INTEGRATED CIRCUIT CONTINUOUS TIME FILTERS

Outline - Sections

SECTION 1 - INTRODUCTION TO CONTINUOUS TIME FILTERS

Integrated Filter Categories

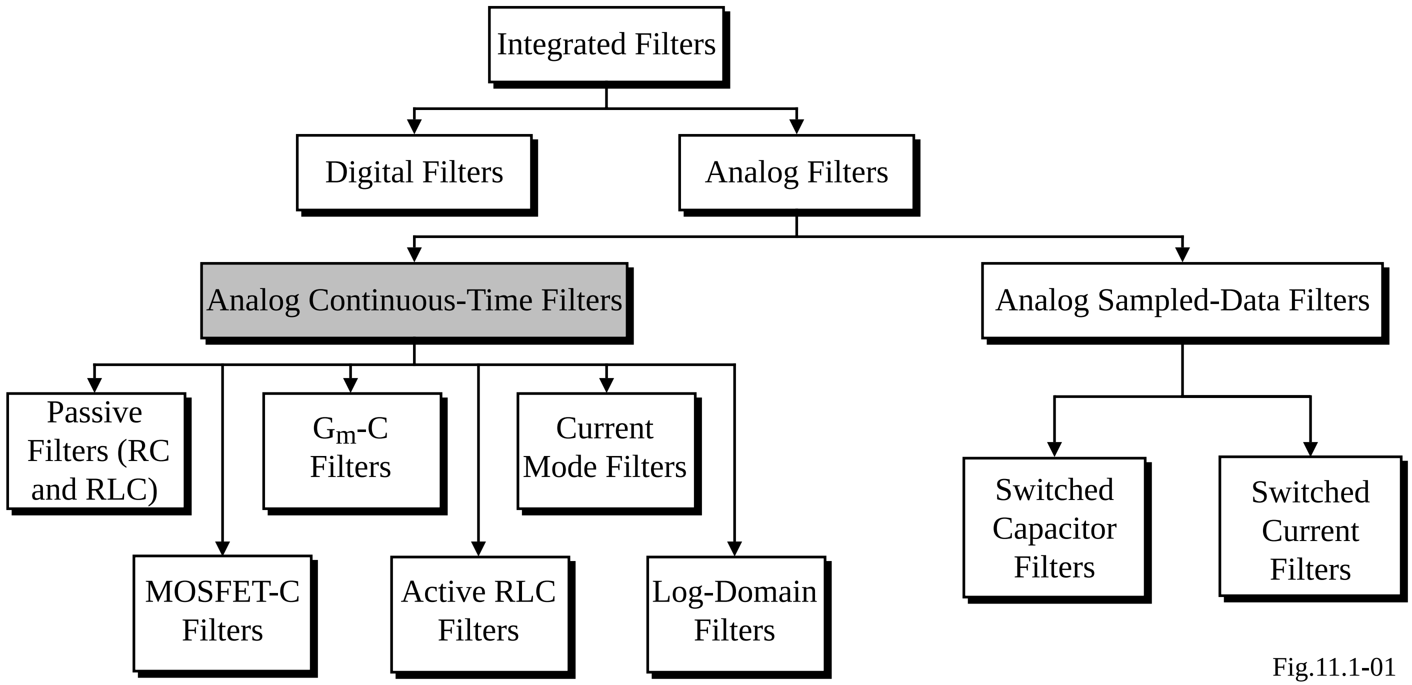


Fig.11.1-01

Types of Filters

- A continuous time filter is a filter whose variables are continuous both in time and in amplitude.
- A discrete-time (analog sampled-data) filter is a filter whose variables are continuous in amplitude but not in time.
- A digital filter is a filter whose variables are discrete both in time and in amplitude.

Example:

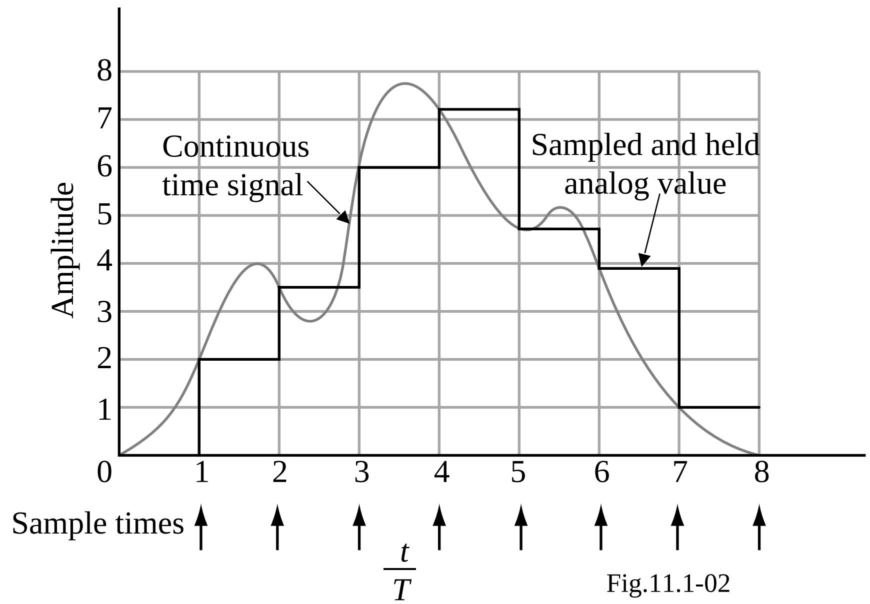


Fig.11.1-02

A digital signal would only have the amplitude values of 0, 1, 2, through 8.

Digital Filters

Current technology constrains digital filtering to 5-10MHz or less.

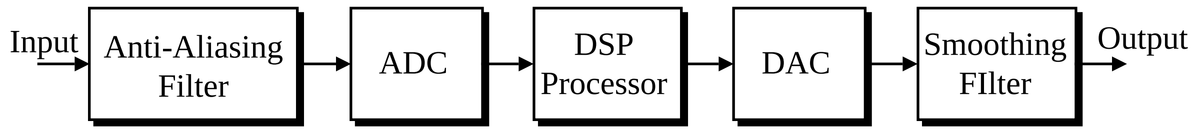


Fig. 10-5

Programmability is the key feature as well as unlimited resolution.

Practical problems include:

- The need for analog anti-aliasing filters
- Large chip area requirements
- Electromagnetic compatibility with low level analog signals
- Requirements for a high resolution, high speed analog to digital converter (ADC)
- Power consumption at high frequencies

Analog Continuous-Time Filters

Continuous-Time Filters

Advantages:

- No clock feedthrough
- No oversampling requirement
- Less dissipation than digital filters
- Higher frequency capability than switched capacitor filters
- No anti-aliasing filter required
- Uses a large body of classical filter theory and data

Disadvantages:

- Accuracy of the time constants must be achieved by tuning
- Linearity of resistor or transconductance implementations

Discrete-Time Filters

Advantages:

- Very accurate, tuning not required
- Switched capacitors are a well-known technique

Disadvantages:

- Clock feedthrough
- Limited in frequency by the oversampling ratio

- Requires an anti-aliasing filter

Dynamic Range of Filters

Dynamic Range:

$$DR = \frac{P_{max}}{P_N} = \frac{V_{sat}^2}{V_N^2}$$

where

P_{max} (V_{sat}) = maximum signal power (voltage) at the filter output (typically for 1% THD)

P_N (V_N) = integrated noise power (voltage) in the bandwidth of interest at filter output

Noise Performance:

6th-order continuous-time filter[1]-

$$V_N^2 = \frac{3kTQ}{C_{int}} \quad \Rightarrow \quad DR = \frac{V_{sat}^2 C_{int}}{3kTQ}$$

Optimum DR for continuous-time filters[2]

$$DR_{opt} = \frac{V_{sat}^2 C_{total}}{4kTFQ}$$

where

[1] H. Khorramabadi and P.R. Gray, "HIgh Frequency CMOS Continuous-Time Filters", *IEEE J. Solid-State Circuits*, pp. 939-948, Dec. 1984.

