3 - HIGHER-ORDER LADDER PASSIVE AND ACTIVE FILTERS

In Fig. 2-1, two approaches to designing active filters were shown. One of the approaches uses cascaded, first- and second-order active filters and was discussed in Sec. 2. The other approach uses a ladder configuration and is presented in this section. Both approaches are used to design successful higher-order active filters. The cascade approach is easier to design and tune but does not have the insensitivity to component variation found in the ladder approach.

The concepts of normalization and frequency transformation developed for cascade active filters will also be used in the design of ladder active filters. The starting point for a ladder design is an RLC passive circuit rather than a transfer function or the roots of a transfer function. This means that our design manipulations will be with circuit elements rather than with roots. These two approaches are among the more widely used approaches for designing active filters.

Sensitivity of an Active Filter

Sensitivity is a measure of the dependence of the filter performance upon the passive and active components of the filter design. Sensitivity is generally expressed as percentage change of some performance aspect to the percentage change in an active or passive component. For example, we can express the *relative sensitivity* of a performance aspect designated as "p" to the variation of some component "x" as

$$S_x^p = \frac{\frac{\partial p}{p}}{\frac{\partial x}{x}} = \frac{x}{p}\frac{\partial p}{\partial x} \quad . \tag{3-1}$$

For example, the relative sensitivity of the pole-Q of the Tow-Thomas, low-pass realization of Fig. 1-16 to R_1 can be found by letting p = Q and x = R_1 and applying Eq. (3-1) to Eq. (1-59) to get

$$S_{R_1}^Q = \frac{R_1}{R_1\sqrt{\frac{C_1}{R_2R_3C_2}}}\left(\sqrt{\frac{C_1}{R_2R_3C_2}}\right) = 1 \quad . \tag{3-2}$$

Furthermore, we can show that the sensitivity of the pole-Q of Fig. 1-16 to R_2 is -0.5. Another interpretation of sensitivity is as follows. If the component x changes by dx/x, then the performance changes by the product of S_x^p times dx/x. For example, the pole-Q will change by +10% if R_1 changes by +10%. If the relationship between p and x is nonlinear, the sensitivity measure is only good for small changes. Consequently, changes much over 10% should be calculated using nonlinear sensitivity methods or point sensitivity calculations.

Sensitivity calculations can become complex if the expression relating the performance to the component is complicated. Fortunately we only wish to use the concept of sensitivity to compare the two different approaches to higher-order active filter design shown in Fig. 2-1. Although we have not yet developed the ladder filter, it can be distinguished from the cascaded filter by its dependence upon the passive and active components of the filter realization. The performance of the ladder filter always depends on more than one passive or active component. As a result, the performance is not strongly dependent upon any one component but rather on all components. However, the performance of the cascaded filter may depend upon only one component. If this component is inaccurate or off in value, the cascade filter performance can be strongly influenced while the same component change may not be noticed in the performance of the ladder filter. This advantage of the ladder filter becomes a disadvantage when tuning the filter. It is almost impossible to tune a ladder filter because no single component dominates the performance of the filter.

Normalized, RLC, Low-Pass Ladder Filters

RLC, low-pass ladder filters are the result of modern network synthesis techniques and are based on techniques well known in circuit theory†. The resulting realizations of these synthesis techniques always start with a load resistor of 1 ohm and works toward the

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† M.E. Van Valkenburg, *Introduction to Modern Network Synthesis*, Chapter 10, John Wiley & Sons, Inc., New York, 1960.

input of the filter. Fig. 3-1 shows the form for a singly-terminated RLC filter for the case of even and odd order functions with the numbering of components going from the output to the input of the filter.

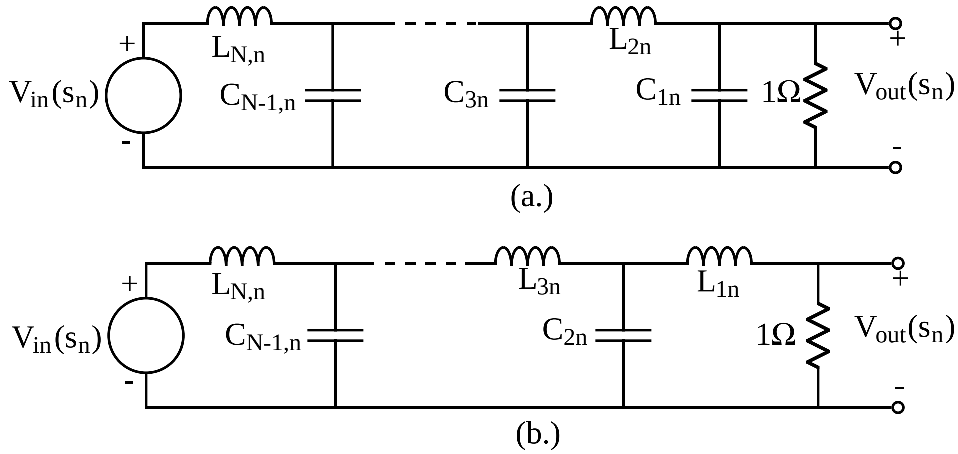

Figure 3-1 - Singly-terminated, RLC filters. (a.) N even. (b.) N odd.

The RLC ladder filters of Fig. 3-1 are normalized to a passband of 1 rps. The denormalizations of Table 2-1 are applicable to the elements of Fig. 3-1. Fig. 3-2 shows the normalized ladder filters for doubly-terminated, RLC filters. It is seen that these filters are similar to those of Fig. 3-1 except for a series source resistance.

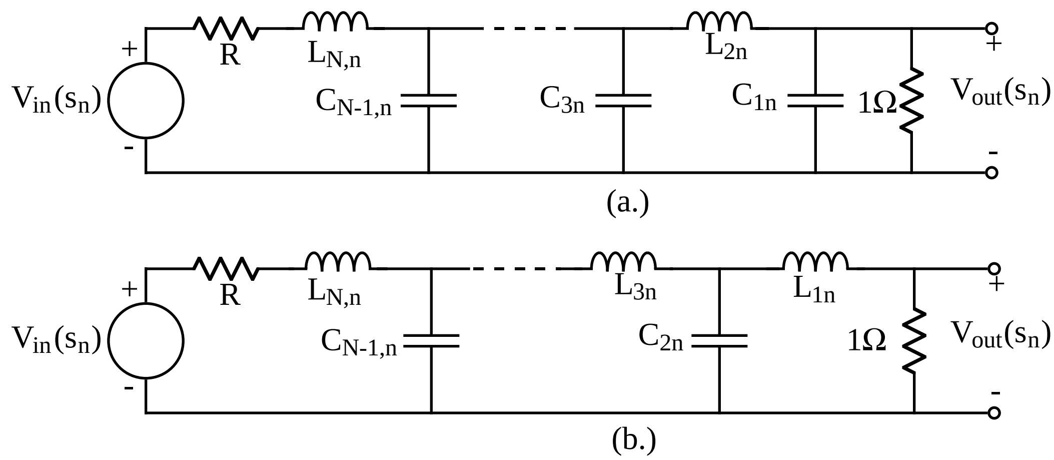

Figure 3-2 - Doubly-terminated, RLC filters. (a.) N even. (b.) N odd.

The tabular information for the design of RLC filters consists of the normalized component values of Figs. 3-1 and 3-2. Each of the many different types of filter approximations have been tabulated for values of N up to 10 or more† . Tables 3-1 and 3-2 are typical of this tabularized information for the Butterworth and 1-dB Chebyshev approximation for the singly-terminated and doubly-terminated, RLC filters of Figs. 3-1 and 3-2.

	Use these component designations for even order of Fig. 3-1a and Fig. 3-2a.									
N	C_{1n}	L_{2n}	C_{3n}	L_{4n}	C_{5n}	L_{6n}	C_{7n}	L_{8n}	C_{9n}	L_{10n}
2	0.7071	1.4142								
3	0.5000	1.3333	1.5000			Butterworth (1 rps passband)				
4	0.3827	1.0824	1.5772	1.5307						
5	0.3090	0.8944	1.3820	1.6944	1.5451					
6	0.2588	0.7579	1.2016	1.5529	1.7593	1.5529				
7	0.2225	0.6560	1.0550	1.3972	1.6588	1.7988	1.5576			
8	0.1951	0.5576	0.9370	1.2588	1.5283	1.7287	1.8246	1.5607		
9	0.1736	0.5155	0.8414	1.1408	1.4037	1.6202	1.7772	1.8424	1.5628	
10	0.1564	0.4654	0.7626	1.0406	1.2921	1.5100	1.6869	1.8121	1.8552	1.5643
2	0.9110	0.9957								
3	1.0118	1.3332	1.5088		1-dB ripple Chebyshev (1 rps passband)					
4	1.0495	1.4126	1.9093	1.2817						
5	1.0674	1.4441	1.9938	1.5908	1.6652					
6	1.0773	1.4601	2.0270	1.6507	2.0491	1.3457				
7	1.0832	1.4694	2.0437	1.6736	2.1192	1.6489	1.7118			
8	1.0872	1.4751	2.0537	1.6850	2.1453	1.7021	2.0922	1.3691		
9	1.0899	1.4790	2.0601	1.6918	2.1583	1.7213	2.1574	1.6707	1.7317	
10	1.0918	1.4817	2.0645	1.6961	2.1658	1.7306	2.1803	1.7215	2.1111	1.3801
	L_{1n}	C_{2n}	L_{3n}	C_{4n}	L_{5n}	C_{6n}	L_{7n}	C_{8n}	L_{9n}	C_{10n}
	Use these component designations for odd order of Fig. 3-1b.									

