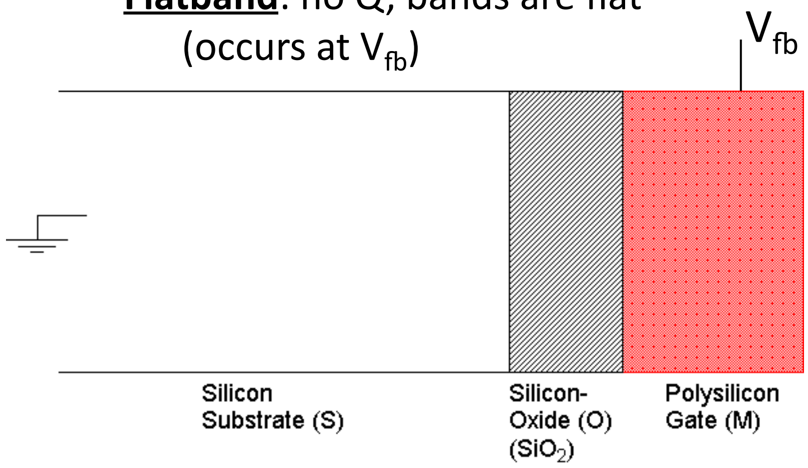
- subthreshold operation = fundamental case

- Above VT: $I > I_{th}$, $\Psi(x)$ in channel, $\Psi(x)$ set by current level, terminal voltages



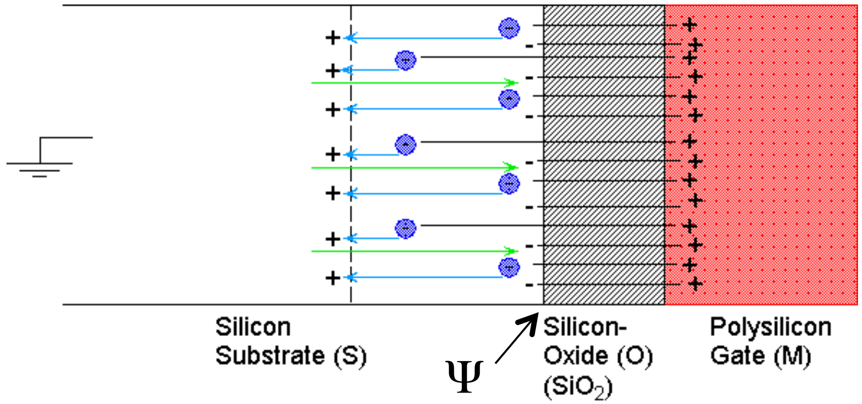
MOS Capacitor Behavior

Flatband: no Q, bands are flat
(occurs at V_{fb})

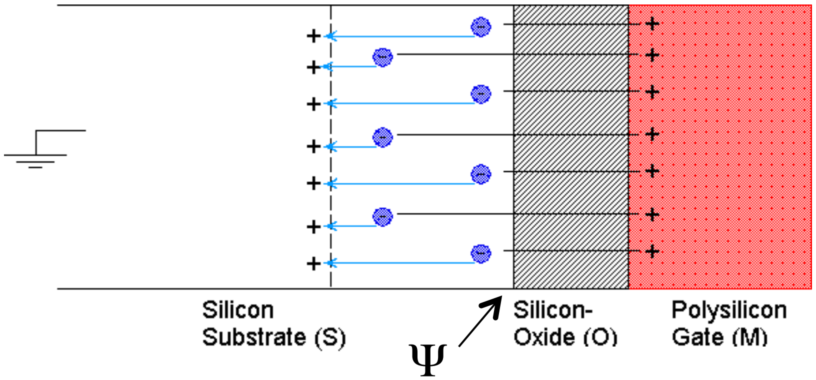


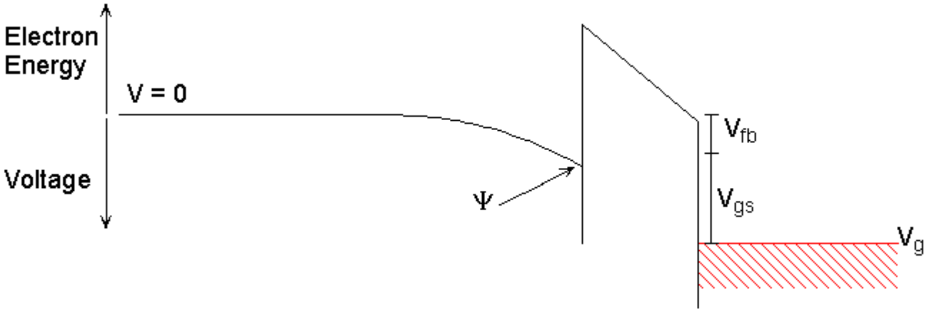
Increasing Gate Voltage:
Flatband (V_{fb}) → Depletion → Inversion

Inversion: further gate charge is terminated by carriers at the silicon--silicon-dioxide interface

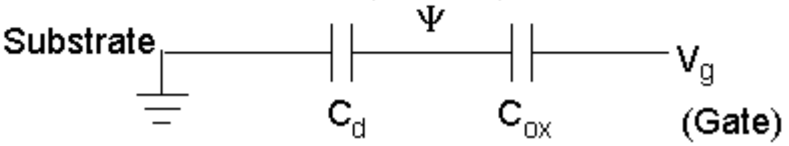


Depletion: gate charge is terminated by charged ions in the depletion region



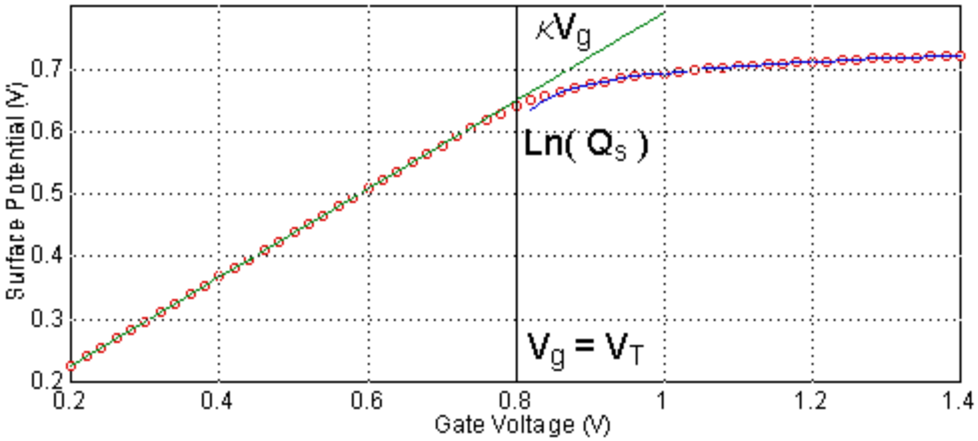


$$\Delta\Psi = \kappa\, \Delta V_g$$



Free Q parameter set by V_{fb}

MOSFET Operating Regions



$$Q_s = e^{(\Psi - V_s)/U_T}$$

Solution: transcendental equation

(Simultaneous solution of Drift-Diffusion Equation)

Depletion ($\kappa(V_g - V_{T0}) - V_s < 0$)

$$Q_s = e^{(\kappa(V_g - V_{T0}) - V_s)/U_T}$$

Inversion ($\kappa(V_g - V_T) - V_s > 0$)

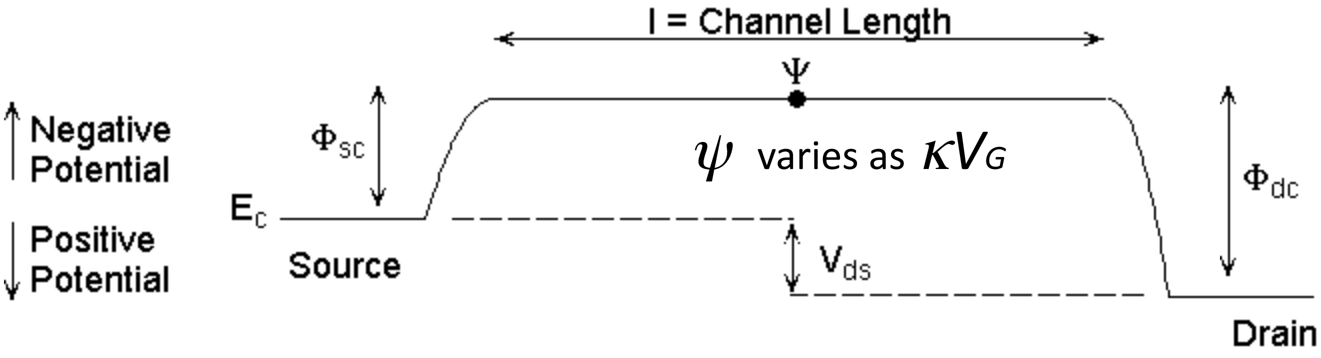
$$Q_s = (\kappa(V_g - V_T) - V_s)/U_T$$

$$Q_s = \ln(1 + e^{(\kappa(V_g - V_{T0}) - V_s)/U_T})$$

(EKV modeling)

	Below Threshold	Above Threshold
Field Lines from gate charges	End on mobile charges in channel	End on mobile charges in channel
Charge boundary condition at source	Set by Fermi Distribution	$C_{ox}(\kappa(V_g-V_T)-V_s)$
Approximate surface potential	κV_g	$\ln(Q_s)$
Channel current flows	Diffusion	Drift

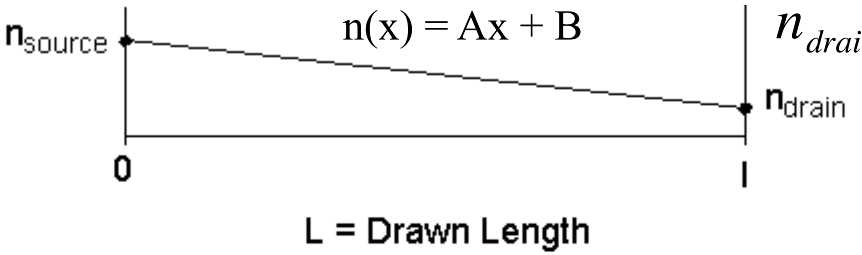
Sub V_T Drain Current Derivation



ψ varies as κV_G

$\Phi_{sc} = V_s - \Psi$ $\Phi_{dc} = V_d - \Psi$

$n_{source} \propto e^{-\phi_{sc}/U_T}$



n(x) = Ax + B

$n_{drain} \propto e^{-\phi_{dc}/U_T}$

Channel Current is constant ⟶ Diffusion: $J_n = q D_n \frac{dn}{dx} = q D_n \frac{n_{source} - n_{drain}}{l}$