[2] G. Groenewold, "The Design of High Dynamic Range Continuous-Time Integratable Bandpass Filters," *IEEE Trans. Circuits and Systems,* pp. 838-852, Aug. 1991.

C_{total} = total capacitance of the filter's integrators

F = noise factor used to account for possible excess noise contributions by nonideal devices

Dynamic Range- Continued

Upper bound on *DR* for a general high-Q bandpass filter

$$DR_{opt} \le \frac{V_{sat}{}^2 C_{total}}{2\pi k T F Q}$$

The dependence of *DR* on power dissipation and bandwidth are[†]

$$DR_{opt} = \frac{\eta P_{diss}}{4\pi k T F B Q^2}$$

where

η = efficiency factor relating the power consumed by the filter to the maximum signal output power

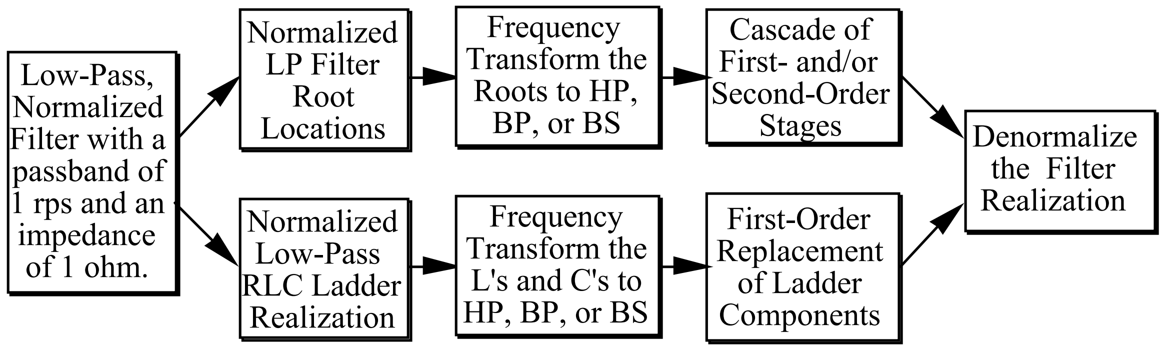
B = filter bandwidth

In general, the mechanism responsible for limiting the *DR* in high Q filters is the regenerative gain associated with the high-Q poles.

Typical *DR* ≈ 70-90dB depending upon the architecture (AGC's, etc.)

[†] W.B. Kuhn, "Design of Integrated, Low Power, Radio Receivers in BiCMOS Technologies," Ph.D. Dissertation, Electrical Engineering Dept., Virginia Polytechnic Institute and State University, Blacksburg, VA, 1995.

General Approach for Continuous and SC Filter Design



All designs start with a normalized, low pass filter with a passband of 1 radian/second and an impedance of 1Ω that will satisfy the filter specification.

1.) Cascade approach - starts with the normalized, low pass filter root locations.

2.) Ladder approach - starts with the normalized, low pass, *RLC* ladder realizations.

Follow-the-Leader Feedback Design (FLF)[†]

Besides the cascade and ladder approaches, there is a third approach called follow-the-leader feedback.

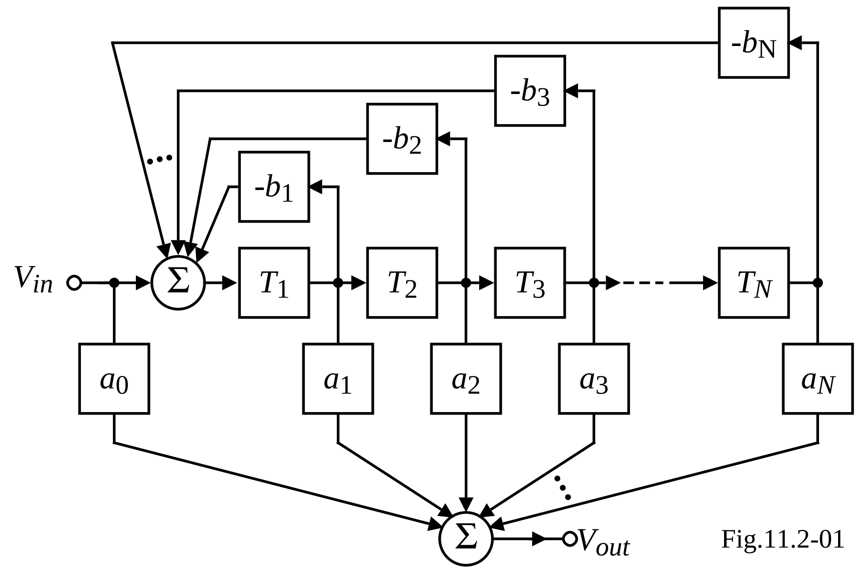


Fig.11.2-01

The various T_i are second-order bandpass transfer functions

This structure can realize both zeros and poles.

† A.S. Sedra and P.O Bracket, 1978, *Filter Theory and Design: Active and Passive*, London: Pitman.

Primary Resonator Block

Similar to FLF except that there is no feedforward paths and consequently it cannot realize complex zeros.

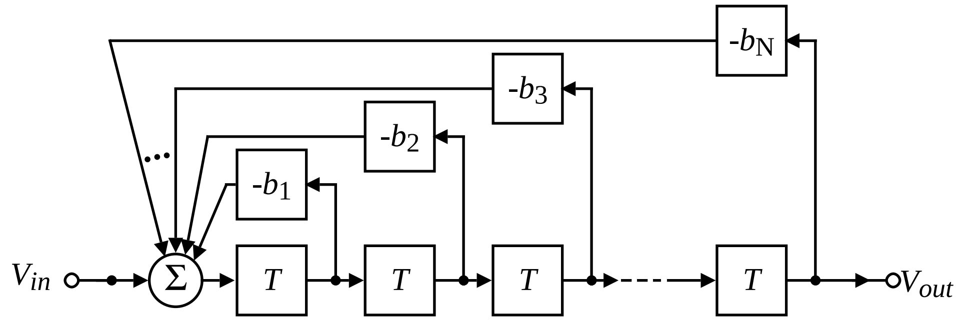


Fig.11.2-02

Easier to design and uses identical second-order blocks.