Table 3-1 - Normalized component values for Fig. 3-1 for the Butterworth and Chebyshev singly-terminated, RLC filter approximations.

Example 3-1 - Use of the Table 3-1 to Find a Singly-Terminated, RLC Low-pass Filter

Find a singly-terminated, normalized, RLC filter for a 4th-order Butterworth filter.

Solution

Using Table 3-1 and using the component designations at the top of the table gives Fig. 3-3.

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† L. Weinburg, *Network Analysis and Synthesis*, McGraw-Hill Book Co., 1962, R.E. Krieger Publishing Co., Huntington, NY, 1975.
A.I. Zverev, *Handbook of Filter Synthesis*, John Wiley & Sons, Inc., NY, 1967.

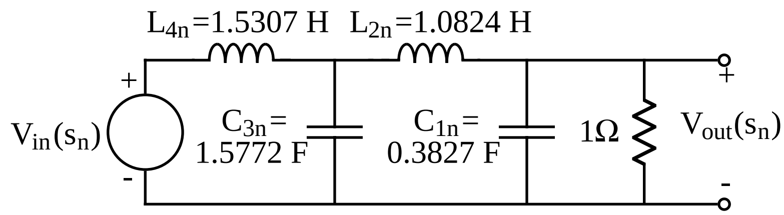

Figure 3-3 - Fourth-order, Butterworth, normalized low-pass RLC filter realization.

	Use these component designations for even order of Fig. 3-2a, R = 1Ω.									
N	C_{1n}	L_{2n}	C_{3n}	L_{4n}	C_{5n}	L_{6n}	C_{7n}	L_{8n}	C_{9n}	L_{10n}
2	1.4142	1.4142								
3	1.0000	2.0000	1.0000			Butterworth (1 rps passband)				
4	0.7654	1.8478	1.8478	0.7654						
5	0.6180	1.6180	2.0000	1.6180	0.6180					
6	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2740	0.4450			
8	0.3902	1.1111	1.6629	1.9616	1.9616	1.6629	1.1111	0.3902		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129
3	2.0236	0.9941	2.0236		1-dB ripple Chebyshev (1 rps passband)					
5	2.1349	1.0911	3.0009	1.0911	2.1349					
7	2.1666	1.1115	3.0936	1.1735	3.0936	1.1115	2.1666			
9	2.1797	1.1192	3.1214	1.1897	3.1746	1.1897	3.1214	1.1192	2.1797	
	L_{1n}	C_{2n}	L_{3n}	C_{4n}	L_{5n}	C_{6n}	L_{7n}	C_{8n}	L_{9n}	C_{10n}
	Use these component designations for odd order of Fig. 3-2b, R = 1Ω.									

Table 3-2 - Normalized component values for Fig. 3-2 for the Butterworth and 1-dB Chebyshev doubly-terminated, RLC approximations.

We note that no solution exists for the even-order cases of the doubly-terminated, RLC Chebyshev approximations for R = 1 Ω. This is a special result for R = 1 Ω and is not true for other values of R. We also can see that the gain in the passband will be no more than -6 dB because of the equal source and load resistances causing an gain of 0.5 at low frequencies where the inductors are short-circuits and the capacitors are open-circuits.

<u>Example 3-2 - Use of Table 3-2 to Find a Doubly-Terminated, RLC Low-pass Filter</u>

Find a doubly-terminated, RLC filter for a fifth-order Chebyshev filter approximation having 1 dB ripple in the passband and a source resistance of 1 Ω.

<u>Solution</u>

Using Table 3-2 and using the component designations at the bottom of the table gives Fig. 3-4.

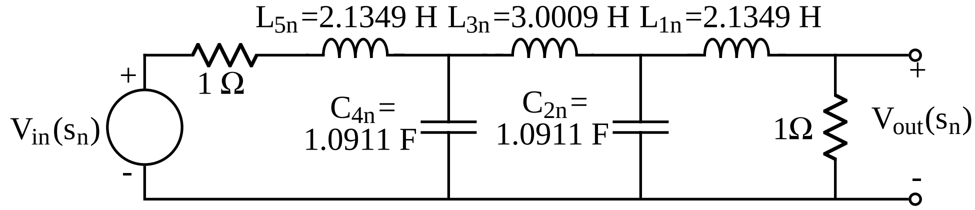

Figure 3-4 - Fifth-order, doubly-terminated, normalized, Chebyshev low-pass RLC filter realization.

Low-Pass, RLC Filter Design

In some cases, such as high frequency, the designer may choose a passive realization over an active realization. In this case, the filter has no active elements such as op amps. The design procedure is simply to find the normalized, low-pass, filter approximation which satisfies the filter specifications and then denormalize the frequency and the impedance. An example will illustrate the procedure.

<u>Example 3-3 - Design of a Passive, Low-Pass Filter</u>

A low-pass filter using the Butterworth approximation is to have the following specifications:

$$T_{PB} = -3 \text{ dB}, T_{SB} = -40 \text{ dB}, f_{PB} = 500 \text{ kHz, and } f_{SB} = 1 \text{ MHz}.$$

Design this filter using a passive RLC realization and denormalize the impedances by 10^3.

<u>Solution</u>

First we see that $\Omega_n = f_{SB}/f_{PB} = 2$ and then try various values of N in Eq. (2-11) using $\varepsilon = 1$ to get $T_{SB} = -30.1$ dB for N = 5, $T_{SB} = -36.1$ dB for N = 6, and $T_{SB} = -42.14$ dB for N = 7. From Table 3-1 for N = 7 for the Butterworth approximation we get L_{1n} = 0.2225 H, C_{2n} = 0.6560 F, L_{3n} = 1.0550 H, C_{4n} = 1.3972 F, L_{5n} = 1.6588 H, C_{6n} = 1.7988 F, and L_{7n} = 1.5576 H. Next, we use the denormalizations in the bottom row of Table 2-1 to get $L_1 = (10^3)(0.2225\text{H})/(\pi\text{x}10^6) = 0.071$ mH, $C_2 = (0.6560\text{F})/(\pi\text{x}10^9)$ = 209 pF, $L_3 = (10^3)(1.0550\text{H})/(\pi\text{x}10^6) = 0.336$ mH, $C_4 = (1.3972\text{F})/(\pi\text{x}10^9)$ = 445 pF,

$L_5 = (10^3)(1.6588\text{H})/(\pi\text{x}10^6) = 0.528$ mH, $C_6 = (1.7988\text{F})/(\pi\text{x}10^9) = 573$ pF, and $L_7 = (10^3)(1.5576\text{H})/(\pi\text{x}10^6) = 0.496$ mH. The load resistor becomes 1 kΩ. The final realization is shown in Fig. 3-5.

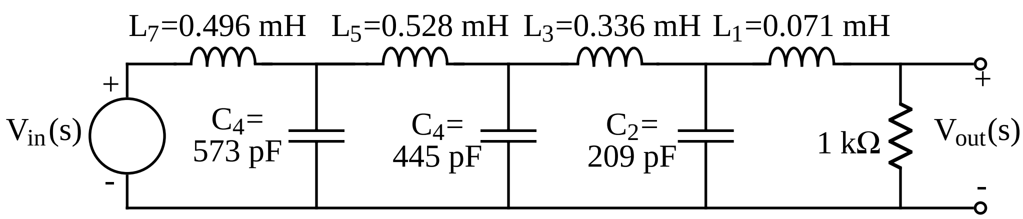

Figure 3-5 - Seventh-order, Butterworth filter realization of Ex. 3-3.

We can see by the previous example, that the design of RLC filters is very simple. However, the actual implementation of the filter requires careful consideration and effort. In order to get the desired results, we must have the exact values of components that were calculated in Ex. 3-3. The cost of such accurate components is generally prohibitive. Therefore, one adjusts each component before inserting it into the filter by the following means. For each capacitor, a small variable capacitor (i.e. 50 pF) can be placed in parallel with a larger fixed capacitor whose value is slightly less than the desired value. This arrangement can be taken to a capacitor bridge or capacitance meter and the variable capacitor trimmed until the desired value is achieved.

The inductors are typically made by winding low-resistance wire around a cylindrical insulator. Inside the cylindrical insulator is a magnetic cylinder whose position is adjusted by a screw which causes the inductance of the inductor to vary. The inductors are custom made by winding the necessary number of turns to achieve an inductance less than desired with the magnetic core removed. Then the magnetic core is inserted and adjusted to achieve the desired inductance.

The above procedures work well if there are no parasitic elements that will effect the filter performance. The important parasitics are the capacitors from each node to ground and the resistance of the inductors. For precise filter applications, these influences must be incorporated into the design phase. This results in a more complex filter design procedure which is beyond the scope of our treatment.