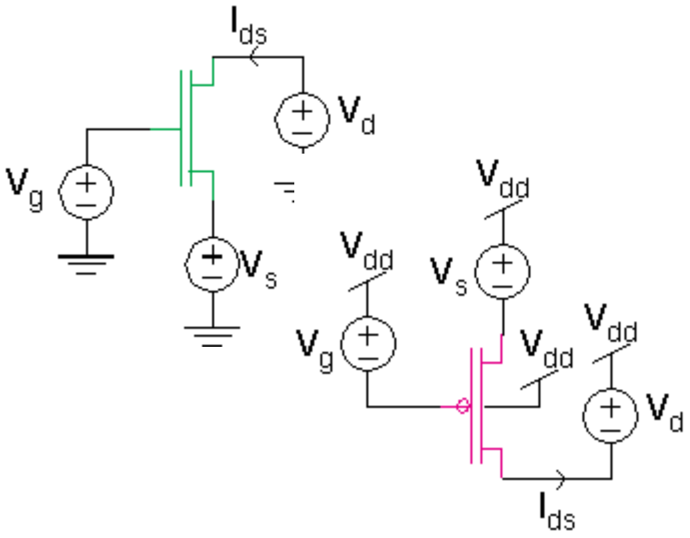
No recombination in channel

$$\cancelto{0}{\frac{dn}{dt}} = D_n \frac{d\Delta n^2}{dx^2} + \cancelto{0}{G} - \cancelto{0}{R}$$

$$I_s = I_{th} \left(e^{(\kappa(V_g - V_{T0}) - V_s)/U_T} - e^{(\kappa(V_g - V_{T0}) - V_d)/U_T} \right)$$

$\longrightarrow$ n(x) = Ax + B

MOSFET Current-V Expressions



$$I_s = I_{th} \left(e^{(\kappa(V_g - V_{T0}) - V_s)/U_T} - e^{(\kappa(V_g - V_{T0}) - V_d)/U_T} \right)$$

$$I_s = I_{th}\ e^{(\kappa(V_g - V_{T0}) - V_s)/U_T} \left(1 - e^{-V_{ds}/U_T}\right)$$

$$I_s = I_{th}\ e^{(\kappa(V_g - V_{T0}) - V_s)/U_T} \qquad (V_{ds} > 4U_T) \quad \text{“Saturation”}$$

$$\boxed{I_0 = I_{th}\ e^{-\kappa V_{T0}/U_T}}$$

Sometimes written

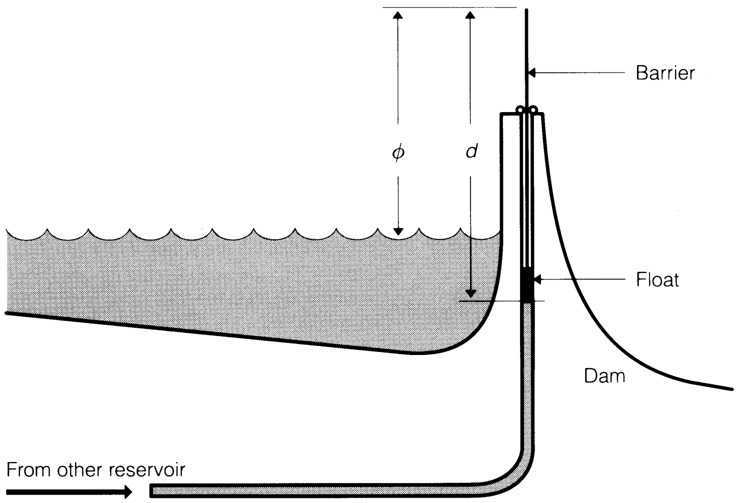
$$I_{ds} = I_o\ e^{\kappa V_g/U_T} \left(e^{-V_s/U_T} - e^{-V_d/U_T} \right)$$

$$= I_o\ e^{(\kappa V_g - V_s)/U_T} \left(1 - e^{-V_{ds}/U_T} \right)$$

$$= I_o\ e^{(\kappa V_g - V_s)/U_T} \qquad (V_{ds} > 4\ U_T)$$

"Saturation"

Water Analogy of a MOSFET

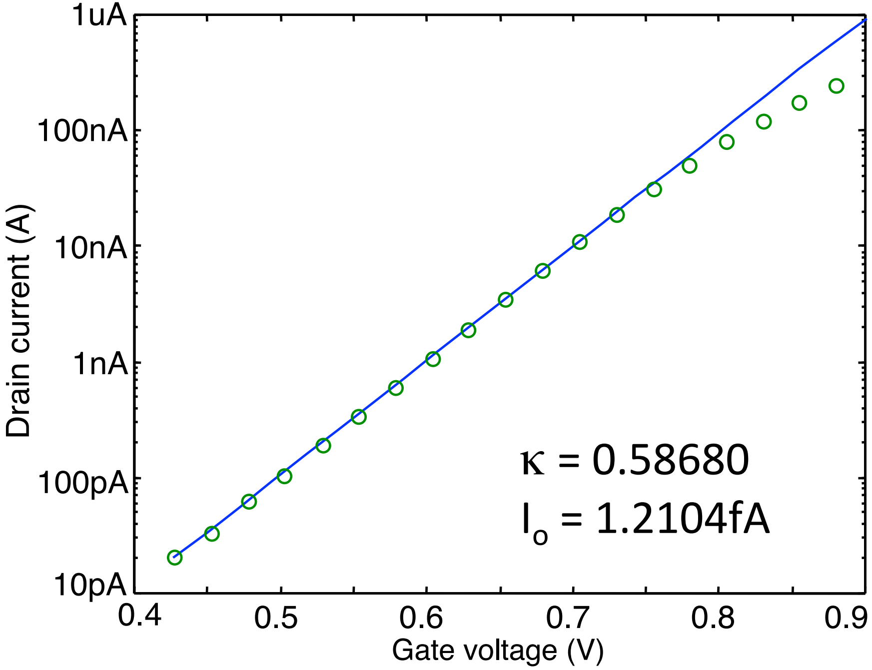


In saturation, including Early effect $(\sigma = V_A/U_T)$

$$I_s = I_{th}\ e^{(\kappa(V_g - V_{T0}) - Vs + \sigma V_s)/U_T} \qquad (V_{ds} > 4U_T)$$

MOSFET SubV_T I Measurements

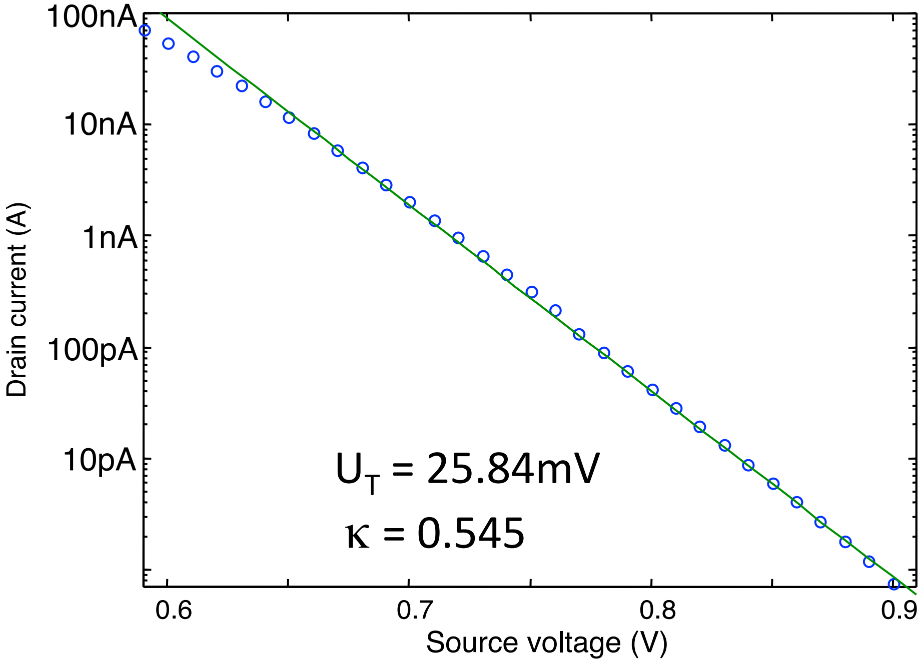
I vs Gate Voltage



$\kappa = 0.58680$

$I_o = 1.2104\text{fA}$

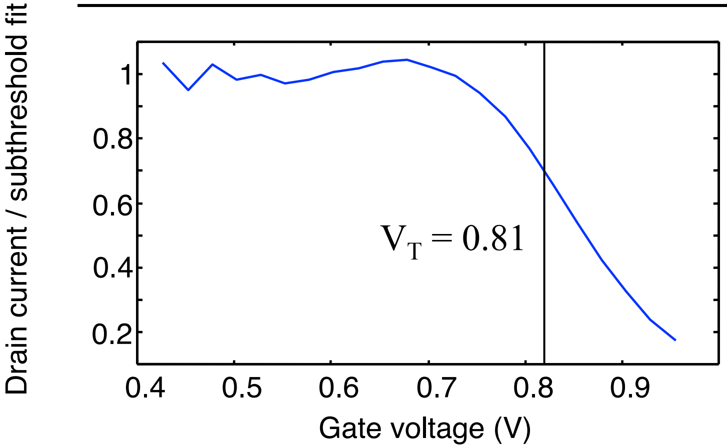
I vs Source Voltage



$U_T = 25.84\text{mV}$

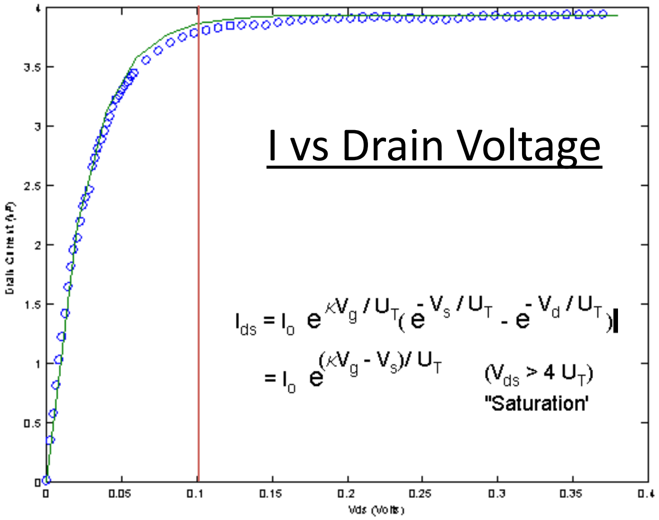
$\kappa = 0.545$

Determination of Threshold Voltage



$V_T = 0.81$

I vs Drain Voltage

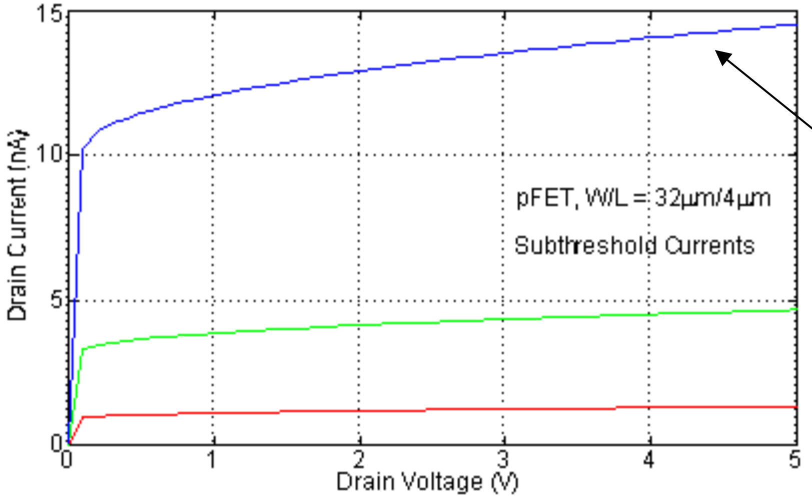


$$I_{ds} = I_o\ e^{\kappa V_g / U_T}(e^{-V_s / U_T} - e^{-V_d / U_T})$$

$$= I_o\ e^{(\kappa V_g - V_s)/ U_T} \quad (V_{ds} > 4\ U_T)$$

"Saturation'

Current versus Drain Voltage



$$I_{ds} = I_o\, e^{\kappa V_g / U_T}(e^{-V_s / U_T} - e^{-V_d / U_T})$$

$$= I_o\, e^{(\kappa V_g - V_s)/ U_T} \qquad (V_{ds} > 4\, U_T)$$

"Saturation'

Why is this not flat?