Denormalization of Filter Realizations

All filters are designed assuming a cutoff frequency of 1 radian/sec. and an impedance level of 1 ohm. In order to move the filter frequency to the desired frequency a denormalization must be performed. For active filters, the impedance denormalization is a free parameter that can be used to adjust the final values of the components.

Frequency and Impedance Denormalizations:

Denormalization	Denormalized Resistance, R	Denormalized Conductance, G_m	Denormalized Capacitor, C	Denormalized Inductor, L
Frequency: $s = \omega_{PB} s_n = \Omega_n s_n$	$R = R_n$	$G_m = G_{mn}$	$C = \dfrac{C_n}{\Omega_n}$	$L = \dfrac{L_n}{\Omega_n}$
Impedance: $Z = z_o Z_n$	$R = z_o R_n$	$G_m = \dfrac{G_{mn}}{z_o}$	$C = \dfrac{C_n}{z_o}$	$L = z_o L_n$
Frequency and Impedance: $Z(s) = z_o Z_n(\Omega_n s_n)$	$R = z_o R_n$	$G_m = \dfrac{G_{mn}}{z_o}$	$C = \dfrac{C_n}{z_o \Omega_n}$	$L = \dfrac{z_o L_n}{\Omega_n}$

Note that the design of switched capacitors is done in such a manner that denormalization is not necessary.

SECTION 2 - PASSIVE FILTERS

Comparison of Active and Passive Filters

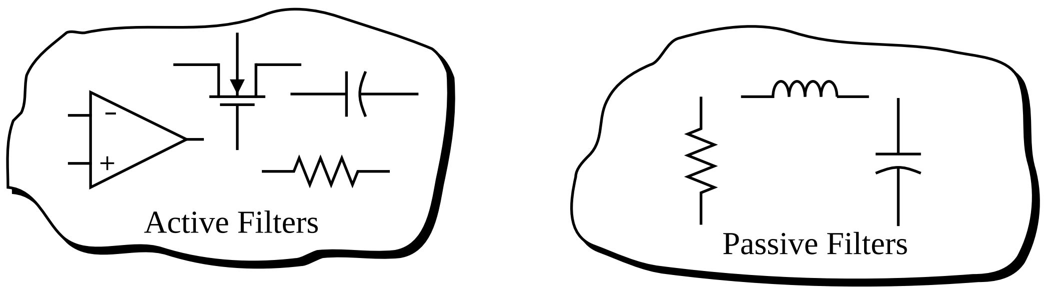


Fig.11.1-03

Characteristic	Active Filters	Passive Filters
Dissipation	Requires power	Dissipationless
Linearity	Limited	Linear
Noise	Active plus thermal	Noiseless (except for R's)
Size	Small	Large
Compatibility with integration	Good	Poor
Midband gain	Not constrained	Equal to less than unity

Passive Filters

Categories:

Discrete ceramic (piezoelectric)

Crystal

Acoustic wave

Surface (SAW)

Bulk (BAW)

LC

General Characteristics:

- Fractional bandwidths are small (0.1% to 3%)
- Shape factors are moderate (16 to 20dB of attenuation at 2 to 3 times the nominal bandwidth)
- Insertion loss is moderate (1.5 to 6dB)
- Cost is low ($0.3 to $3) when purchased in large quantities
- No power dissipation
- Low noise figure

Passive Filter Performance

Part No.	Type	Application	Frequency	BW	Shape Factor	IL	Price
Toko HCFM8-262B	Ceramic	AM Broad-cast IF	262 kHz	6kHz (2.3%)	-16dB @ ±9kHz	6dB	$1
Toko CFMR-455B	Ceramic	AM Broad-cast IF	455kHz	6kHz (1.3%)	-16dB @ ±9kHz	6dB	$1
MuRata SFP450F	Ceramic	Pager IF	450kHz	6kHz (1.3%)	-40dB @ ±12kHz	6dB	-
MuRata SFE4.5MBF	Ceramic	Television Sound IF	4.5MHz	120kHz (2.7%)	-20dB @ ±270kHz	6dB	-
MuRata	Ceramic	FM Broad-cast IF	10.7MHz	230kHz (2.1%)	-20dB @ ±290kHz	6dB	$0.3
ECS-10.7-15B	MCF	Cellular Phone IF	10.7MHz	25kHz (0.2%)	-40dB @ ±25kHz	2.5dB	$3
Siemens B4535	SAW	DECT IF	110MHz	1.1MHz (1%)	-20dB @ ±1.5MHz	-	$3
MuRata LFC30-01B0881B025	LC	Cellular RF	881MHz	25MHz (2.8%)	-20dB @ ±78MHz	3.5dB	-
Toko 6DFA-881E-11	Dielectric	Cellular RF	881MHz	25MHz (2.8%)	-20dB @ ±78MHz	1.8dB	-

Toko 6DFA-914A-14	Dielectric	Cordless Phone RF	914MHz	1MHz (0.1%)	-24dB @ ±45MHz	2.2dB	-

Passive RLC Filters

Passive filters consist of:

- Inductors
- Capacitors

Realizations:

1.) Geometrically centered

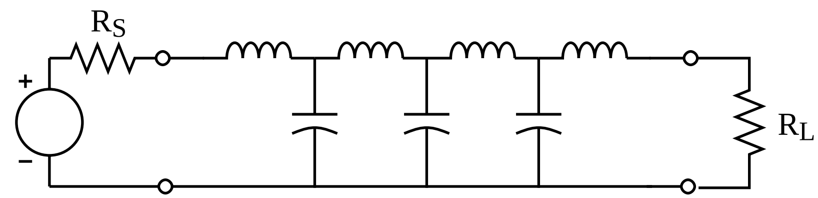
2.) Arbitrary (difficult)

Advantages of passive filters:

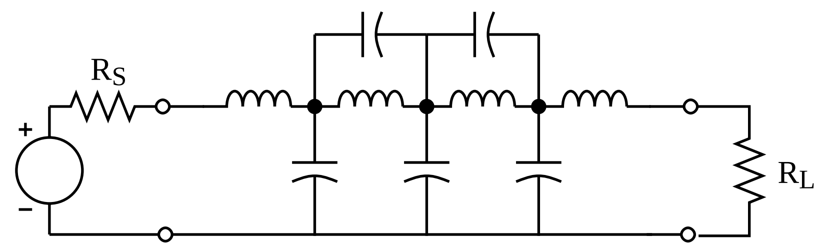
- Low noise (no noise)
- High frequency - up to the GHz range

Disadvantage of passive filters:

- Poor IC compatibility
- Difficult to tune



Lowpass Filter with zeros at infinity



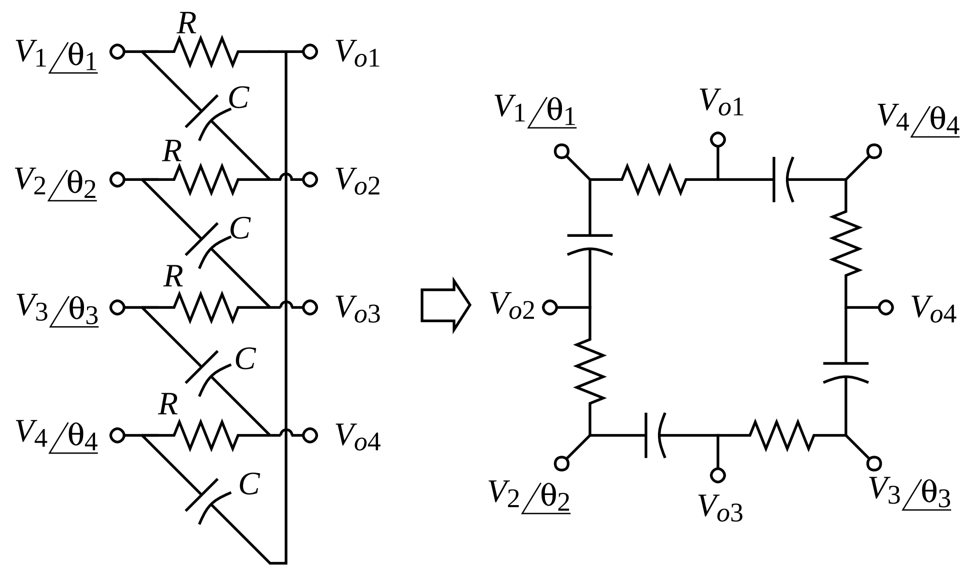
Lowpass Filter with jω zeros

Fig.12.6-4

RC Filters (Polyphase Filters)

A polyphase filter is a fully symmetric RC network with multiple inputs.

- Depending on the phase and amplitude relation of the inputs, it rejects some inputs and passes others.
- Every input vector set can be decomposed into the unit vector sets.



Input frequency = $\frac{1}{2\pi RC}$

Fig10-PF1

Lag: $T(j\omega) = \frac{1}{j\omega RC + 1}$

$Arg[j\omega\} = -\tan^{-1}(\omega RC)$

Lead: $T(j\omega) = \frac{j\omega RC}{j\omega RC + 1}$

$Arg[j\omega] = 90° - \tan^{-1}(\omega RC)$

RC Polyphase Filters - Continued

Polyphase filters will add the counter-clockwise phase sequences and reject the clockwise phase sequences.

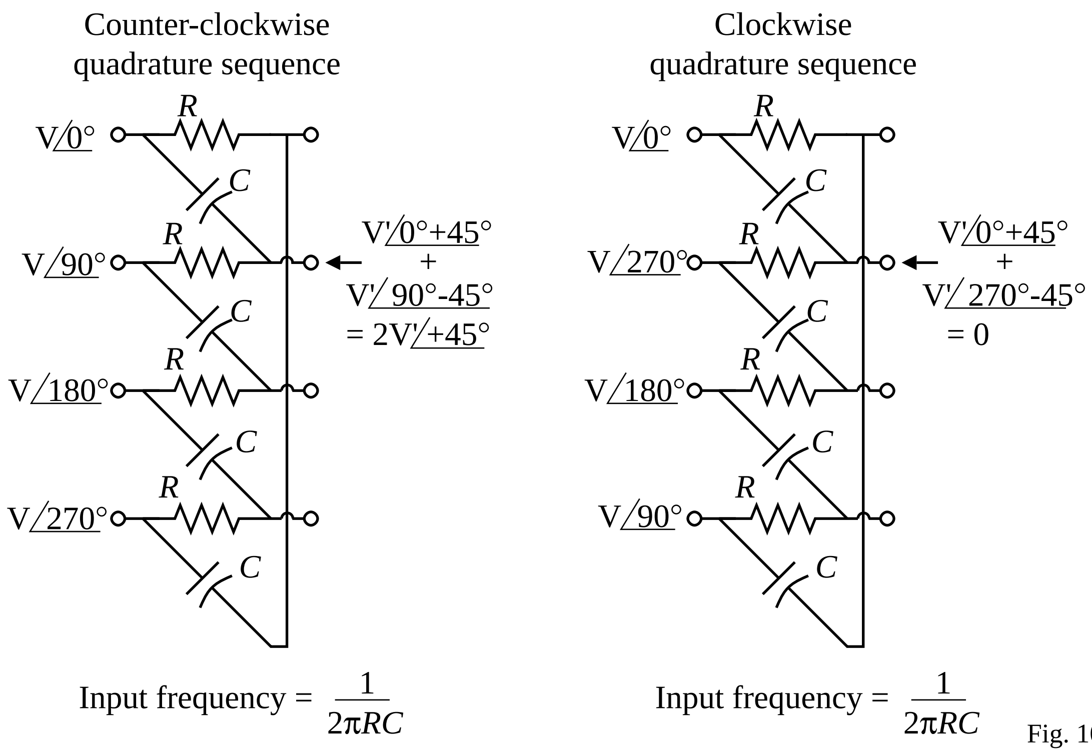


Fig. 10-PF2

Applications of Polyphase Filters

1.) Quadrature Signal Generation:

Quadrature signals are generated from a differential signal by means of the polyphase filter.