High-Pass, RLC, Ladder Filter Design

The design of high-pass and bandpass, RLC ladder filters is made simple by the frequency transformations of the previous section. The frequency transformation from the normalized, low-pass to normalized high-pass was given by Eq. (2-23). If we apply this transformation to an inductor of a normalized, low-pass realization, we obtain

$$s_{ln}L_{ln} = \left(\frac{1}{s_{hn}}\right) L_{ln} = \frac{1}{s_{hn}C_{hn}} \quad . \tag{3-3}$$

Similarly, if applying the transformation to a capacitor, C_{ln}, of a normalized, low-pass realization, we obtain

$$\frac{1}{s_{ln}C_{ln}} = \left(\frac{s_{hn}}{1}\right)\frac{1}{C_{ln}} = s_{hn}L_{hn} \; . \tag{3-4}$$

From Eqs. (3-3) and (3-4), we see that the normalized, low-pass to normalized, high-pass frequency transformation takes an inductor, L_{ln}, and replaces it by a capacitor, C_{hn}, whose value is $1/L_{ln}$. This transformation also takes a capacitor, C_{ln}, and replaces it by an inductor, L_{hn}, whose value is $1/C_{ln}$. Fig. 3-6 illustrates these important relationships.

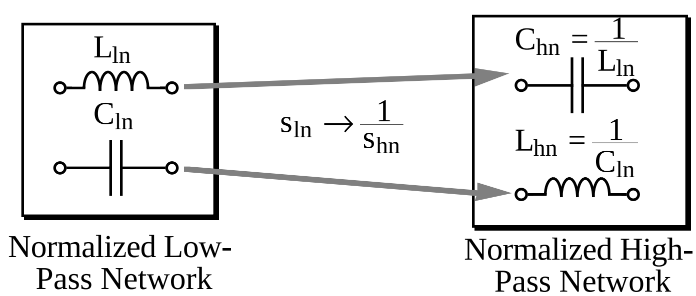

Figure 3-6 - Influence of the normalized, low-pass to normalized, high-pass frequency transformation on the inductors and capacitors.

From the above results, we see that to achieve a normalized, high-pass RLC filter, we replace each inductor, L_{ln}, with a capacitor, C_{hn}, whose value is $1/L_{ln}$ and each capacitor, C_{ln}, with an inductor, L_{hn}, whose value is $1/C_{ln}$. Once, the elements have been transformed, then both a frequency and impedance denormalization can be performed to result in the final filter design.

Example 3-4 - Design of a High-Pass, RLC Filter

A filter is to be designed to provide a passband with a ripple of no more than 3 dB above 100 kHz and a stopband attenuation of at least 45 dB below 50 kHz. The output of this filter will be connected to a 50-ohm cable and will serve as the load resistor. Select a suitable RLC filter approximation and give the values of all components.

Solution

We know from Eq. (2-26) that $\Omega_n = 2$. Using Eq. (2-11) gives an N = 8 (T_{SB} = -48.2 dB) for a Butterworth approximation. Eq. (2-19) gives N = 5 (T_{SB} = -45.3 dB) for a Chebyshev approximation. Let us choose the Chebyshev approximation since there are almost half the components. We must choose the singly-terminated filter because the doubly-terminated filter has a maximum passband gain of -6 dB. Therefore, from Table 3-1 we have L_{1ln} = 1.0674 H, C_{2ln} = 1.4441 F, L_{3ln} = 1.9938 H, C_{4ln} = 1.5908 F, and L_{5ln} = 1.6652 H.

Next, we apply the normalized, low-pass to normalized, high-pass frequency transformation to each element to get $C_{1hn} = 1/L_{1ln}$ = 1/1.0674 = 0.9369 F, $L_{2hn} = 1/C_{2ln}$ = 1/1.4441 = 0.6925 H, $C_{3hn} = 1/L_{3ln}$ = 1/1.9938 = 0.5016 F, $L_{4hn} = 1/C_{4ln}$ = 1/1.5908 = 0.6286 H, and $C_{5hn} = 1/L_{5ln}$ = 1/1.6652 = 0.6005 F. Finally, we denormalize the frequency by 2πx100 kHz = $2\pi \text{x} 10^5$ and the impedance by 50 in order to represent the connection of the 50-ohm cable to the output of the filter. The actual values for the filter are $C_{1h} = 0.9369/(50\text{x}2\pi\text{x}10^5)$ = 59.6 nF, $L_{2h} = 0.6925\text{x}(50)/(2\pi\text{x}10^5)$ = 110 μH, $C_{3h} = 0.5016/(50\text{x}2\pi\text{x}10^5)$ = 31.9 nF, $L_{4h} = 0.6286\text{x}(50)/(2\pi\text{x}10^5)$ = 100 μH, and $C_{5h} = 0.6005/(50\text{x}2\pi\text{x}10^5)$ = 38.3 nF. Fig. 3-7 shows the final realization for this example.

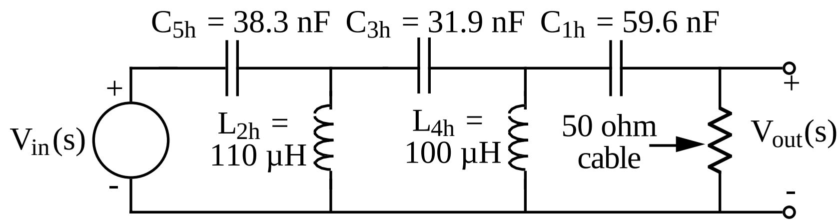

Figure 3-7 - Denormalized, Chebyshev, high-pass filter realization for Ex. 3-4.

Bandpass, RLC, Ladder Filter Design

The design of RLC bandpass ladder filter follows exactly the same approach as used in the previous section except the transformations are applied to the passive components of the normalized low-pass filter rather than the roots. The first step in designing a bandpass filter is to apply the bandpass normalization of Eq. (2-32) to the components of the low-pass filter. Next, we apply the normalized, low-pass to normalized, bandpass transformation of Eq. (2-33). Both of these steps are incorporated in Eq. (2-31). Lastly, we denormalize the normalized, bandpass filter by ω_r and by the desired impedance denormalization.

Let us first consider the inductor, L_{ln}, of a normalized, low-pass filter. Let us simultaneously apply the bandpass normalization and the frequency transformation by using Eq. (2-31). The normalized, inductance L_{ln} can be expressed as

$$s_{ln}L_{ln} = \left[\left(\frac{\omega_r}{BW}\right)\left(s_{bn} + \frac{1}{s_{bn}}\right)\right] L_{ln}$$

$$= s_{bn}\left(\frac{\omega_r\, L_{ln}}{BW}\right) + \frac{1}{s_{bn}}\left(\frac{\omega_r\, L_{ln}}{BW}\right) = s_{bn}L_{bn} + \frac{1}{s_{bn}C_{bn}} \quad . \tag{3-5}$$

Thus we see that the bandpass normalization and frequency transformation takes an inductance, L_{ln}, and replaces it by an inductor, L_{bn}, in series with a capacitor, C_{bn}, whose values are given as

$$L_{bn} = \left(\frac{\omega_r}{BW}\right) L_{ln} = \frac{L_{ln}}{\Omega_b} \tag{3-6}$$

and

$$C_{bn} = \left(\frac{BW}{\omega_r}\right)\frac{1}{L_{ln}} = \frac{\Omega_b}{L_{ln}} \quad . \tag{3-7}$$

Now we apply Eq. (2-31) to a normalized capacitance, C_{ln}, to get

$$\frac{1}{s_{ln}C_{ln}} = \frac{1}{\left[\left(\frac{\omega_r}{BW}\right)\left((s_{bn} + \frac{1}{s_{bn}})\right)\right] C_{ln}}$$

$$= \frac{1}{s_{bn}\left(\frac{\omega_r}{BW}\right) C_{ln} + \frac{1}{s_{bn}}\left(\frac{\omega_r C_{ln}}{BW}\right)} = \frac{1}{s_{bn}C_{bn} + \frac{1}{s_{bn}L_{bn}}} \quad . \tag{3-8}$$

From Eq. (3-8), we see that the bandpass normalization and frequency transformation takes a capacitor, C_{ln}, in a low-pass circuit and transfors to a capacitor, C_{bn}, in parallel with an inductor, L_{bn}, whose values are given as

$$C_{bn} = \left(\frac{\omega_r}{BW}\right) C_{ln} = \frac{C_{ln}}{\Omega_b} \tag{3-9}$$

and

$$L_{bn} = \left(\frac{BW}{\omega_r}\right)\frac{1}{C_{ln}} = \frac{\Omega_b}{C_{ln}} \quad . \tag{3-10}$$

Eqs. (3-6), (3-7), (3-9), and (3-10) are very important in the design of RLC bandpass filters and are illustrated in Fig. 3-8.

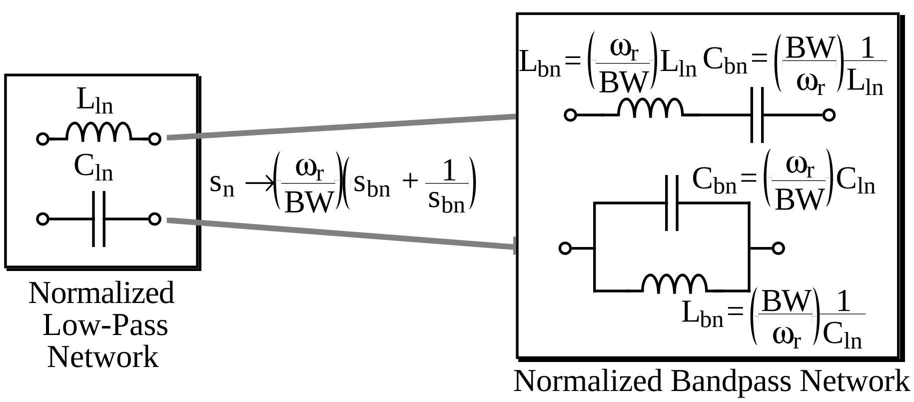

Figure 3-8 - Illustration of the influence of the normalized, low-pass to the normalized, bandpass transformation of Eq. (2-31) on an inductor and capacitor of a low-pass filter.