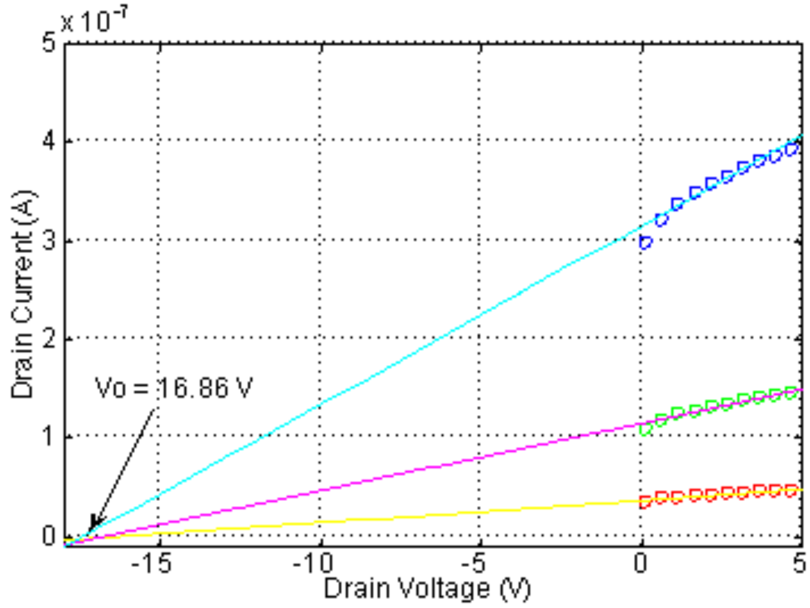
Effect is called the Early Effect

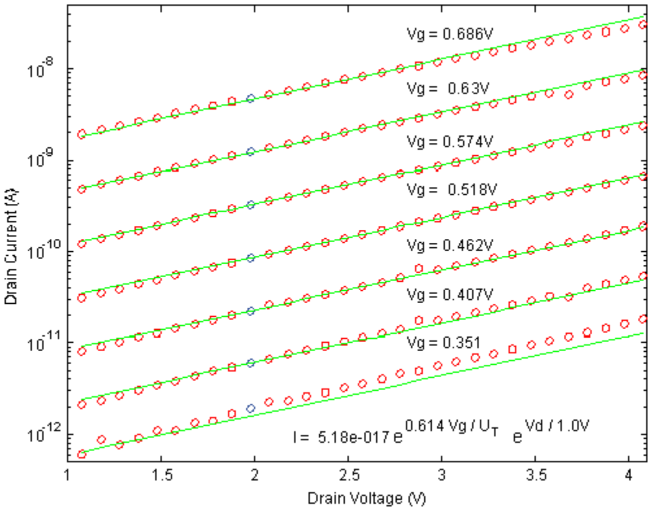
- first found in BJT devices (Jim Early)
- limits transistor gain

$$I_d = I_d(\mathrm{sat})\,(1 + (V_d/V_A))$$

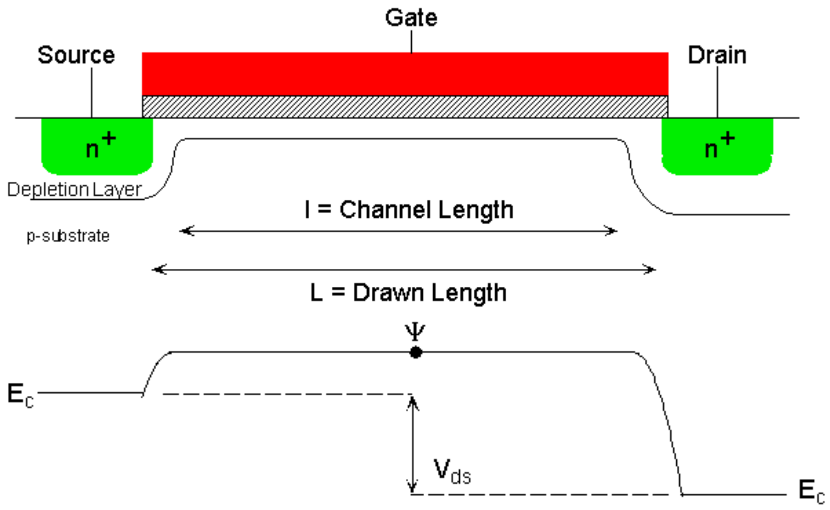
$$V_A = U_T / \sigma = \text{Early voltage} = 1/\lambda$$

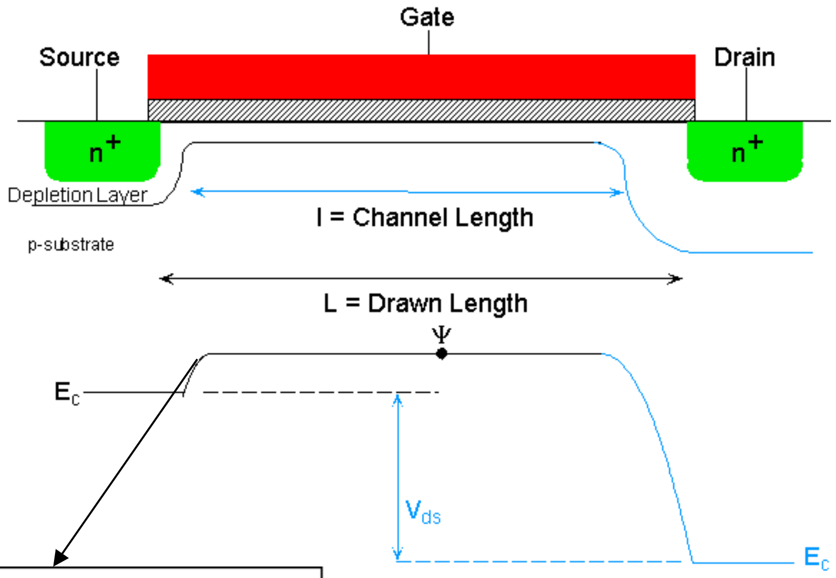
$$\boxed{I_d = I_d(\mathrm{sat})\, e^{\sigma V_d/U_T}}$$





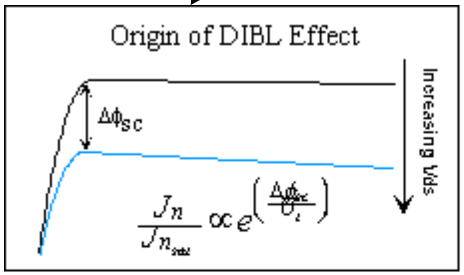
Origin of Drain Current Dependencies

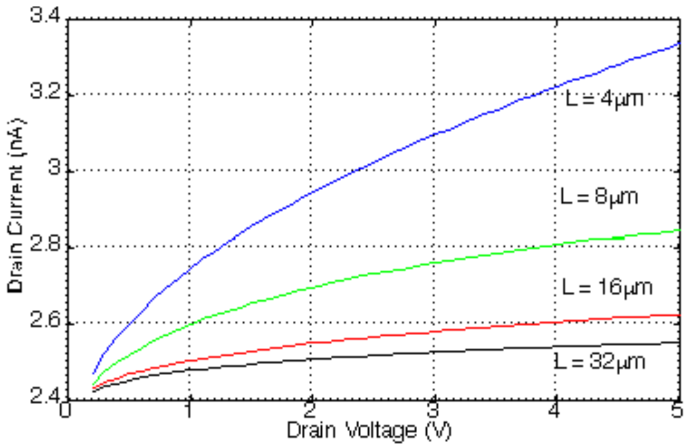




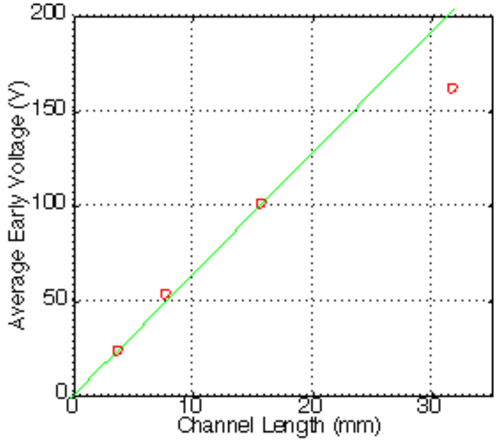
Increasing Vd effects the drain-to-channel region:

- increases barrier height
- increases depletion width





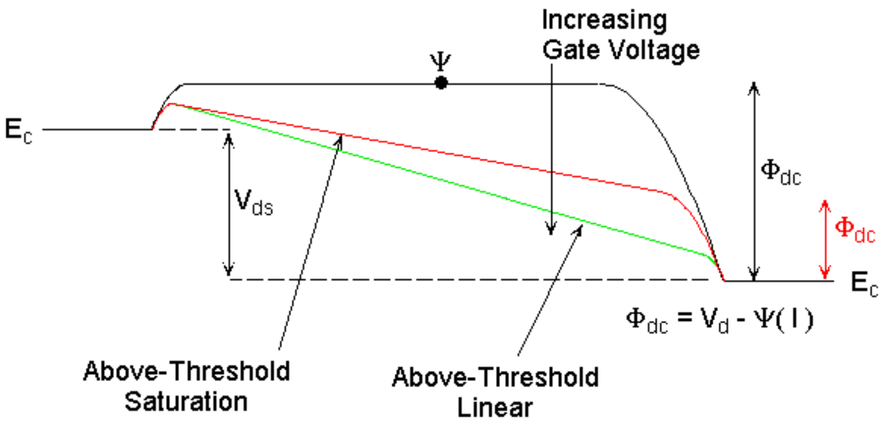
Width of depletion region depends on doping, not L