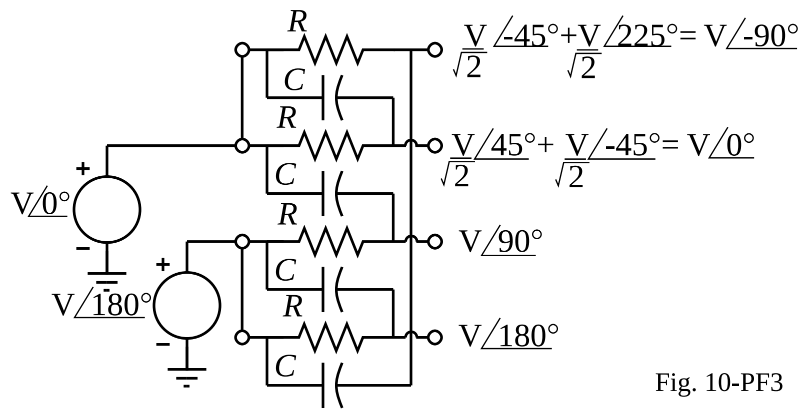


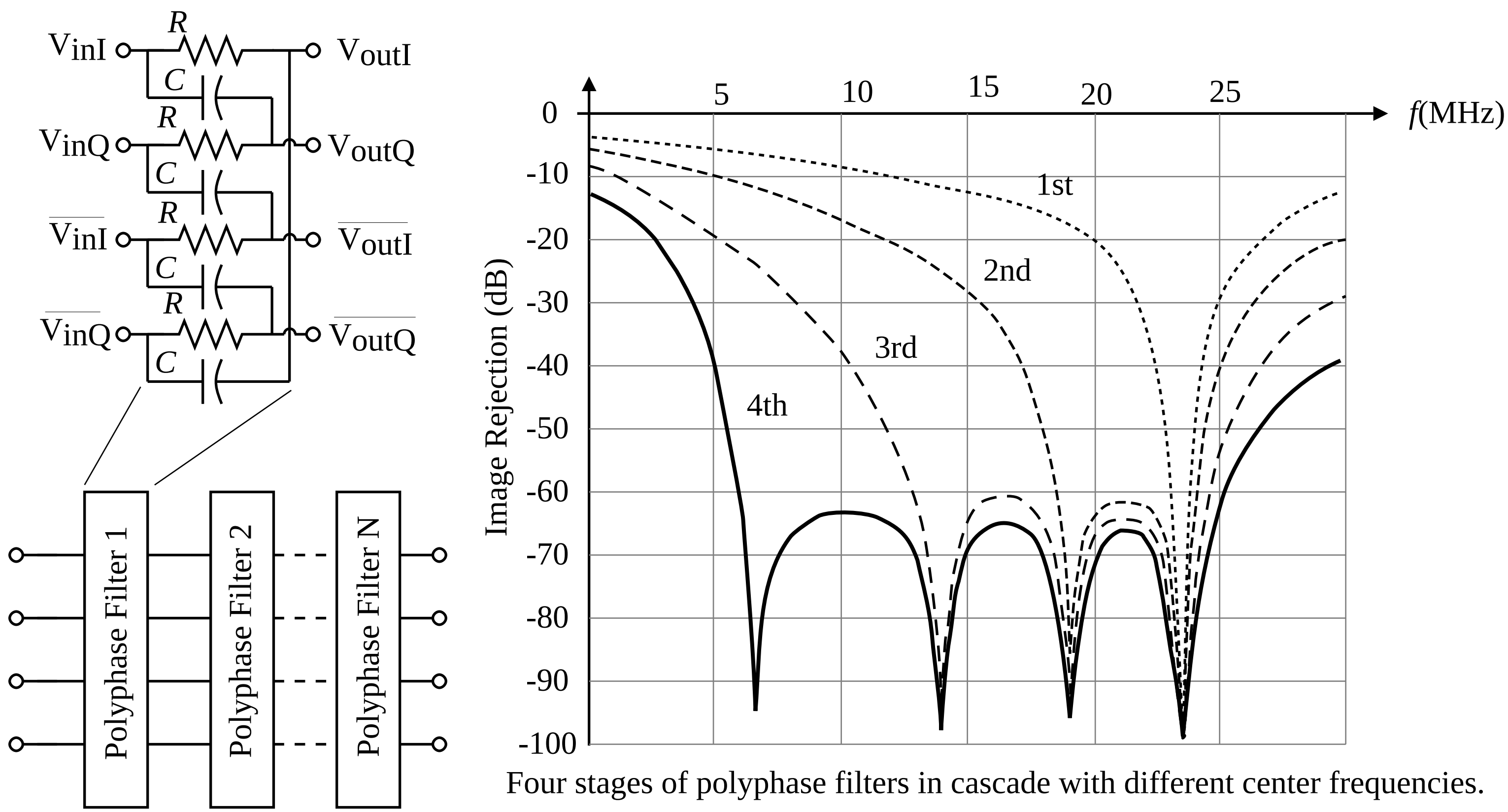
Fig. 10-PF3

2.) Unwanted Signal Rejection:

If the desired signal and the unwanted signal have opposite sequences, the unwanted signal can be rejected by the polyphase filter.

Wideband Polyphase Filters

Staggered polyphase filters:



Four stages of polyphase filters in cascade with different center frequencies.

Fig10-PF4

- Wideband image rejection can be obtained with staggering several polyphase stages
- Wider polyphase filter band ⇒ Large number of stages ⇒ More loss

Practical Design Issues of Polyphase Filters

1.) Component Matching:

From Monte Carlo simulations it has been found that:

Ultimate Image Rejection > $-20\cdot\log_{10}(\sigma_{Component})$

(For 60dB image rejection a 0.1% matching between R's and C's is required)

The matching requirements will set the minimum area of the R's and C's

Comments-

- In general, $\sigma^2_{Component} \propto \frac{1}{Area}$
- Measurements on poly resistors of a 1µm CMOS process gave $\sigma < 0.1\%$ for a resistor area of $2800\mu m^2$
- Approximate matching of capacitors is $\sigma \approx 0.1\%$ for a capacitor area of $220\mu m^2$

2.) Cutoff Frequency of the Resistors:

At high frequencies, the polysilicon acts as a transmission line and does not provide the desired phase shift.

$$f_{Resistor} = \frac{1}{2\pi L^2 R\_ C_{ox}}$$

where L is the length of the resistor and R_ is the sheet resistance of the polysilicon and C_{ox} is the oxide capacitance. This property of the resistors sets the maximum length of the resistors.

Practical Design Issues of Polyphase Filters - Continued

3.) Capacitive Loading Mismatch:

- Symmetric loading has no effect on the image rejection property of the polyphase filter.
- Low impedance loads (Z_{Load}) increases the polyphase filter loss.
- Matching between the output loads is required

  Normally, the load is either another polyphase filter which is sufficiently matched by the gate capacitance of the amplifier following the polyphase filter.

Rule of thumb from worst case simulations:

If $C_{Load} < C_{Poly}$, the ultimate image rejection $> -20\log_{10}\left(\frac{\sigma_{Cload} \cdot C_{Load}}{C_{Poly}}\right)$

If $C_{Load} > C_{Poly}$, the ultimate image rejection $> -20\log_{10}(\sigma_{Cload})$

Practical Design Issues of Polyphase Filters - Continued

4.) Differential Output instead of Quadrature Differential Output:

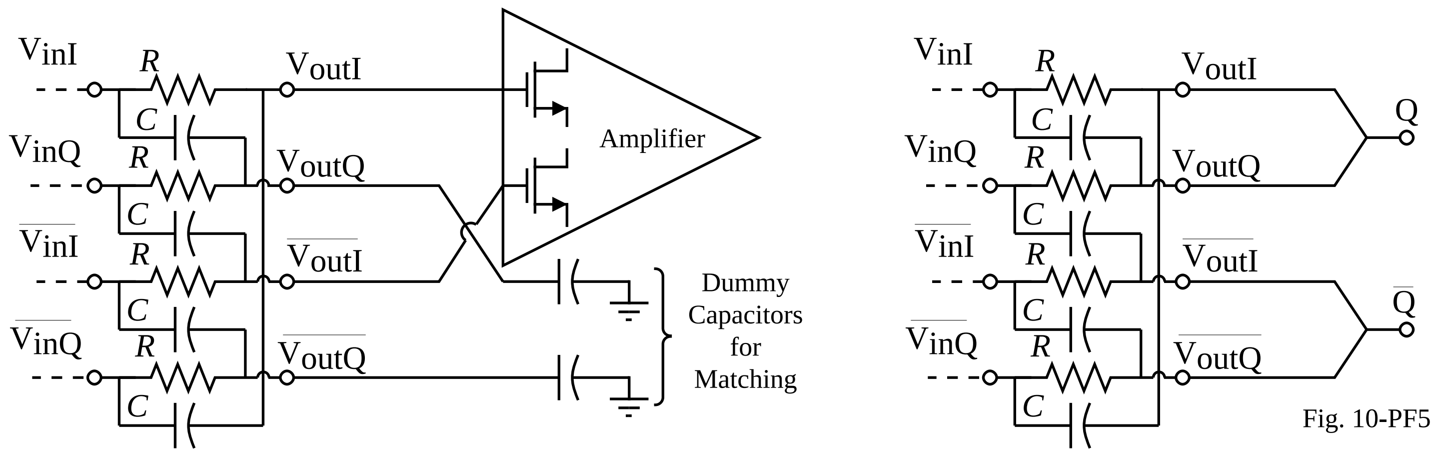


Fig. 10-PF5

- Last polyphase stage can provide up to 3dB of gain with small loading capacitors
- Dummy capacitors try to match the output loading on all branches
- Because of the matching issue, this is useful in high image rejection systems only if $C_{Load} << C_{Poly}$

- Outputs load each other, therefore the maximum gain is 0 dB
- This is an order of magnitude less sensitive to load mismatches if $C_{Load} << C_{Poly}$ and almost zero sensitivity to load mismatch if $C_{Load} > C_{Poly}$

Practical Design Issues of Polyphase Filters - Continued

5.) Polyphase Input and Output Impedance at Pole Frequency:

At the polyphase pole frequency, the input and output impedance of the polyphase filter is R||(1/sC) which is independent of the load and source impedance.