Once the normalized, low-pass realization has been transformed to a normalized, bandpass realization, then all that is left is to denormalize the passive elements. An example will serve to illustrate the design approach.

Example 3-5 - Design of an RLC Bandpass Filter

A 1-dB ripple, Chebyshev approximation is to be used to design a bandpass filter with a passband of 10 kHz and a stopband of 50 kHz geometrically centered at 100 kHz. The attenuation in the stopband should be at least 45 dB. Assume that the load resistor is 300 ohms. Give a final filter schematic with the actual component values.

Solution

From the specification and Eq. (2-41), we know that $\Omega_n = 5$. Substituting N = 3 into Eq. (2-19) gives an attenuation of 47.85 dB. The values of the third-order, normalized Chebyshev approximation are $L_{1ln} = 1.0118$ H, $C_{2ln} = 1.3332$ F, and $L_{3ln} = 1.5088$ H.

The value of Ω_b, defined in Eq. (2-35), is $\Omega_b = 10/100 = 0.1$. Applying the normalized, low-pass to normalized, bandpass transformation gives the following results. For L_{1ln} we get $L_{1bn} = 1.0118/0.1 = 11.018$ H and $C_{1bn} = 0.1/0.11018 = 0.090761$ F. For C_{2ln} we get $C_{2bn} = 1.3332/0.1 = 13.332$ F and $L_{2bn} = 0.1/0.13332 = 0.075008$ H. Finally, for L_{3ln} we get $L_{3bn} = 1.5088/0.1 = 15.088$ H and $C_{3bn} = 0.1/1.5088 = 0.066278$ F.

Finally, we frequency and impedance denomalize each element by $2\pi x10^5$ and 300, respectively. The resulting bandpass filter is shown in Fig. 3-9.

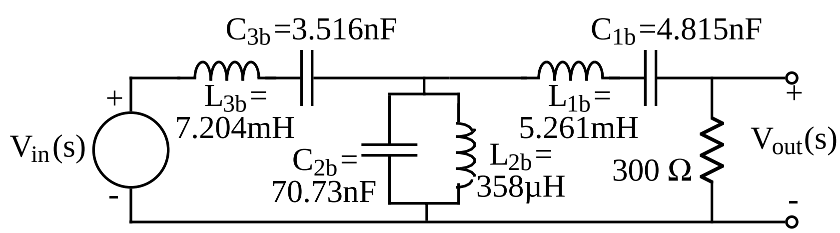

Figure 3-9 - Chebyshev, bandpass realization for Ex. 3-5.

Design of Other Types of RLC Filters

There are several types of RLC filters that we will not cover in this section but we should briefly mention them. One type are filters that have $j\omega$ axis zeros such as the one shown in Fig. 2-18. Normalized, low-pass RLC filters have been developed and tabulated for this type of filter. They are similar in form to the ladder filters of Figs. 3-1 and 3-2 except that the series inductor is replaced by a parallel inductor and capacitor or the shunt capacitor is replaced by a series capacitor and inductor.

There is also bandstop filters which we have not considered. These filters can be developed by first applying the normalized, low-pass to normalized, high-pass transformation to the low-pass normalized realization. Next, the low-pass to bandpass transformation of Eq. (2-31) is applied to the normalized high-pass filter. The result is a bandstop filter having a stopband of BW geometrically centered at ω_r. The passband is geometrically centered around ω_r and has the width of Ω_nBW where BW is the bandwidth of a bandpass filter. If Ex. 3-5 were repeated for a bandstop filter with a stopband of 10 kHz and a passband of 50 kHz geometrically centered at 100 kHz, the result would be similar to Fig. 3-9 except L_{1b} and C_{1b} (L_{3b} and C_{3b}) would in parallel and C_{2b} and L_{2b} would be in series. The values would be different because of the normalized, low-pass to normalized, high-pass transformation (see PA3-1).

State Variable Equations for RLC Ladder Filters

The next step in the design of ladder filters is to show how to use active elements and resistors and capacitors to realize a low-pass ladder filter. Let us demonstrate the approach by using an example. Consider the doubly-terminated, fifth-order, RLC, low-pass ladder filter of Fig. 3-10. Note that we have reordered the numbering of the components to start with the source and proceed to the load. We are also dropping the "l" from the component subscripts because we will only be dealing with low-pass structures in this discussion.

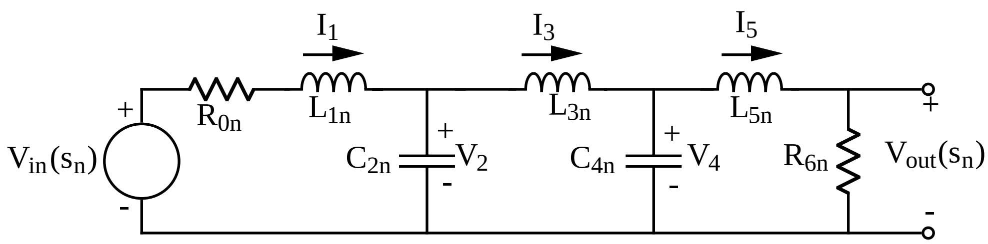

Figure 3-10 - A fifth-order, low-pass, normalized RLC ladder filter.

The first step in realizing the RLC filter of Fig. 3-10 by active RC elements is to assign a current I_j to every j-th series element (or combination of elements in series) of the ladder filter and a voltage V_k to every k-th shunt element (or combination of elements in shunt) ofthe ladder filter. These currents and voltages for the example of Fig. 3-10 are shown on the figure. These variables are called *state variables*.

The next step is to alternatively use loop (KVL) and node (KCL) equations expressed in terms of the state variables only. For example, we begin at the source of Fig. 3-10 and write the loop equation

$$V_{in}(s) - I_1(s)R_{0n} - sL_{1n}I_1(s) - V_2(s) = 0 \quad . \tag{3-11}$$

Next, we write the nodal equation

$$I_1(s) - sC_{2n}V_2(s) - I_3(s) = 0 \quad . \tag{3-12}$$

We continue in this manner to get the following state equations.

$$V_2(s) - sL_{3n}I_3(s) - V_4(s) = 0 \tag{3-13}$$

$$I_3(s) - sC_{4n}V_4(s) - I_5(s) = 0 \tag{3-14}$$

and

$$V_4(s) - sL_{5n}I_5(s) - R_{6n}I_{5n}(s) = 0 \tag{3-15}$$

Eqs. (3-11) through (3-15) constitute the state equations which completely describe the ladder filter of Fig. 3-10. A supplementary equation of interest is

$$V_{out}(s) = I_5(s)R_{6n} \quad . \tag{3-16}$$

Once, the state equations for a ladder filter are written, then we define a *voltage analog*, V_j' *of current* I_j as

$$V_j' = R'I_j \tag{3-17}$$

where R' is an arbitrary resistance (normally 1 ohm). The voltage analog concept allows us to convert from impedance and admittance functions to voltage transfer functions which is a useful step in the active-RC implementation of the ladder filter. Now if for every current in the state equations of Eq. (3-11) through Eq. (3-15) we replace currents I_1, I_3, and I_5 by their voltage analogs, we get the following modified set of state equations.

$$V_{in}(s) - \left(\frac{V_1'(s)}{R'}\right)(R_{0n} + sL_{1n}) - V_2(s) = 0 \quad (3\text{-}18)$$

$$\left(\frac{V_1'(s)}{R'}\right) - sC_{2n}V_2(s) - \left(\frac{V_3'(s)}{R'}\right) = 0 \quad (3\text{-}19)$$

$$V_2(s) - sL_{3n}\left(\frac{V_3'(s)}{R'}\right) - V_4(s) = 0 \quad (3\text{-}20)$$

$$\left(\frac{V_3'(s)}{R'}\right) - sC_{4n}V_4(s) - \left(\frac{V_5'(s)}{R'}\right) = 0 \quad (3\text{-}21)$$

and

$$V_4(s) - \left(\frac{V_5'(s)}{R'}\right)(sL_{5n} + R_{6n}) = 0 \quad (3\text{-}22)$$

The next step is to use the 5 equations of Eqs. (3-18) through (3-22) to solve for each of the state variables. The result is

$$V_1'(s) = \frac{R'}{sL_{1n}}\left[V_{in}(s) - V_2(s) - \left(\frac{R_{0n}}{R'}\right)V_1'(s)\right] \quad (3\text{-}23)$$

$$V_2(s) = \frac{1}{sR'C_{2n}}\left[V_1'(s) - V_3'(s)\right] \quad (3\text{-}24)$$

$$V_3'(s) = \frac{R'}{sL_{3n}}\left[V_2(s) - V_4(s)\right] \quad (3\text{-}25)$$

$$V_4(s) = \frac{1}{sR'C_{4n}}\left[V_3'(s) - V_5'(s)\right] \quad (3\text{-}26)$$

and

$$V_5'(s) = \frac{R'}{sL_{5n}}\left[V_4(s) - \frac{R_{6n}}{R'}V_5'(s)\right] . \quad (3\text{-}27)$$

However, we would prefer to have the variable $V_{out}(s)$ used in place of $V_5'(s)$. From Eq. (3-15) we get

$$V_{out}(s) = \left(\frac{R_{6n}}{R'}\right) V_5'(s) \quad . \tag{3-28}$$

Combining Eqs. (3-26) and (3-27) with (3-28) gives

$$V_4(s) = \frac{1}{sR'C_{4n}} \left[V_3'(s) - \left(\frac{R'}{R_{6n}}\right) V_{out}(s)\right] \tag{3-29}$$

$$V_{out}(s) = \frac{R_{6n}}{sL_{5n}} [V_4(s) - V_{out}(s)] \ . \tag{3-30}$$

Design of Low-Pass, Ladder Filters using Active-RC Elements

Finally, we are ready to synthesize an active-RC realization of Fig. 3-10. This is done by realizing that Eqs. (3-23), (3-24), (3-25), (3-29) and Eq. (3-30) are simple integrators. In each case, the state variable is expressed as the integration of itself or other state variables. The next step would be to go back to Sec. 4.1 and realize each of the five equations with 5 positive integrators capable of summing more that one input. However, as we will recall, the positive integrator did not provide the flexibility and usefulness (i.e. summing) found in the negative or inverting integrator. We could realize the state equations using positive integrators of the form shown in Fig. 3-11.