V_A varies linearly with l

Above-Threshold Derivation

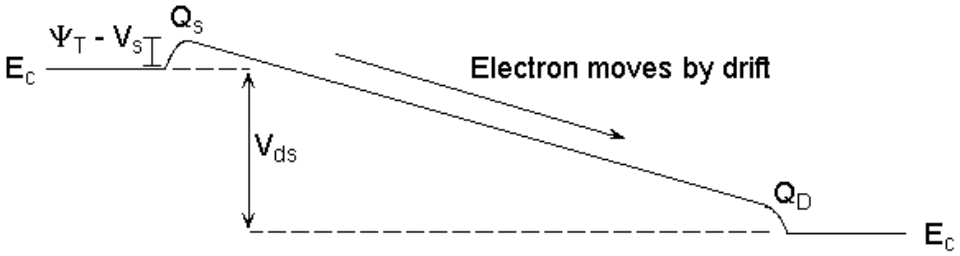
Band-Diagram MOSFET Picture



Conduction band bends due to electrostatic force of the electrons moving through the channel

Band-diagram
picture moving
from subthreshold to
above-threshold

Intuitive Above-Threshold Derivation



Drift Current: $I = \mu\ Q\ E$

$$Q = \frac{Q_s + Q_D}{2} \qquad E = \frac{Q_s - Q_D}{\varepsilon_{si}}$$

Current is proportional to $(Q_s + Q_D)(Q_s - Q_D) =$

$$Q_s^2 - Q_D^2$$

$$Q_s = C_T\ (\ \kappa(V_g - V_T) - V_s), \qquad Q_d = C_T\ (\ \kappa(V_g - V_T) - V_d)$$

$$I = (K/2\kappa)\ ((\ \kappa(V_g - V_T) - V_s\ )^2 - ((\ \kappa(V_g - V_T) - V_d\ )^2\ )$$

Above-Threshold Derivation

Current moves by Drift

$$I = \mu_n Q(x) E(x) = \mu_n Q(x) \frac{dV(x)}{dx}$$

$$Q(x) = C_T (\kappa(V_g - V_T) - \Psi(x))$$

$$C_T = C_D + C_{ox}$$

$$- \frac{d V(x)}{dx} = (1 / C_T) \frac{d Q(x)}{dx}$$

We know Q at Source and Drain edges of the channel

$$Q_s = C_T (\kappa(V_g - V_T) - V_s), \qquad Q_d = C_T (\kappa(V_g - V_T) - V_d)$$

$$(\kappa = C_{ox} / C_T)$$

$$I = (\mu / C_T) Q(x) \frac{d Q(x)}{dx}$$

(Current is constant;
no Q loss in channel)

$$\int_0^l I dx = \mu \int_{Qs}^{Qd} Q(x) dQ(x)$$

$$I = (\mu / 2 C_T) (1 / L) (Q_s^2 - Q_d^2)$$

$$I = \frac{K}{2\kappa}\left(\underbrace{\left(\kappa(V_g - V_{T0}) - V_S\right)^2}_{\propto Q_S^2} - \underbrace{\left(\kappa(V_g - V_{T0}) - V_d\right)^2}_{\propto Q_D^2} \right)$$

$$K = \mu C_{ox} (W/L)$$

Saturation: Above Threshold

Q_D is positive, but by this simple model, could become negative.

Where this model breaks down defines the saturation region for above threshold bias currents.

When $Q_D = 0$, the MOSCAP at the drain at the boundary of depletion and inversion. Further increases in drain voltage push this MOSCAP at the drain into depletion.

⇨ for sufficiently large V_d , $Q_D = 0$

$$I = \frac{K}{2\kappa}\left(\kappa(V_g - V_T) - V_S\right)^2$$

$$V_d = \kappa\left(V_g - V_T\right)$$

Above Threshold MOSFET Equations

$$I = (K/2\kappa)\ (\ (\kappa(V_g - V_T) - V_s)^2 - (\kappa(V_g - V_T) - V_d)^2\ )$$

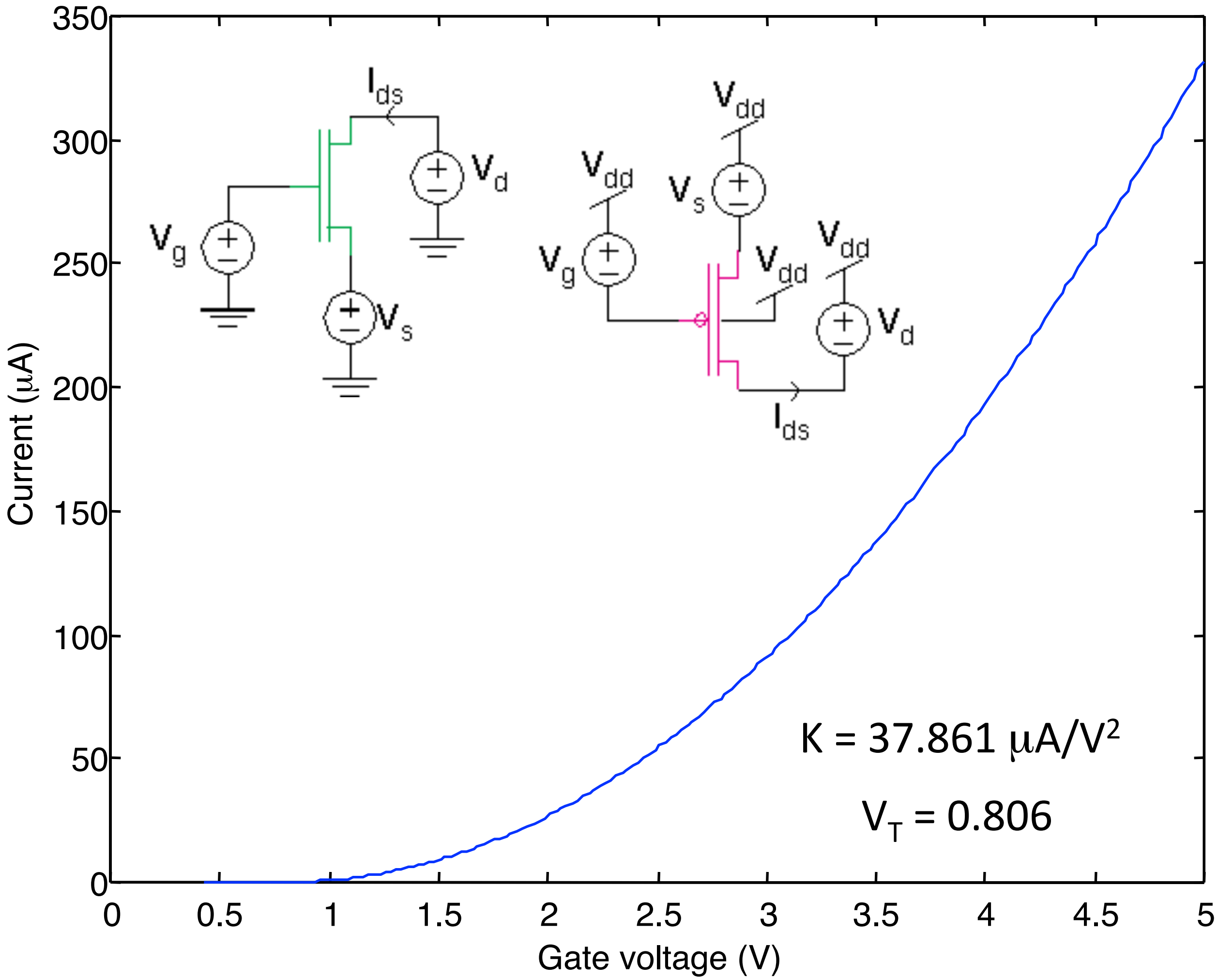
Saturation: $Q_d = 0$

$$V_d > \kappa\ (\ V_g - V_T\ )$$

$$I = (K/2\kappa)\ (\ (\kappa(V_g - V_T) - V_s)^2$$

If $\kappa = 1$ (ignoring back-gate):