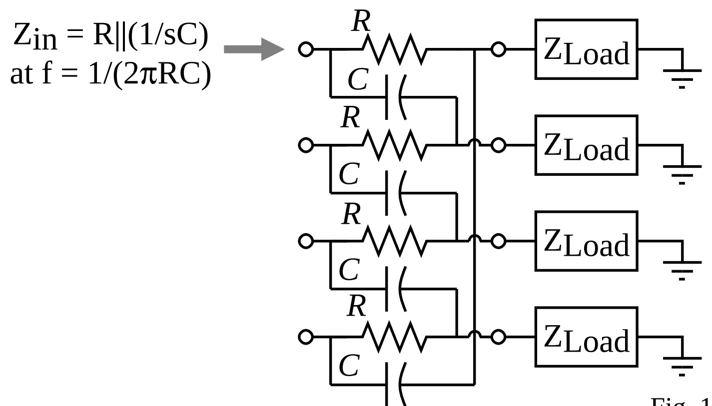


Fig. 10-PF6

6.) Input Impedance at DC:

If the load is a capacitor, then the input impedance at DC is high.

If the polyphase filter is driven by a current source a peak is experienced at low frequencies. (Should be driven by a low impedance source)

Practical Design Issues of Polyphase Filters - Continued

7.) Polyphase Noise:

- The resistors in the polyphase filters generate noise at the output ($\overline{v_n^2} = 4kTR_{poly}$)
- If several polyphase filters are in cascade, the noise of the previous stage is attenuated by the following. Therefore, the last stage noise becomes dominant.
- There is a tradeoff between the loading of the polyphase filter on the preceding stage (which sets the loss) and the noise of the polyphase filter which establishes the resistor.

8.) Polyphase Voltage Gain and Loss:

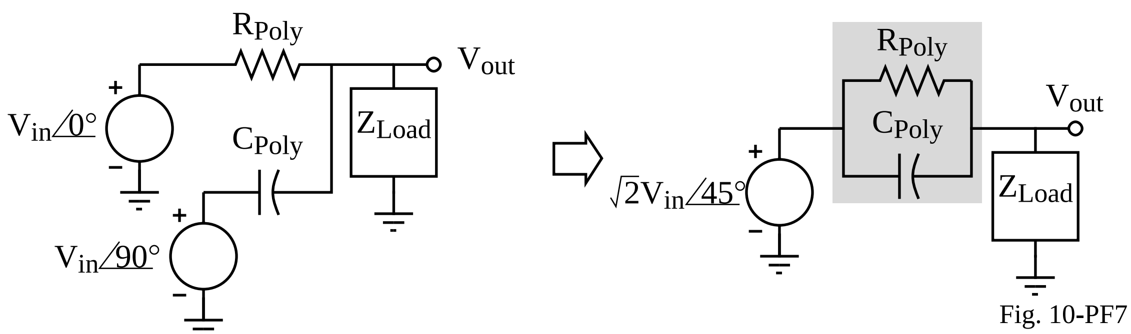


Fig. 10-PF7

Therefore,

$$|V_{out}| = \sqrt{2}\left|\frac{Z_{Load}}{Z_{Load}+Z_{Poly}}\right| |V_{in}|$$

a.) Output open circuit: $Z_{Load} >> Z_{Poly} \Rightarrow$ 3dB gain

b.) $Z_{Load} = Z_{Poly} \Rightarrow$ 3dB loss c.) At high frequencies, capacitive loads cause severe loss

Layout Issues for Polyphase Filters

- 60dB image rejection requires 0.1% matching among *R*'s and *C*'s of the polyphase filter.
- Therefore, careful layout is vital to preserve the matching property.

Comments:

1.) Common centroid layout is used to cancel the process gradients

2.) The dummy elements eliminate systematic mismatch

3.) Minimized total width of the layout further decreases the gradient.

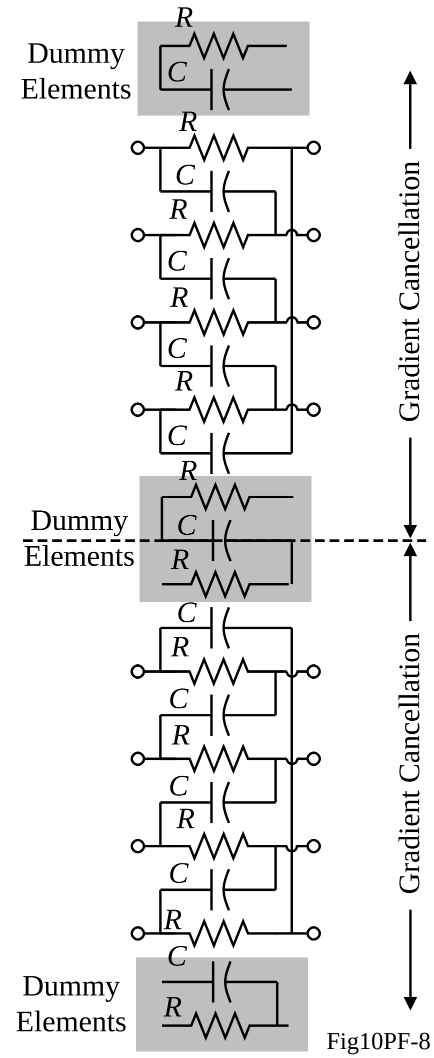


Fig10PF-8

Layout of Polyphase Filters - Continued

- Each interconnect line has the same length and the same number of corners using the following serpentine structure:

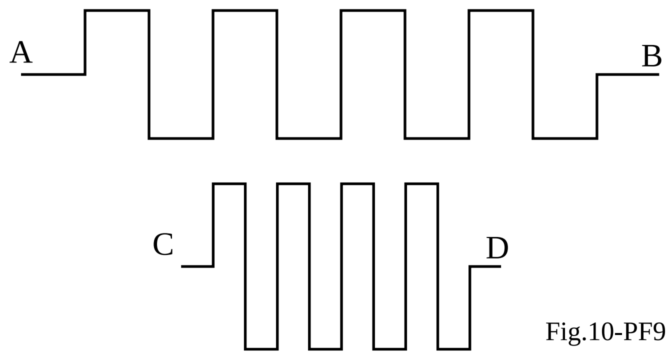


Fig.10-PF9

- The capacitance on interconnect lines is balanced.