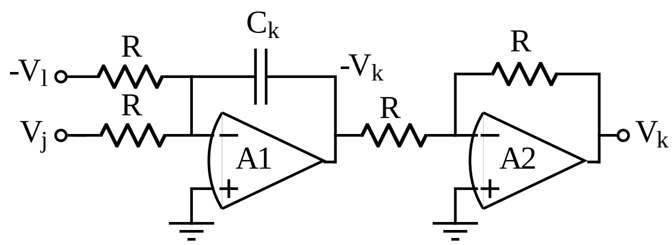

Figure 3-11 - Positive integrator realization of the k-th integrator.

We can show that the output of Fig. 3-11 is given as

$$V_k(s) = \frac{1}{sRC_k}\,[V_j(s) - V_l(s)] \tag{3-31}$$

where $V_j(s)$ is the input from the previous stage and $-V_l(s)$ is the negative output of the next stage. Note that the inverter at the output of the integrator allows us to get either the positive or negative output variable.

Fig. 3-11 works well for Eqs. (3-24), (3-25), and (3-29). However, a different approach is necessary for Eqs. (3-23) and (3-30) because the variable is a function of itself. Solving for $V_1'(s)$ and $V_{out}(s)$ in Eqs. (3-23) and (3-31) gives

$$V_1'(s) = \left(\frac{\frac{R'}{L_{1n}}}{s + \frac{R_{0n}}{L_{1n}}}\right)\{V_{in}(s) + [-V_2(s)]\} \tag{3-32}$$

and

$$V_{out}(s) = \left(\frac{\frac{R_{6n}}{L_{5n}}}{s + \frac{R_{0n}}{L_{5n}}} \right) V_4(s) \quad , \tag{3-33}$$

respectively. We see that Eqs. (3-32) and (3-33) are nothing more than the first-order, low-pass filter of Fig. 4.2-6c preceeded by a gain of -1 stage which also serves as a summer for Eq. (3-32). The implementation of Eq. (3-32) is shown in Fig. 3-12.

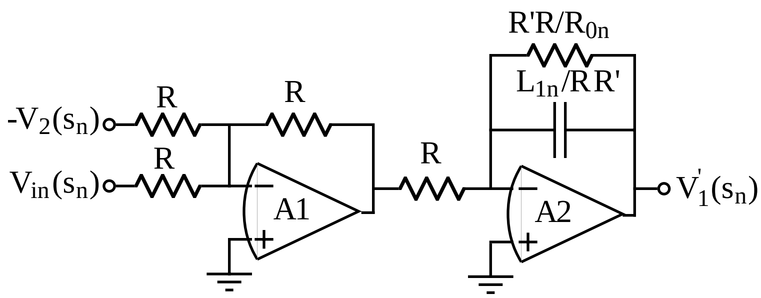

Figure 3-12 - Realization of Eq. (3-32).

A similar realization can be found for Eq. (3-33). This realization will only have one input to the inverter.

Unfortunately, the straight-forward application of Figs. 3-11 and 3-12 to realize the state equations uses more op amps than is necessary. A more efficient realization is achieved by modification of the state equations. One modification of Eqs. (3-23), (3-24), (3-25), (3-29) and Eq. (3-30) is shown below.

$$V_1'(s) = \frac{R'}{sL_{1n}}\left[V_{in}(s) - V_2(s) - \left(\frac{R_{0n}}{R'}\right)V_1'(s)\right] \tag{3-34}$$

$$-V_2(s) = \frac{-1}{sR'C_{2n}} \left[V_1'(s) - V_3'(s)\right] \tag{3-35}$$

$$-V_3'(s) = \frac{R'}{sL_{3n}} \left[-V_2(s) + V_4(s)\right] \tag{3-36}$$

$$V_4(s) = \frac{-1}{sR'C_{4n}} \left[-V_3'(s) + \left(\frac{R'}{R_{6n}}\right) V_{out}(s)\right] \tag{3-37}$$

and

$$V_{out}(s) = \frac{R_{6n}}{sL_{5n}} \left[V_4(s) - V_{out}(s)\right] . \tag{3-38}$$

Another modification of the state equations is given as

$$-V_1'(s) = \frac{-R'}{sL_{1n}}\left[V_{in}(s) - V_2(s) - \left(\frac{R_{0n}}{R'}\right)V_1'(s)\right] \tag{3-39}$$

$$-V_2(s) = \frac{1}{sR'C_{2n}}\ [-V_1'(s) + V_3'(s)] \tag{3-40}$$

$$V_3'(s) = \frac{-R'}{sL_{3n}}\ [-V_2(s) + V_4(s)] \tag{3-41}$$

$$V_4(s) = \frac{1}{sR'C_{4n}}\ \left[V_3'(s) - \left(\frac{R'}{R_{6n}}\right) V_{out}(s)\right] \tag{3-42}$$

and

$$-V_{out}(s) = \frac{-R_{6n}}{sL_{5n}}\ [V_4(s) - V_{out}(s)]\ . \tag{3-43}$$

Fig. 3-13a shows the realization of the state equations of Eqs. (3-34) through (3-38). Fig. 3-13b shows the realization of the state equations of Eqs. (3-39) through (3-43). The second modification has one less op amp but only $-V_{out}(s)$ is available. Fig. 3-13a saves two op amps over the case where every integrator consists of two op amps and Fig. 3-13b saves three op amps over the case where every integrator consists of two op amps. It is interesting to note that Fig. 3-13 turns out to be the interconnection of four, second-order, Tow-Thomas filters of Fig. 1-16. The inside filters have a Q of infinity because neither integrator is damped. The intercoupled second-order filters are distinctly different from the cascade realization in that there is more than one path from the input to the output of the filter. This type of realization was first published by Girling and Good† and were called "leapfrog" filters.

† F.E.J. Girling and E.F. Good, "Active Filters 12: The Leap-Frog or Active-Ladder Synthesis," *Wireless World*, vol. 76, July 1970, pp. 341-345.