$$I = (K/2)\ (\ 2(V_{gs} - V_T)\ V_{ds} - V_{ds}^2\ )$$

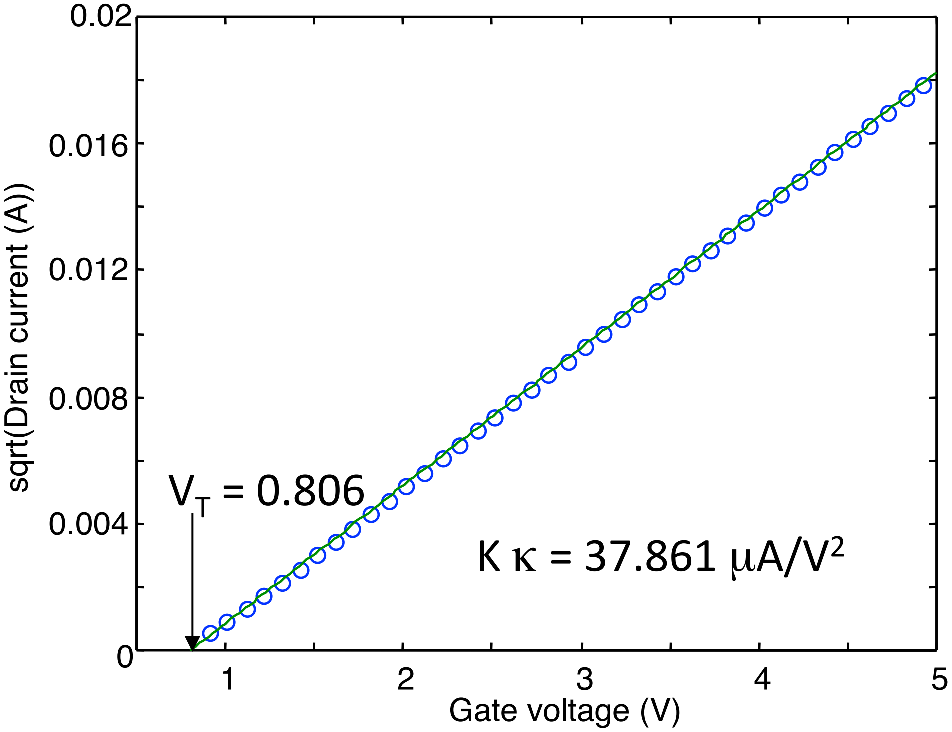


K = 37.861 μA/V^2

$V_T = 0.806$

MOSFET Above V_T I Measurements

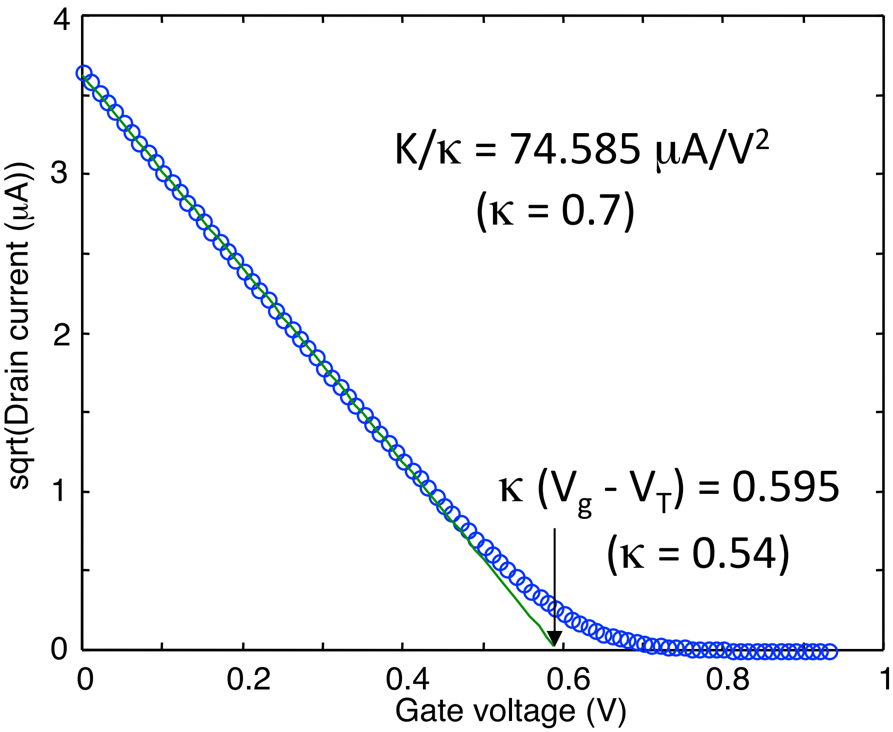
Current vs. Gate Voltage



$V_T = 0.806$

$K\,\kappa = 37.861\ \mu A/V^2$

Current vs. Source Voltage



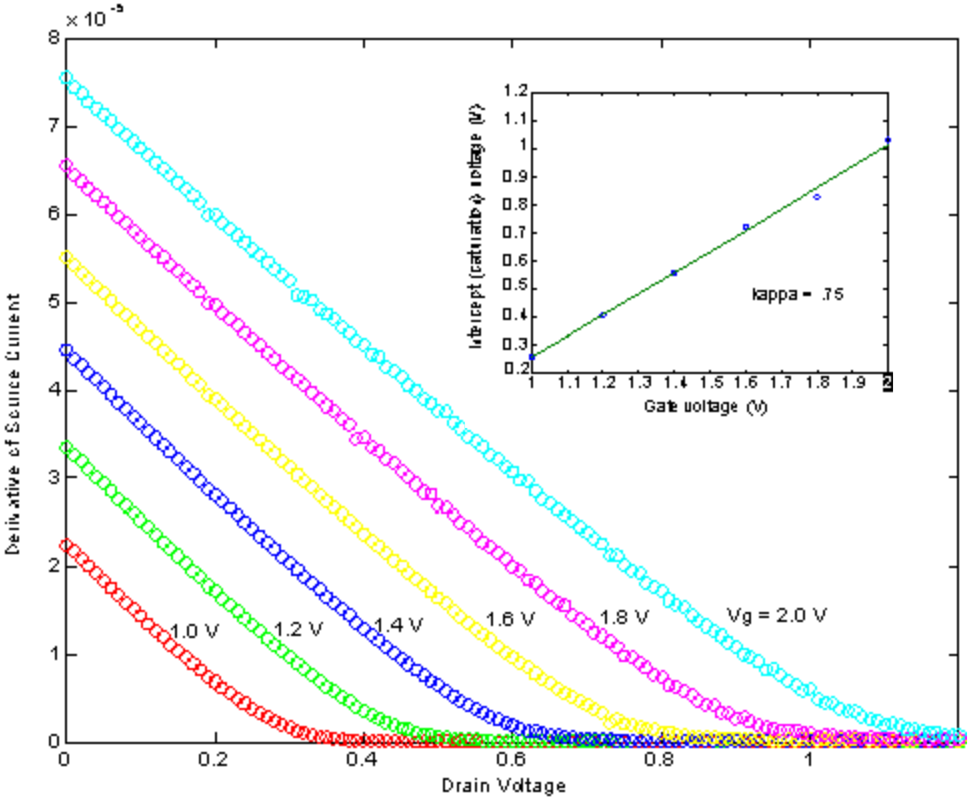
$K/\kappa = 74.585\ \mu A/V^2$

$(\kappa = 0.7)$

$\kappa\,(V_g - V_T) = 0.595$

$(\kappa = 0.54)$

More Ohmic Region Data



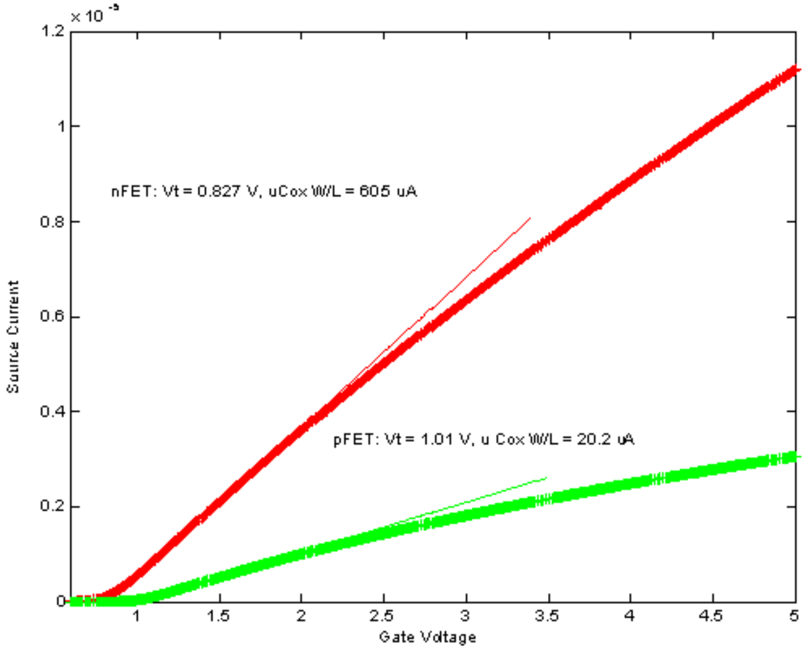
$$I = (K/2\kappa)\ (\ (\kappa(V_g - V_T) - V_s)^2 - (\kappa(V_g - V_T) - V_d)^2\ )$$

Take the derivative of I with respect to V_d ($V_s = 0$)

$$dI / d V_d = (K/2\kappa)(\ 0 - (-2)\ (\kappa(V_g - V_T) - V_d)\ )$$
$$= (K/2\kappa)(\kappa(V_g - V_T) - V_d)$$

Above Threshold MOSFET Data

An Ohmic Device



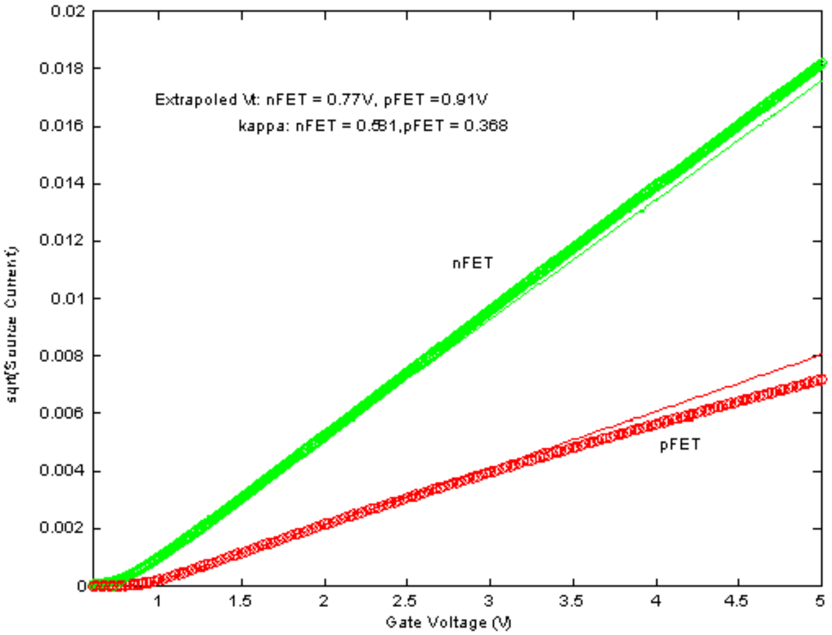
(V_d - V_s = 50mV)

$$I = (K/2\kappa)\ (\ (\kappa(V_g - V_{T0}) - V_s)^2$$
$$-\ (\kappa(V_g - V_{T0}) - V_d)^2\ )$$

If $V_d \sim V_s$, (small difference)