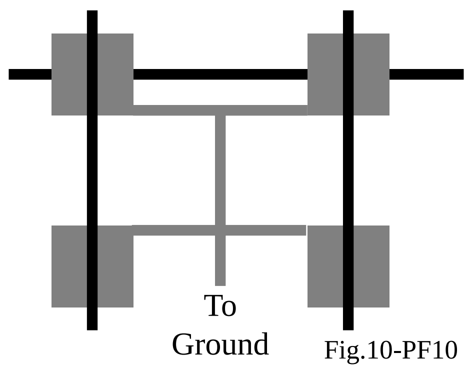


Fig.10-PF10

SECTION 3 - INTEGRATORS

The Role of the Integrator in Active Filters

In most active filters, the summing integrator is the key building block (the primitive).

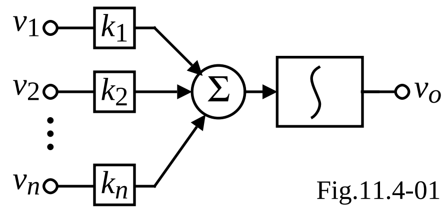


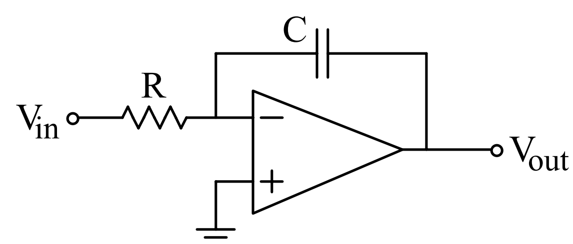
Fig.11.4-01

$$V_o(s) = \pm\frac{k_1}{s}V_1(s) \pm\frac{k_2}{s}V_2(s) \cdots \pm\frac{k_n}{s}V_n(s)$$

Classical realization:

$$H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{-1}{j\omega RC}$$

Damping Resistor



Integrator Nonidealities

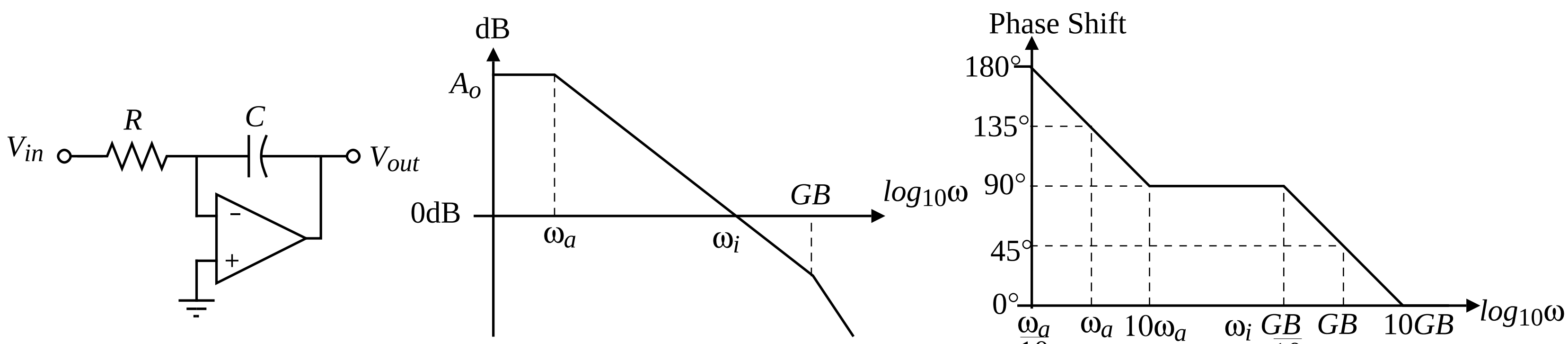


Fig10-10

The op amp is approximated as:

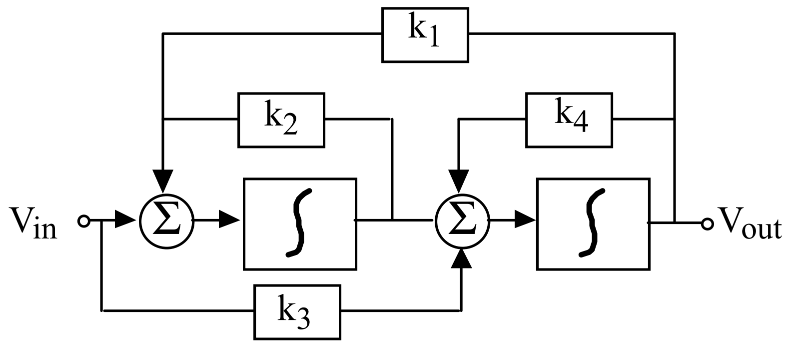
$$A(s) = \frac{A_o}{1 + \frac{s}{\omega_a}} = \frac{GB}{s + \omega_a}$$

The integrator behavior is degraded by:

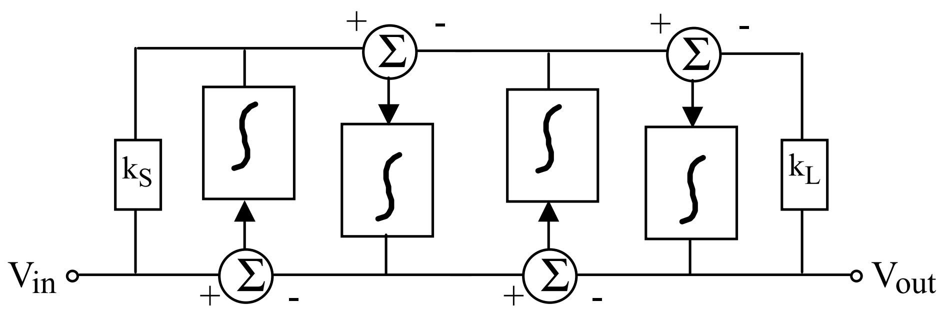
- Finite A_o in the low frequency region
- Finite GB in the high frequency region

Integrators - Continued

Biquad uses ±summing integrator with damping for second-order stages.

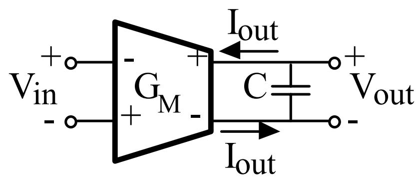


Leapfrog uses direct integrator implementation.