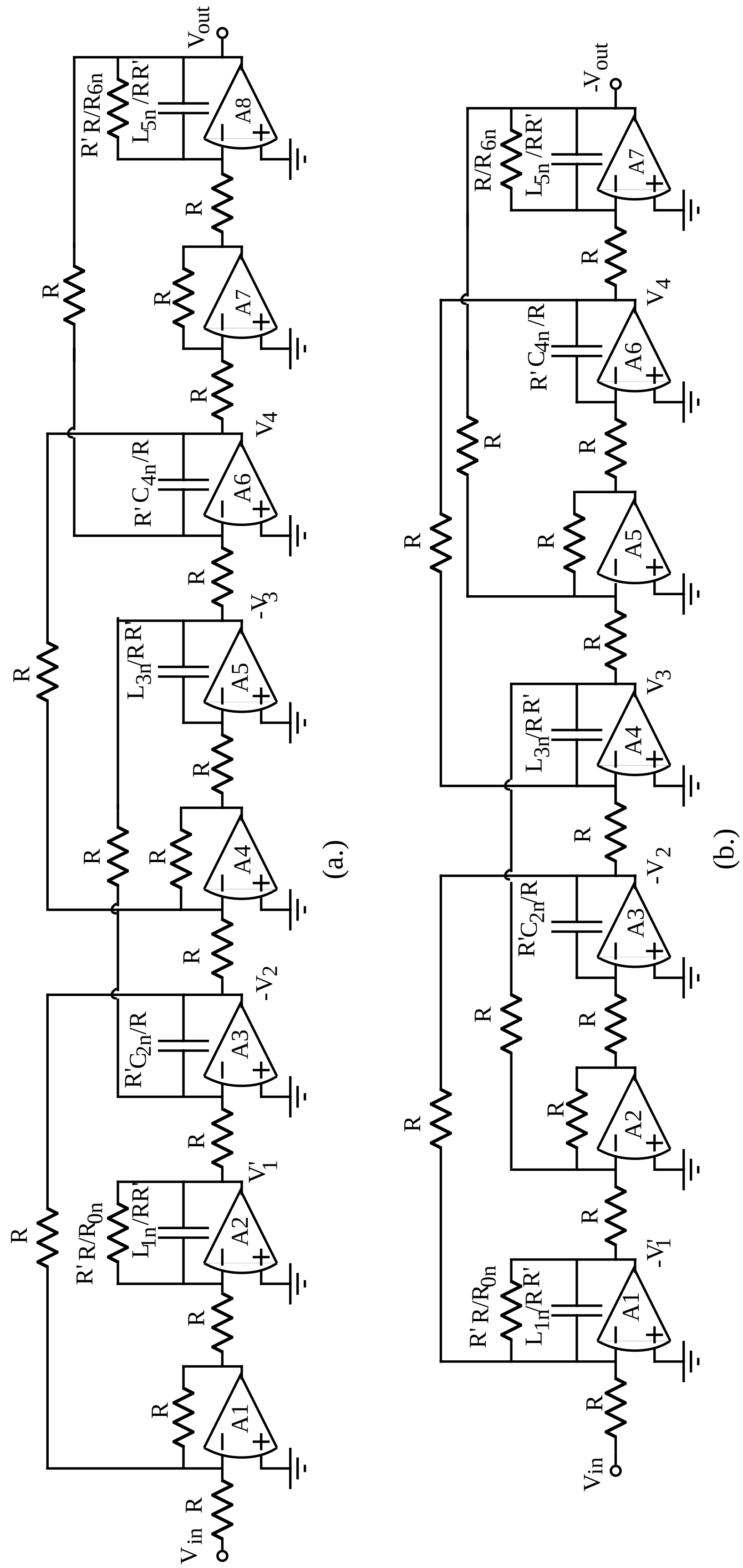

Figure 3-13 - (a.) Realization of Eqs. (3-34) through (3-38). (b.) Realization of Eqs. (3-39) through (3-43).

Example 3-6 - Design of a Low-Pass, Butterworth Ladder Filter using the Active-RC Elements

Design a third-order, Butterworth, low-pass filter having a passband frequency of 1 kHz using the RC-active ladder realization of Fig. 3-13. Assume that the source resistance of the input voltage is zero. Give a realization and the value of all components.

Solution

The normalized, low-pass ladder filter is obtain from Table 3-1 and is shown in Fig. 3-14a. Note that we have reversed the order of the subscripts to correspond with Fig. 3-10. Let us choose Fig. 3-13b as the realization form resulting in Fig. 3-14b where R_{0n} = 0 and R' = R = 1 Ω. Frequency denormalizing this filter by $2\pi x10^3$ and impedance denormalizing by 10^4 (arbitrarily chosen) gives the realization of Fig. 3-14c.

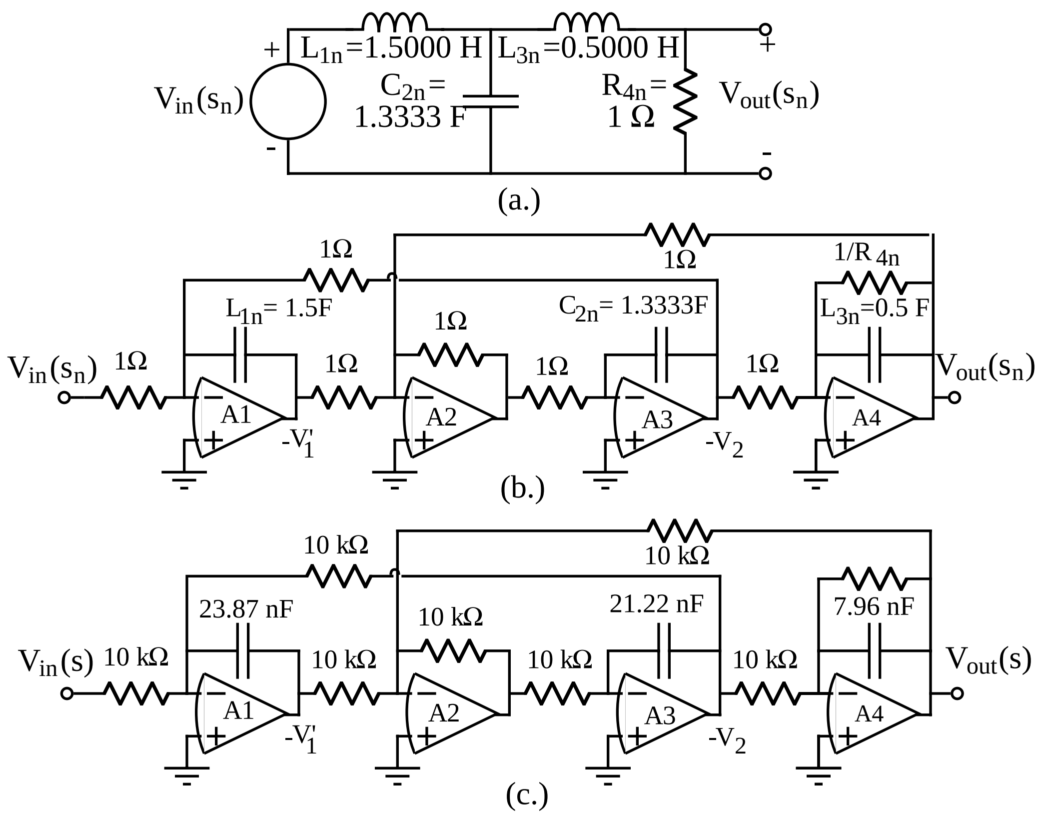

Figure 3-14 - Results of Ex. 3-6. (a.) Normalized, low-pass RLC filter. (b.) Normalized, low-pass, active-RC filter. (c.) Denormalized, low-pass, active-RC filter.

Example 3-7 - A Fifth-Order, 1 dB Chebyshev, Low-Pass, Active-RC Filter Design

A fifth-order, low-pass, Chebyshev filter with a 1 dB ripple in the passband is to be designed for a cutoff frequency of 1 kHz. Design an active-RC ladder realization of this filter. Give a schematic and the value of all components. The input voltage source is assumed to have a source resistance of 1 kΩ.

Solution

From Table 3-2, we get the normalized, low-pass, RLC ladder filter elements of R_{0n} = 1 Ω, L_{1n} = 2.1349 H, C_{2n} = 1.09011 F, L_{3n} = 3.0009 H, C_{4n} = 1.09011, L_{5n} = 2.1349 H, and R_{6n} = 1 Ω. Note that we have reversed the order of the subscripts in Table 3-2 to correspond to that of Fig. 3-10 (although in this particular case it makes no difference). Let us again select Fig. 3-13b as the realization because it has one less op amp. The component values above are directly substituted into Fig. 3-13b to achieve a normalized, low-pass, active-RC, ladder realization. To get the denormalized filter, we select R = 1000 Ω and frequency denormalize by $2\pi x10^3$. The values of the capacitor of each integrator, from input to output are $C_1 = L_{1n}/2\pi x10^6 = 2.1349/2\pi x10^6 = 0.340$ μF, $C_2 = C_{2n}/2\pi x10^6 = 1.09011/2\pi x10^6 = 0.174$μF, $C_3 = L_{3n}/2\pi x10^6 = 3.0009/2\pi x10^6 = 0.478$ μF, $C_4 = C_{4n}/2\pi x10^6 = 1.09011/2\pi x10^6 = 0.174$μF, and $C_5 = L_{5n}/2\pi x10^6 = 2.1349/2\pi x10^6 = 0.340$ μF.

Other Types of Ladder Filters using Active-RC Elements

High-pass, bandpass, and bandstop ladder filters can also be designed using active-RC elements. The design methods follow a simular procedure as for the low-pass filter. Unfortunately, in the high-pass ladder filters, the state variables are realized by differentiators rather than integrators if the state variables are the currents in the series elements and the voltages across the shunt elements. Integrator realization of the state variables is possible if the state variables are the voltages across the series elements and the currents through the shunt elements. However, the equations are not as straight-forward to write as for the low-pass case. Examples of high-pass ladder filter design can be found in other references†.

We will illustrate the design of bandpass ladder filters using active-RC elements because it is a straight-forward extension of the low-pass ladder filter design. Once a normalized, low-pass ladder filter has been found which meets the bandpass filter specifications, we apply the element transformations illustrated in Fig. 3-8. Thus, is the j-th series element consists of a series resistor, R_{jn}, and a series inductor, L_{jln}, become the series RLC circuit shown in Fig. 3-15a. If the k-th shunt element consists of a shunt resistor, R_{kn}, and a shunt capacitor, C_{kln}, the bandpass equivalent is the parallel RLC circuit shown in Fig. 3-15b. Normally, R_{jn} (R_{kn}) is zero (infinity) except for the series or shunt elements which include the terminating resistors of the ladder filter. The values of the series normalized bandpass components are

$$L_{jbn} = \left(\frac{\omega_r}{BW}\right) L_{jln} , \quad C_{jbn} = \left(\frac{BW}{\omega_r}\right)\frac{1}{L_{jln}} \quad . \tag{3-44}$$

The values of the shunt normalized components are

$$C_{kbn} = \left(\frac{\omega_r}{BW}\right) C_{jln} , \quad L_{jbn} = \left(\frac{BW}{\omega_r}\right)\frac{1}{C_{jln}} \quad . \tag{3-45}$$

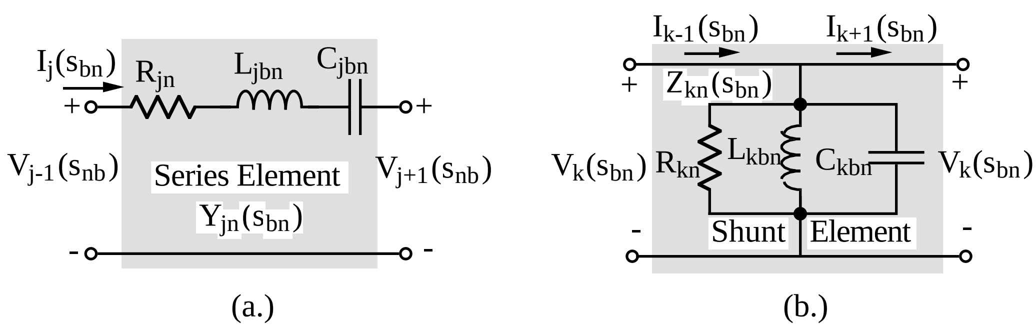

Figure 3-15 - (a.) Series ladder element and (b.) shunt ladder element after the normalized, low-pass to normalized, bandpass transformation of Fig. 3-8.