$$I = K\ (V_g - V_{T0})(V_d - V_s)$$

A Saturated Device



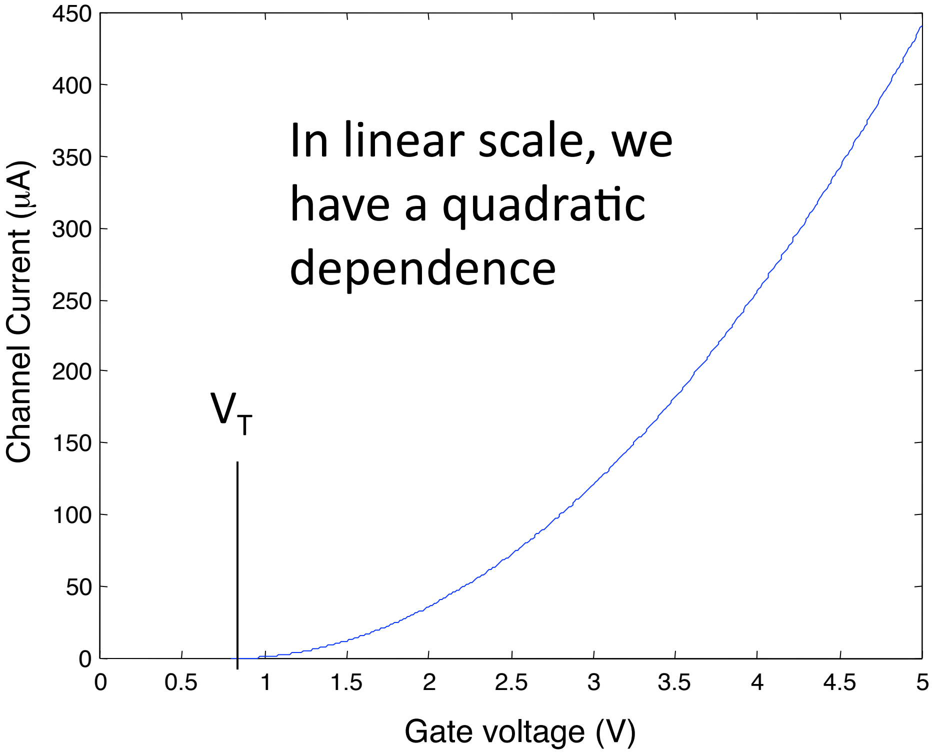
Saturation: $Q_d = 0$

$$I = (K/2\kappa)\ (\ (\kappa(V_g - V_T) - V_s)^2$$

$V_s = 0$ $\qquad$ $\underline{I = (K\kappa/2)\ (V_g - V_T)^2}$

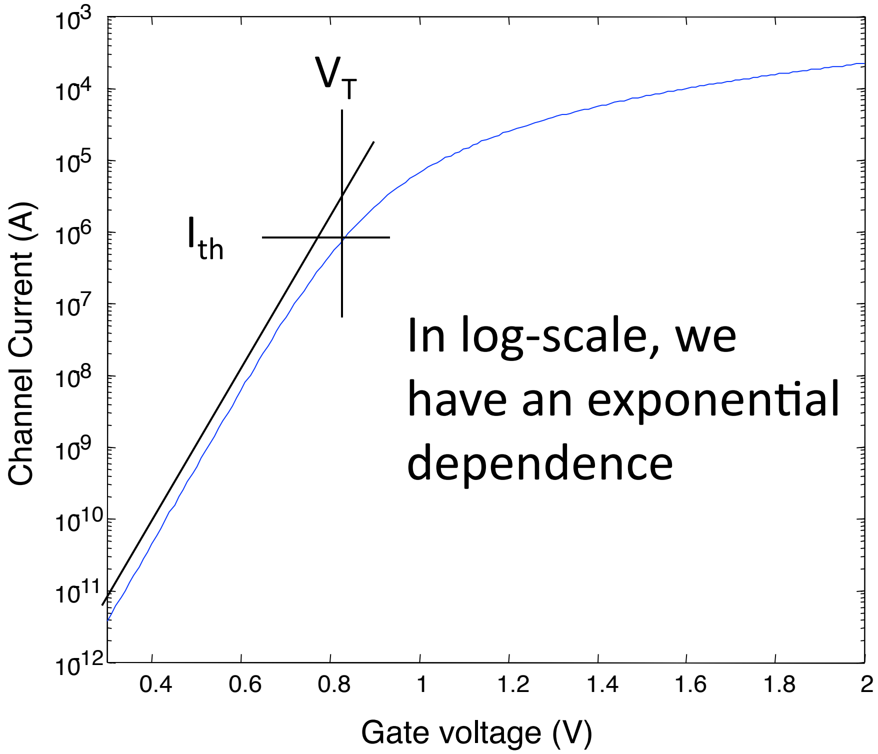
Channel Current vs. Gate Voltage

Above-Threshold



In linear scale, we have a quadratic dependence

Sub-Threshold



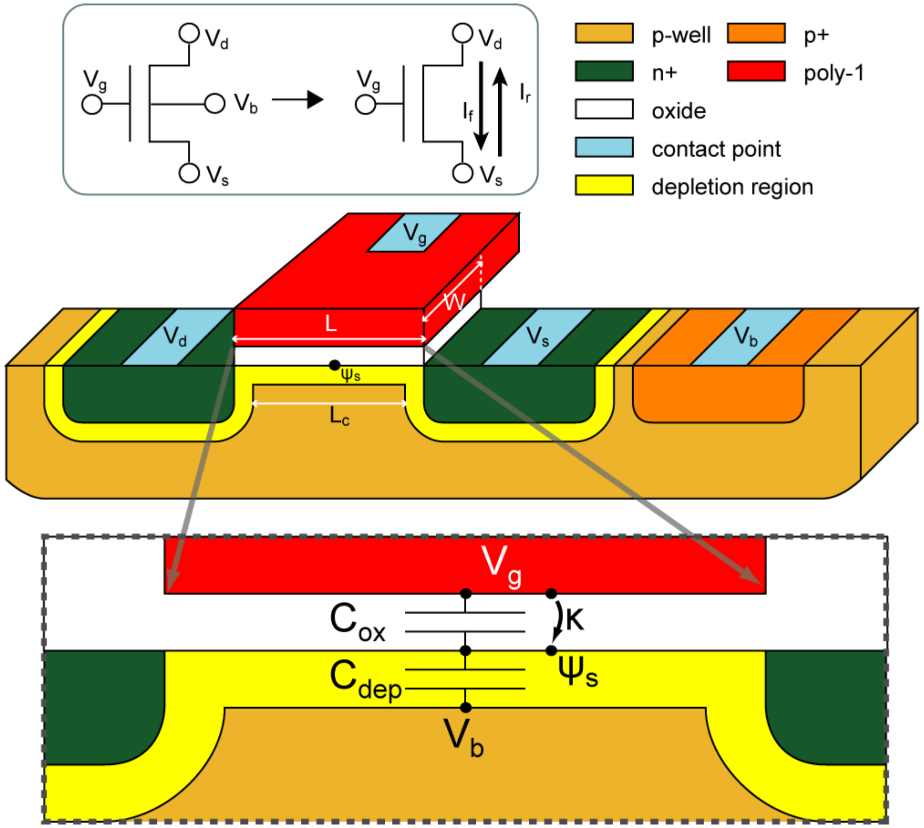
In log-scale, we have an exponential dependence

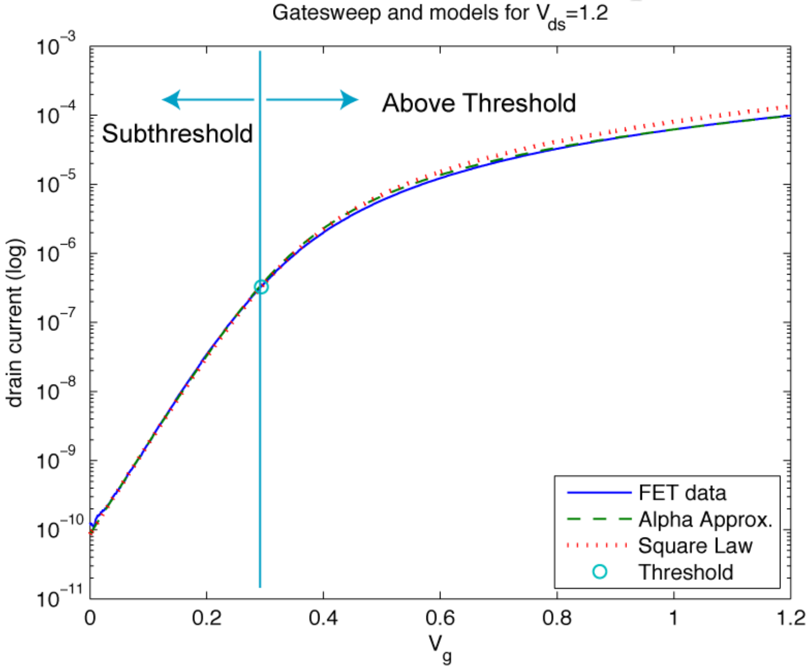
Compact EKV Model

$$I_{f,r} = \frac{W}{L} 2U_T^2 \frac{\mu C_{cox}}{2\kappa} \ln^2\left(1 + e^{\frac{\kappa(V_g - V_{T0}) + (1-\kappa)V_b - V_s + \sigma V_d}{2U_T}}\right)$$

$$I = I_f - I_r$$

$$\kappa = \frac{C_{ox}}{C_{ox} + C_{dep}}$$



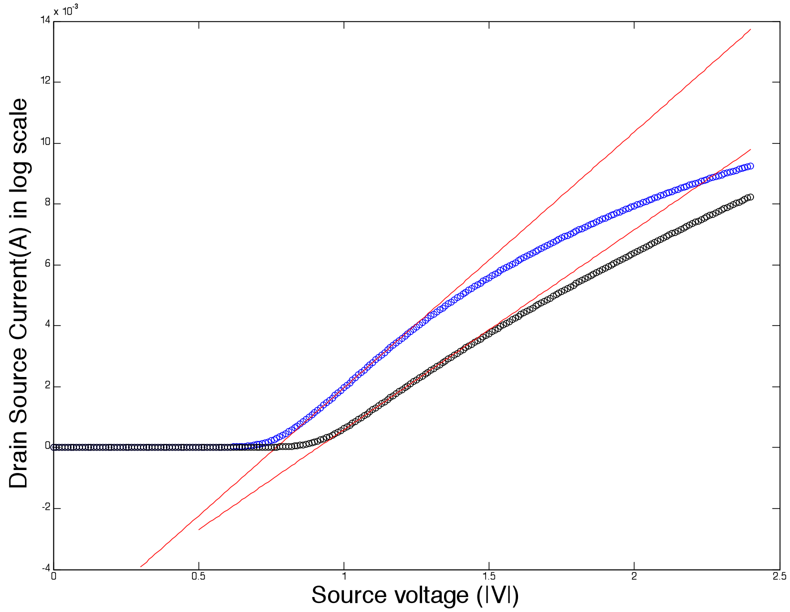

Gatesweep and models for V_{ds}=1.2

EKV Model Extraction

$V_{T0}, K, \text{gamma}, 2\emptyset_f$

$$V_T = V_{T0} + V_{SB}\left(\frac{1}{\kappa} - 1\right)$$

$$V_T = V_{T0} + \gamma\left(\sqrt{2\emptyset_f + V_{SB}} - \sqrt{2\emptyset_f}\right)$$



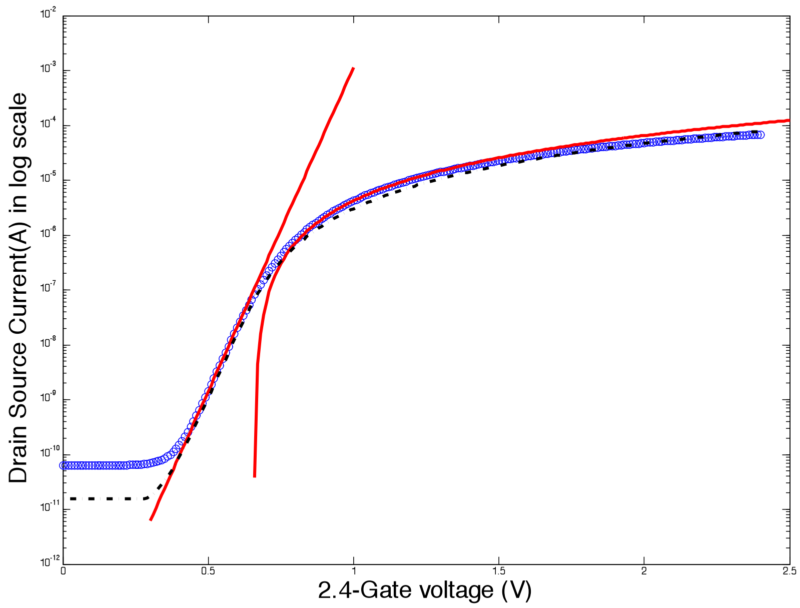
Parameters	NMOS	PMOS
V_{T0} (V)	0.405	-0.620
K' (uA/V^2)	40.6 ➔ 55	27.7
gamma	0.45	0.38
$2\varphi_f$	0.38	0.38