† P.E. Allen and E.Sá nchez-Sinencio, *Switched Capacitor Circuits*, Chapt. 4, Van Nostrand Reinhold Company, Inc., New York, 1984.

Each element of the bandpass filter can be realized by considering the admittance, Y_{jn}, of Fig. 3-15a and the impedance, Z_{kn}, of Fig. 3-15b. These driving-point functions can be written as

$$Y_{jn}(s_{bn}) = \frac{I_j(s_{bn})}{V_j(s_{bn})} = \frac{I_j(s_{bn})}{V_{j-1}(s_{bn})-V_{j+1}(s_{bn})} = \frac{\left(\frac{1}{L_{jbn}}\right)s_{bn}}{s_{bn}^2 + \left(\frac{R_{jn}}{L_{jbn}}\right)s_{bn} + \frac{1}{L_{jbn}C_{jbn}}} \qquad (3\text{-}46)$$

and

$$Z_{kn}(s_{bn}) = \frac{V_k(s_{bn})}{I_k(s_{bn})} = \frac{V_k(s_{bn})}{I_{k-1}(s_{bn})-I_{k+1}(s_{bn})} = \frac{\left(\frac{1}{C_{kbn}}\right)s_{bn}}{s_{bn}^2 + \left(\frac{1}{R_{kn}C_{kbn}}\right)s_{bn} + \frac{1}{L_{jbn}C_{jbn}}} \;. \qquad (3\text{-}47)$$

These driving-point functions can be turned into voltage transfer functions using the concept of voltage analogs for the currents I_j and I_k. Multiplying Eq. (3-46) by R' gives

$$T_{Yj}(s_{bn}) = \frac{R'I_j(s_{bn})}{V_j(s_{bn})} = \frac{V_j'(s_{bn})}{V_{j-1}(s_{bn})-V_{j+1}(s_{bn})} = \frac{\left(\frac{R'}{L_{jbn}}\right)s_{bn}}{s_{bn}^2 + \left(\frac{R_{jn}}{L_{jbn}}\right)s_{bn} + \frac{1}{L_{jbn}C_{jbn}}} \qquad (3\text{-}48)$$

or

$$V_j'(s_{bn}) = \left(\frac{\left(\frac{R'}{L_{jbn}}\right)s_{bn}}{s_{bn}^2 + \left(\frac{R_{jn}}{L_{jbn}}\right)s_{bn} + \frac{1}{L_{jbn}C_{jbn}}}\right)[V_{j-1}(s_{bn})-V_{j+1}(s_{bn})] \;. \qquad (3\text{-}49)$$

If $R_{jn} = 0$, then Eq. (3-49) reduces to

$$V_j'(s_{bn}) = \left(\frac{\left(\frac{R'}{L_{jbn}}\right)s_{bn}}{s_{bn}^2 + \frac{1}{L_{jbn}C_{jbn}}}\right)[V_{j-1}(s_{bn})-V_{j+1}(s_{bn})] \;. \qquad (3\text{-}50)$$

Multiplying Eq. (3-47) by 1/R' gives

$$T_{Zk}(s_{bn}) = \frac{V_k(s_{bn})}{R'I_k(s_{bn})} = \frac{V_k(s_{bn})}{V_{k-1}'(s_{bn})-V_{k+1}'(s_{bn})} = \frac{\left(\frac{1}{R'C_{kbn}}\right)s_{bn}}{s_{bn}^2 + \left(\frac{1}{R_{kn}C_{kbn}}\right)s_{bn} + \frac{1}{L_{jbn}C_{jbn}}} \qquad (3\text{-}51)$$

or

$$V_k(s_{bn}) = \left(\frac{\left(\frac{1}{R'C_{kbn}}\right)s_{bn}}{s_{bn}^2 + \left(\frac{1}{R_{kn}C_{kbn}}\right)s_{bn} + \frac{1}{L_{jbn}C_{jbn}}} \right) [V'_{k-1}(s_{bn}) - V'_{k+1}(s_{bn})] \; . \qquad (3\text{-}52)$$

If $R_{kn} = \infty$, then Eq. (3-52) reduces to

$$V_k(s_{bn}) = \left(\frac{\left(\frac{1}{R'C_{kbn}}\right)s_{bn}}{s_{bn}^2 + \frac{1}{L_{jbn}C_{jbn}}} \right) [V'_{k-1}(s_{bn}) - V'_{k+1}(s_{bn})] \; . \qquad (3\text{-}53)$$

The second-order, bandpass, Tow-Thomas filter of Fig. 1-23 is a perfect realization of Eq. (3-49) or Eq. (3-52). The modification necessary is to add another input which is simple to do because the op amps have their positive input terminals grounded. Fig. 3-16 shows the general second-order realization of either Fig. 3-15a for the j-th series elements. The realization of the k-th shunt element is exactly the same except for the subscript j. For the cases of Eq. (3-50) and Eq. (3-53), $R_{j4} = \infty$. The availability of both positive and negative outputs simplifies the bandpass realization over the previous low-pass realizations.

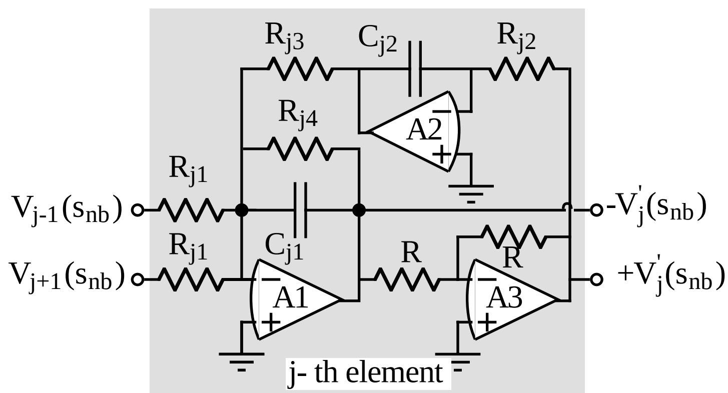

Figure 3-16 - Active-RC realization of either Fig. 3-15a or Fig. 3-15b (replace subscript j by k and interchange primes).

Table 3-3 shows the design relationships for the implementation of Fig. 3-15 by the Tow-Thomas circuit of Fig. 3-16. An example will illustrate the approach to designing active-RC bandpass, ladder filters.

Parameters of Fig. 3-16	Design of Fig. 3-15a	Design of Fig. 3-15b
R_{j1} or R_{k1}	$R_{j1} = \frac{R}{R'}\sqrt{\frac{L_{jbn}}{C_{jbn}}}$	$R_{k1} = R'R\sqrt{\frac{C_{kbn}}{L_{kbn}}}$
R_{j4} or R_{k4}	$R_{j4} = \frac{R}{R_{jn}}\sqrt{\frac{L_{jbn}}{C_{jbn}}}$	$R_{k4} = RR_{kn}\sqrt{\frac{C_{kbn}}{L_{kbn}}}$
Choose $R_{j2} = R_{j3} = R$ or $R_{k2} = R_{k3} = R$, a convenient value: $C_{j1} = C_{j2}$ or $C_{k1} = C_{k2}$	$C_{j1} = C_{j2} = \frac{\sqrt{L_{jbn}C_{jbn}}}{R}$	$C_{k1} = C_{k2} = \frac{\sqrt{L_{kbn}C_{kbn}}}{R}$
Choose $C_{j1} = C_{j2} = C$ or $C_{k1} = C_{k2} = C$, a convenient value: $R_{j2} = R_{j3}$ or $R_{k2} = R_{k3}$	$R_{j2} = R_{j3} = \frac{\sqrt{L_{jbn}C_{jbn}}}{C}$	$R_{k2} = R_{k3} = \frac{\sqrt{L_{kbn}C_{kbn}}}{C}$
R' is the scaling resistance of the voltage analog concept and is normally 1 Ω		

Table 3-3 - Design relationships for the implementation of Fig. 3-15a or Fig. 3-15b by the active-RC filter of Fig. 3-16.

Example 3-8 - Design of a Butterworth, Bandpass, Ladder Active-RC Filter

Design a sixth-order, Butterworth, bandpass, doubly-terminated, ladder filter using active-RC elements. Assume the passband is an octave, centered geometrically about 1 kHz. Impedance denormalize by a factor of 10^4 and give a schematic, or equivalent, with all component values.