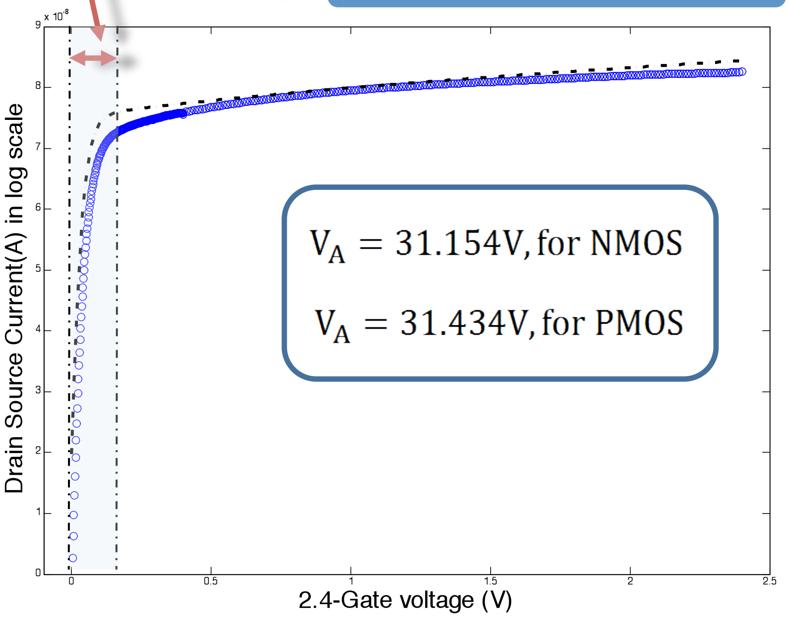
Theory, Simulation, and Data from 0.35um CMOS ICs