Solution

The first thing we must do is find the bandwidth of the bandpass filter. We know from the specification that $f_{PB2} = 2f_{PB1}$. We also know that 1 kHz = $\sqrt{f_{BP2}f_{BP1}}$. Combining these 2 relationships gives $f_{BP1} = 1/\sqrt{2}$ kHz and $f_{PB2} = \sqrt{2}$ kHz. Thus the bandwidth is $1/\sqrt{2}$ kHz.

From Table 3-2, for a third-order, doubly-terminated Butterworth approximation we find that L_{1n} = 1 H, C_{2n} = 2 F, and L_{3n} = 1 H. Since the filter is symmetrical, we don't have to reorder the subscripts but can use them as they are. The resulting, normalized, low-pass filter is shown in Fig. 3-17a.

If we apply the normalized, low-pass to normalized, bandpass transformation of Eqs. (3-44) and (3-45) to Fig. 3-17a we get the following normalized bandpass elements of Fig. 3-17b. $L_{1bn} = (\omega_r/BW)L_{1ln} = (R(2))(1) = \sqrt{2}$ H, $C_{1bn} = (BW/\omega_r/)(1/L_{1ln}) = 1/\sqrt{2}$ F, $C_{2bn} = (\omega_r/BW)C_{2ln} = (\sqrt{2})(2) = 2\sqrt{2}$ F, $L_{2bn} = (BW,\omega_r)(1/C_{2ln}) = 1/2\sqrt{2}$ H, $L_{3bn} = (\omega_r/BW)L_{3ln} = (\sqrt{2})(1) = \sqrt{2}$ H, and $C_{3bn} = (BW/\omega_r)(1/L_{3ln}) = 1/\sqrt{2}$ F. Therefore the values of Y_{1n} are R_{1n} = 1 Ω, $L_{1bn} = \sqrt{2}$ H, and $C_{1bn} = 1/\sqrt{2}$ F. The values for Z_{2n} are $R_{2n} = \infty$, $C_{2bn} = 2\sqrt{2}$ F, and $L_{2bn} = 1/2\sqrt{2}$ H. The values for Y_{3n} are R_{3n} = 1 Ω, $L_{3bn} = \sqrt{2}$ H, and $C_{3bn} = 1/\sqrt{2}$ F.

Finally, we use the design relationships of Table 3-3 to design three, second-order active filters of the form given in Fig. 3-16. We will select R = R' = 1 Ω. For the first element, Y_{1n}, which is series we choose $C_{11} = C_{12}$ = 1 F to get $R_{12} = R_{13}$ = 1 Ω, and $R_{11} = R_{14} = \sqrt{2}$ Ω. For the second element, Z_{2n}, which is shunt we choose $C_{21} = C_{22}$ = 1F to get $R_{22} = R_{23}$ = 1 Ω, $R_{21} = 2\sqrt{2}$ Ω, and $R_{24} = \infty$. For the third element, Y_{3n}, the values are identical to those for Y_{n1}. If we frequency denormalize by $2\pi x10^3$ and impedance denormalize by 10^4 we get the realization of Fig. 3-17c.

The above example illustrates the general method by which bandpass ladder filters are realized using active-RC elements. This method is applicable to any bandpass filter whose passband and stopband are geometrically centered about the frequency, ω_r. Other examples of active-RC bandpass ladder filter design can be found in the problems.

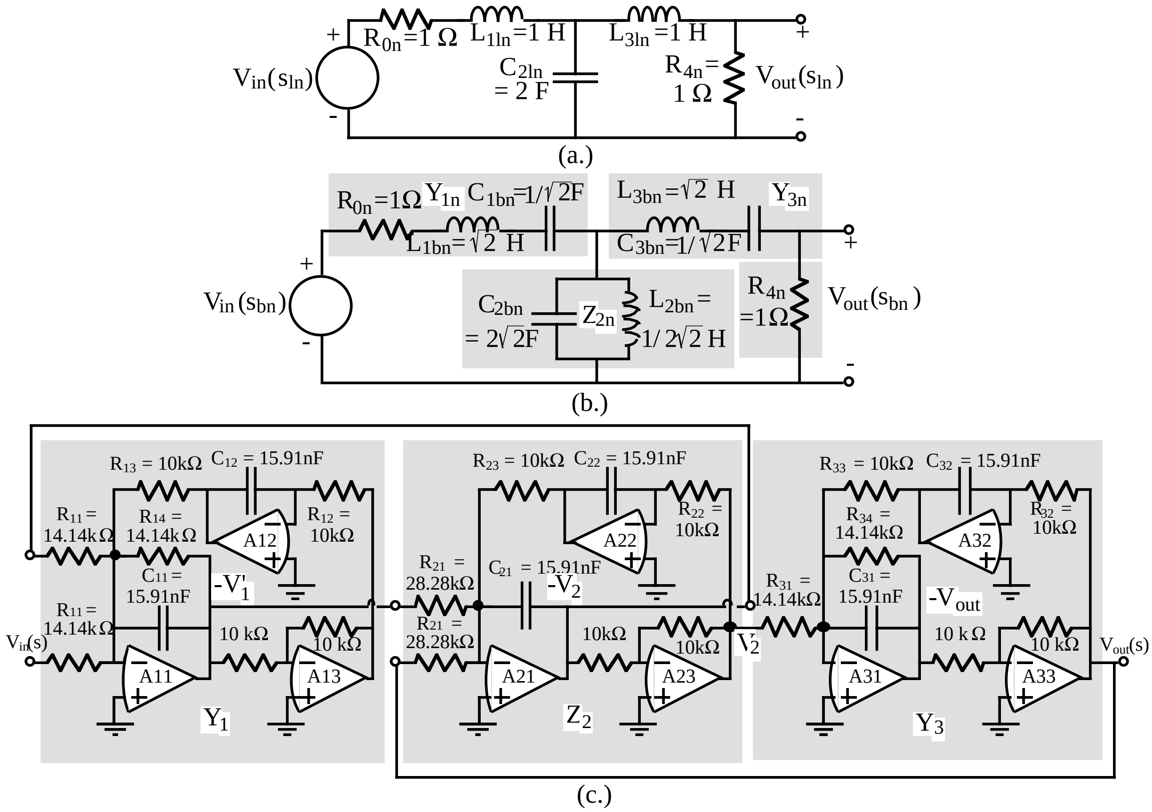

Figure 3-17 - Bandpass, active-RC ladder filter of Ex. 3-8. (a.) Normalized, low-pass, ladder filter. (b.) Normalized, bandpass ladder filter. (c.) Denormalized, active-RC bandpass ladder filter.

Summary

The block diagram in Fig. 2-1 illustrated the two general design approaches normally used to design active filters. The first approach was the cascade of first- or second-order stages and was covered in the previous section. The second approach was the design of ladder filters which has been the subject of this section. Basically, the cascade approach is easier to tune but is more sensitive to component tolerance than the ladder filters.

The starting point of the ladder active filter design is with the normalized, low-pass RLC ladder filter. These filter structures are the result of network synthesis methods and are widely tabulated for the designer. As in Sec. 2, we restricted ourselves to filters whose zeros were at infinity or the origin of the complex frequency plane. The design of high-pass and bandpass filters was achieved by direct application of the normalized frequency transformations to the elements of the normalized, low-pass RLC filter.

To achieve an active-RC realization of the RLC ladder filters, we introduced the concept of state variable. By correctly selecting the state variables and using the concept of voltage analog for current, we were able to synthesize the state variables using summing integrators. The resulting circuits were very similar to interconnected, second-order, low-pass Tow-Thomas filters. We saw that high-pass, active-RC ladder filters required a little cleverness to be able to express all of the state variables so that they could be realized by integrators. Each element of the normalized, low-pass RLC resulted in a second-order bandpass circuit when the normalized low-pass to normalized bandpass frequency transformation was applied. These second-order, bandpass circuits were easily realizable as second-order, bandpass, Tow-Thomas filters.

One of the themes that hopefully has become evident to the reader is how filter design all builds from the normalized, low-pass filters. Using simple normalizations and frequency transformations permits the design of complex filters. One important constraint to remember is that the bandpass and bandstop filters designed by these methods must have passbands and stopbands geometrically centered about a common frequency. Other design methods, not covered here, will allow the design of filters not subject to this constraint.

Some the important points of this section are summarized below for the convenience of the reader.

- Sensitivity is a measure of some aspect of the filter performance on the components of the filter.
- The normalized low-pass to normalized high-pass frequency transformation turns inductors (capacitors) into capacitors (inductors) whose value is the reciprocal of the low-pass component value.
- The normalized low-pass to normalized bandpass frequency transformation turns an inductor into an inductor in series with a capacitor. The value of the inductor is

$(\omega_r/BW)L_{ln}$ and the value of the capacitor is $(BW/\omega_r L_{ln})$. whose value is (inductors) whose value is the reciprocal of the low-pass component value.

- The normalized low-pass to normalized bandpass frequency transformation turns a capacitor into a capacitor in parallel with an inductor. The value of the capacitor is $(\omega_r/BW)C_{ln}$ and the value of the inductor is $(BW/\omega_r C_{ln})$.
- In order to realize the state variables by integrators, the current through an inductor and the voltage across a capacitor should be chosen as state variables.
- The Tow-Thomas second-order realization becomes a very useful active-RC filter for realizing RLC ladder filters.