Vg Sweep

PMOS

$4 \times U_T \cong 100\text{mV}$ ⇨ **Sub Vt Operation**





$V_A = 31.154\text{V}$, for NMOS

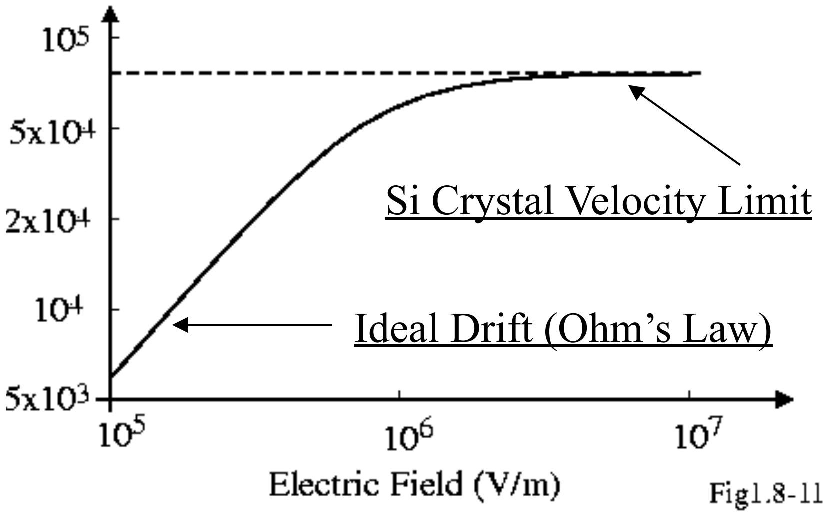
$V_A = 31.434\text{V}$, for PMOS

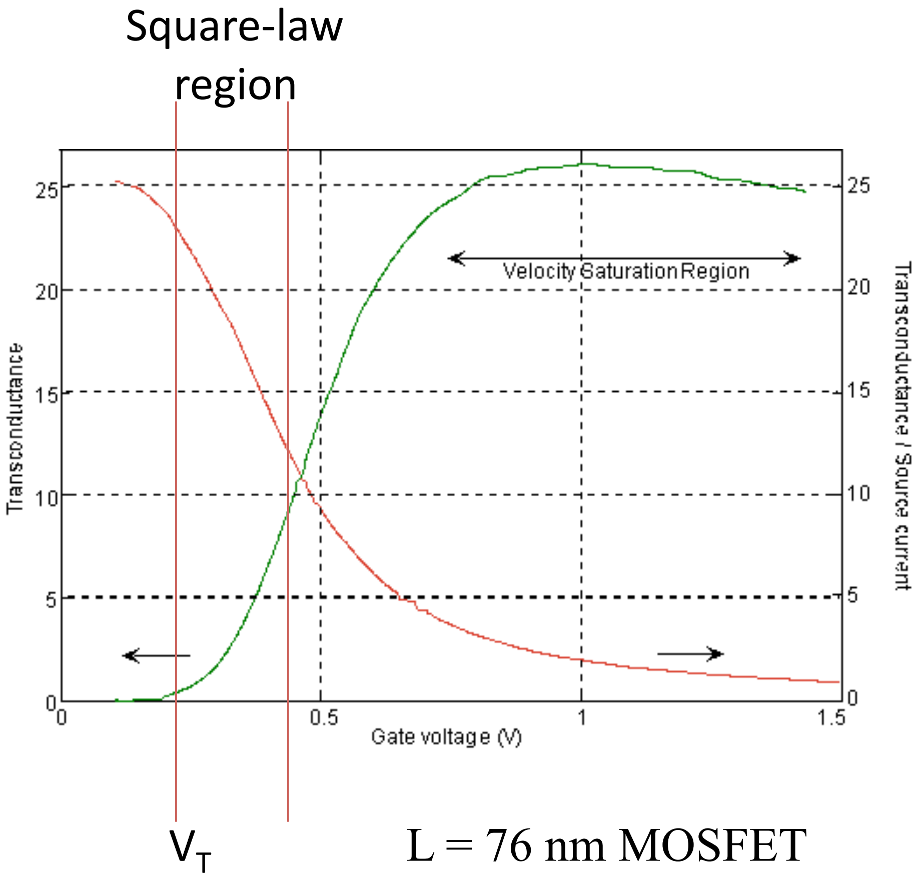
━━ Experiment

▪▪▪▪▪▪▪▪ Simulation

Effect of Velocity Saturation

Velocity Saturation

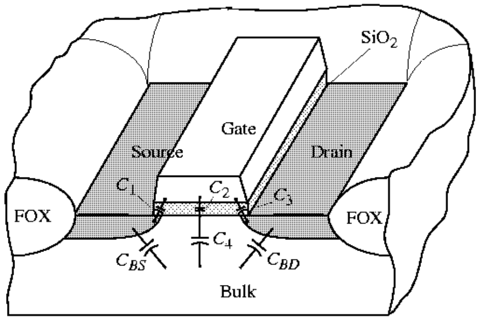


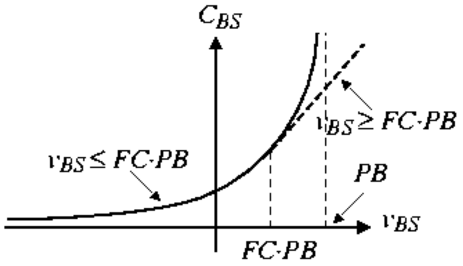


L = 76 nm MOSFET

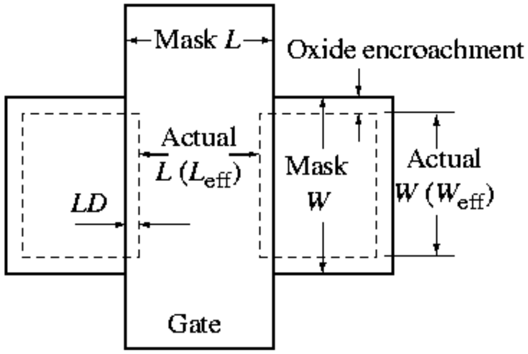
Capacitance Modeling in a MOSFET

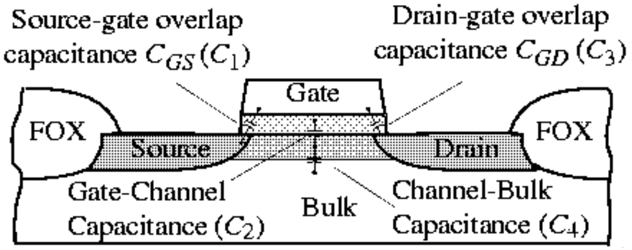
MOSFET Depletion Capacitors



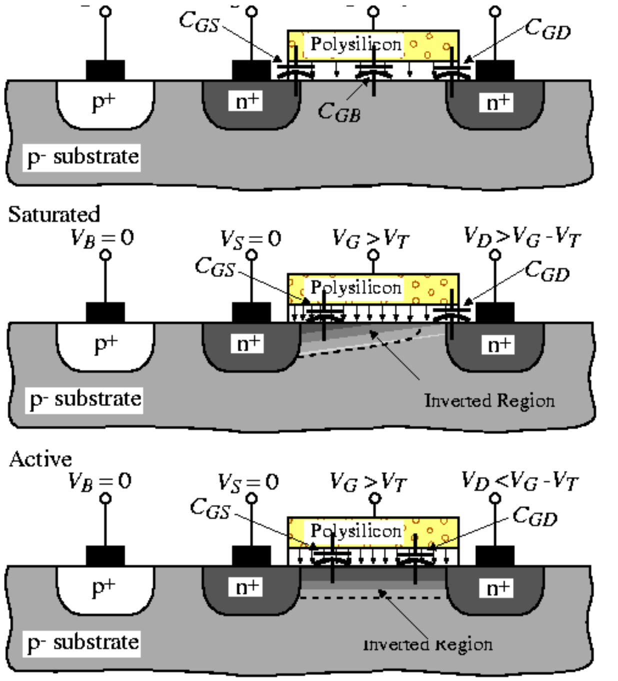


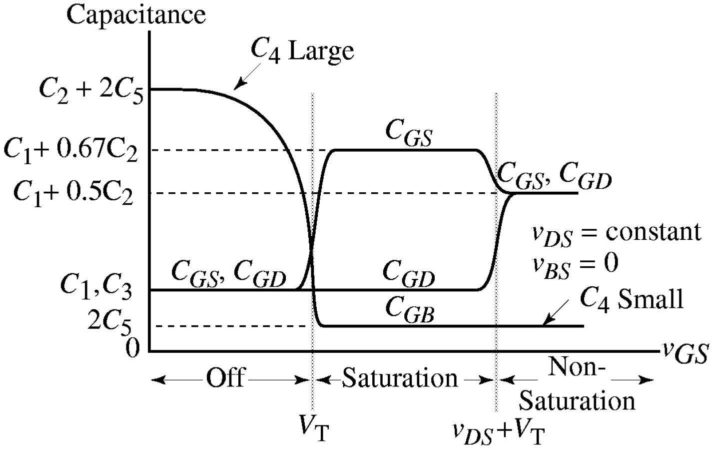
MOSFET Overlap Capacitors



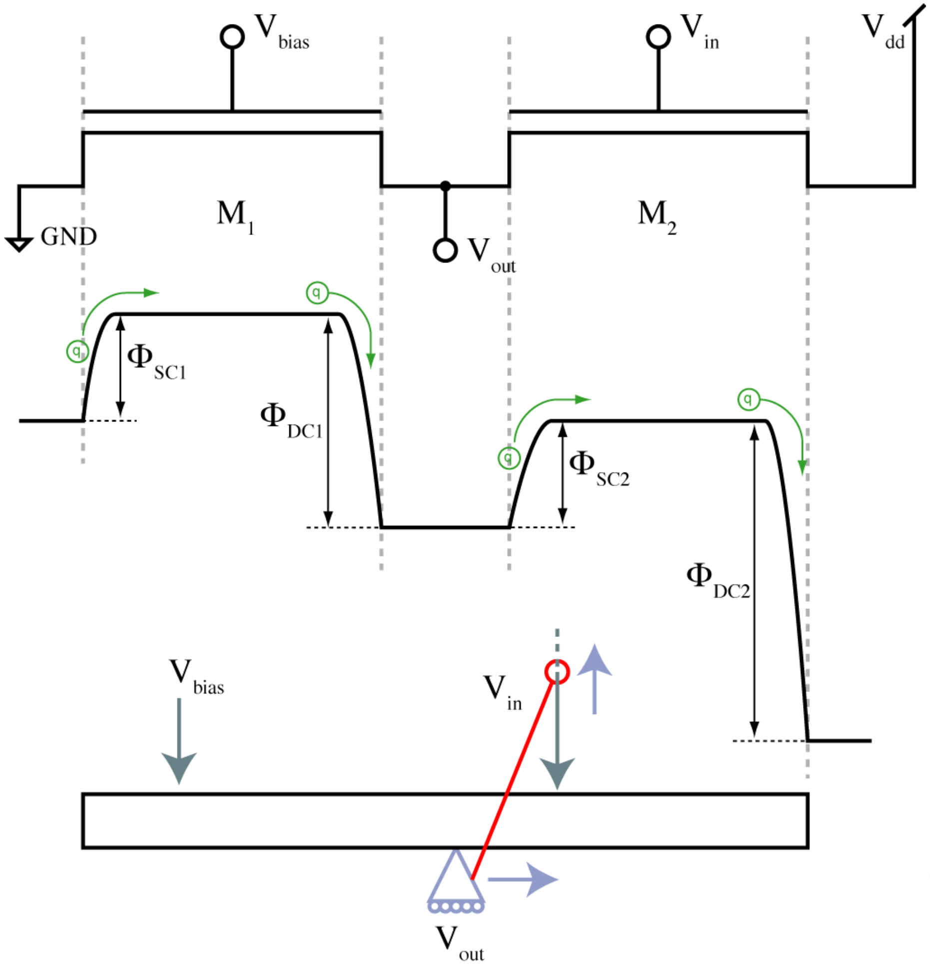


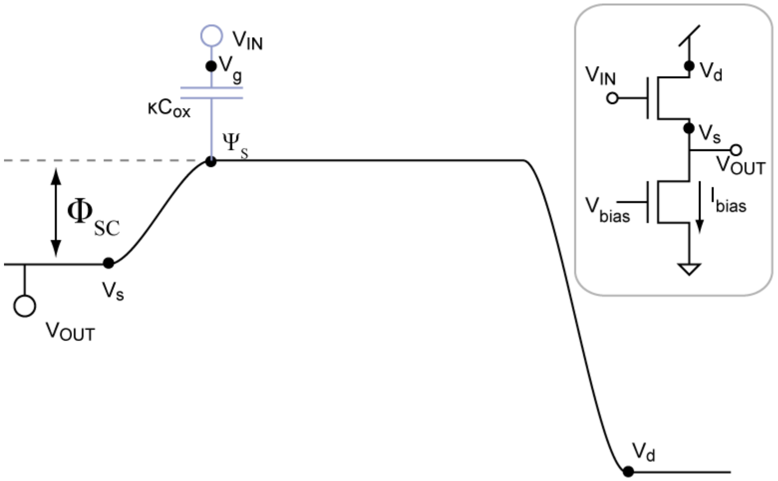
MOSFET Channel Capacitance





Source Follower (Sub V_T)

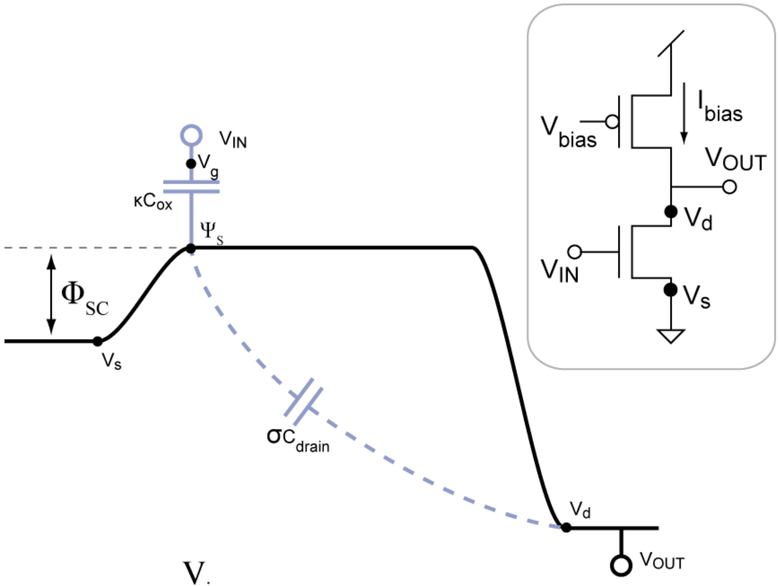




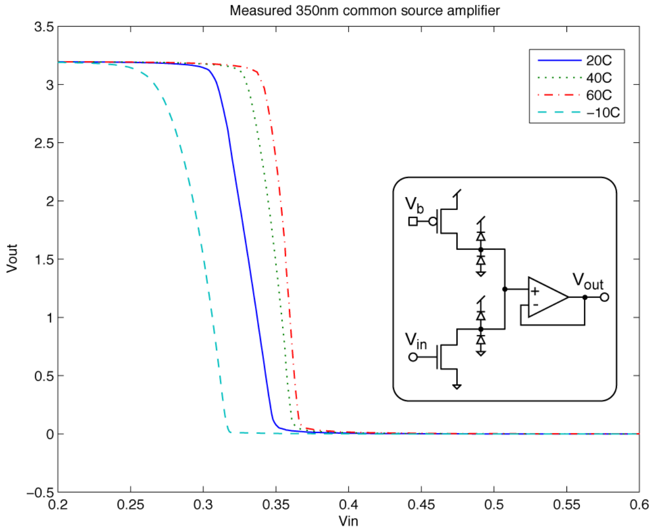
$$I_{sat} = I_{th} e^{\frac{\Phi_{SC}}{U_T}}$$

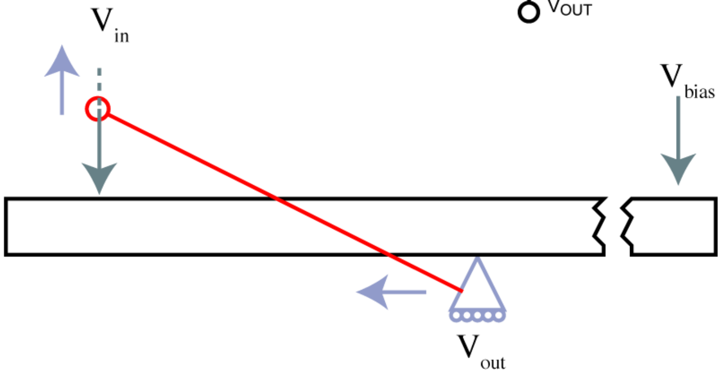
$$
\begin{aligned}
I_{th} e^{\frac{\Phi_{SC1}}{U_T}} &= I_{th} e^{\frac{\Phi_{SC2}}{U_T}} \\
I_{th} e^{\frac{\kappa_1 V_{bias} - 0}{U_T}} &= I_{th} e^{\frac{\kappa_2 V_{in} - V_{out}}{U_T}} \\
\kappa_1 V_{bias} &= \kappa_2 V_{in} - V_{out} \\
\Delta V_{out} &= \kappa \Delta V_{in} - \kappa V_{bias}
\end{aligned}
$$

Common Source Amplifier

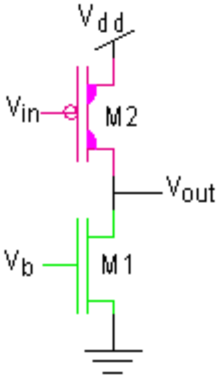


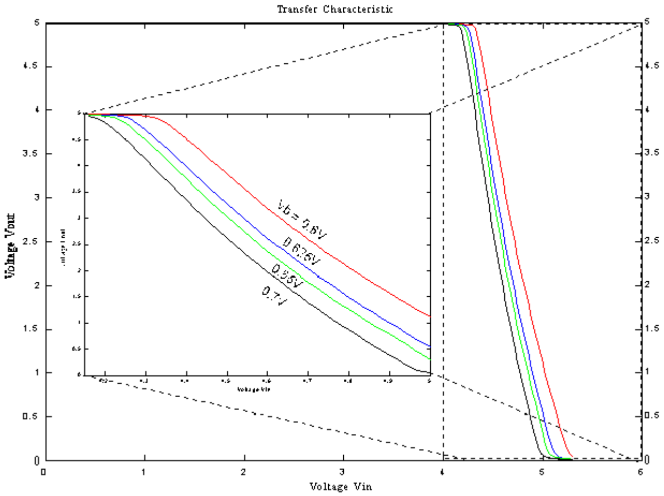
$$\Delta V_{out} = -\frac{\kappa}{\sigma}\Delta V_{in} + V_{const}$$


Measured 350nm common source amplifier



$$I_{thn} f\left(\frac{\kappa_n \left(V_{in}\right) - \sigma_n V_{out}}{2U_T}\right) = I_{thp} f\left(\frac{\kappa_p \left(V_{dd} - V_{bias}\right) - \sigma_p V_{out}}{2U_T}\right)$$




Transfer Characteristic