OTA-C (G_M-C) Integrators

Integrator:

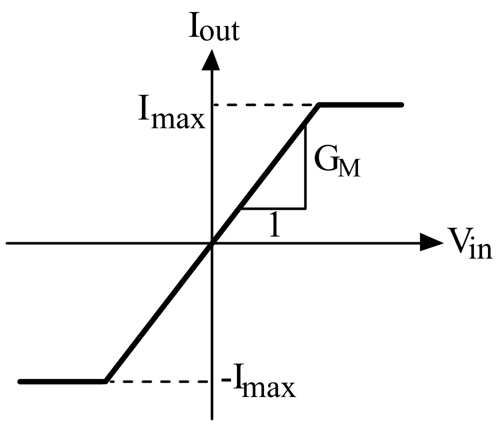


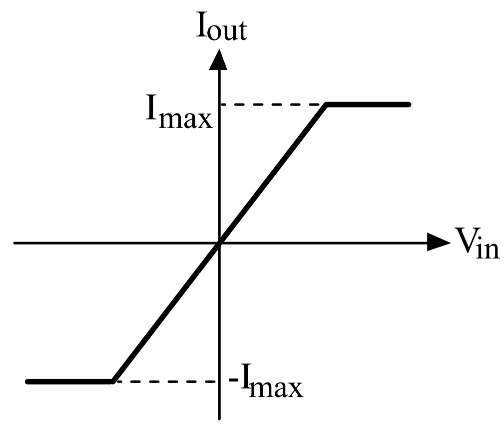
$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{G_M}{sC}$$

What is a G_M?

A transconductance amplifier with a linear relationship between I_{out} and V_{in}.

It must be tunable.





Advantages:

- High frequency
- Simple

Disadvantages:

- G_M must be linear and tunable
- Sensitive to parasitic capacitances at the output nodes

Differential output requires common mode feedback

Nonideal Performance of the OTA-C Integrator

We will consider the single-ended version for purposes of simplicity.

Sources of nonideal behavior are:

1.) Finite output resistance

2.) Frequency dependence of G_m

3.) RHP zero due to feedforward (found in Miller compensation)

Finite Output Resistance of the OTA-C Integrator

Model:

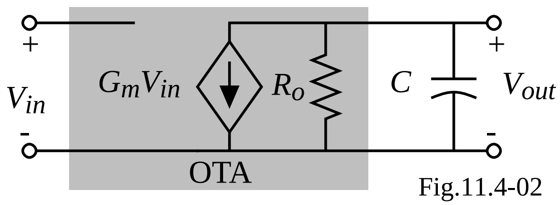


Fig.11.4-02

$$\frac{V_{out}}{V_{in}} = \frac{\frac{-G_mR_o}{sC}}{R_o + \frac{1}{sC}} = \frac{-G_mR_o}{sCR_o+1}$$

Frequency Response:

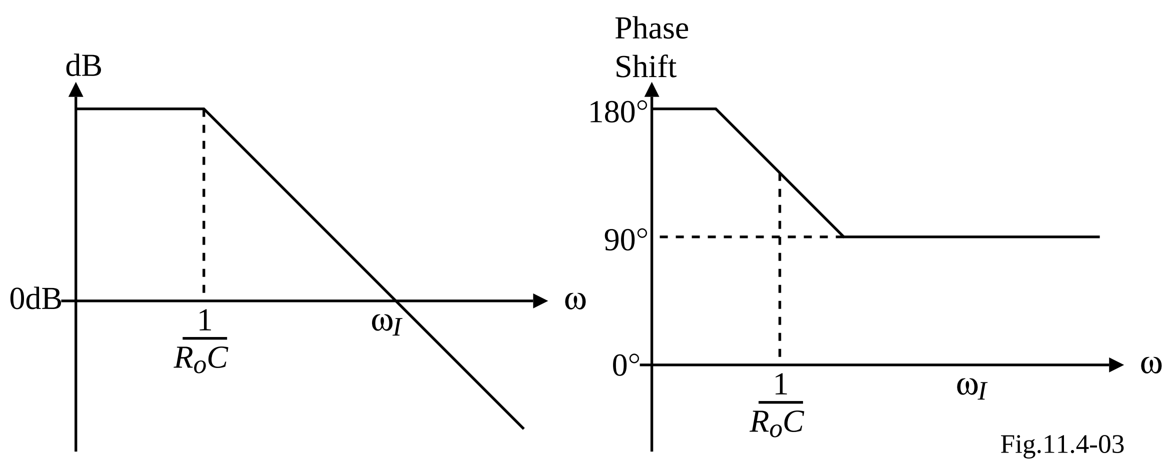


Fig.11.4-03

Solution:

Make R_o large by using cascode configuration

Finite Bandwidth of the OTA-C Integrator

Model:

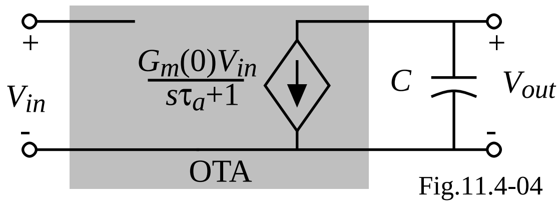


Fig.11.4-04

$$\frac{V_{out}}{V_{in}} = \frac{-G_m(0)}{sC(s\tau_a,+1)}$$

Frequency Response:

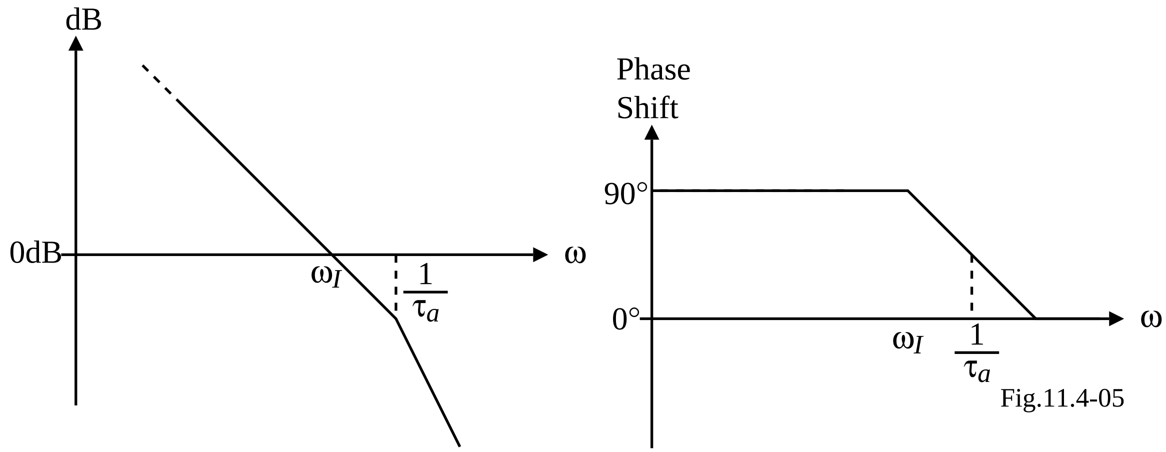


Fig.11.4-05

Solution:

Use higher f_T MOSFETs (smaller channel lengths)

RHP of the OTA-C Integrator

Model:

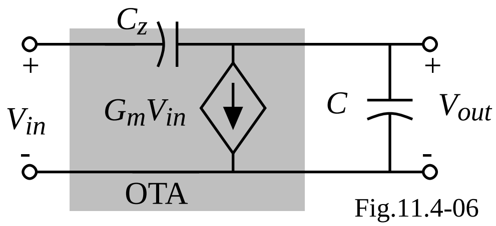


Fig.11.4-06

$$\frac{V_{out}}{V_{in}} = \frac{-G_m - sC_z}{s(C+C_z)} \approx \frac{-(G_m - sC_z)}{sC}$$

Frequency Response:

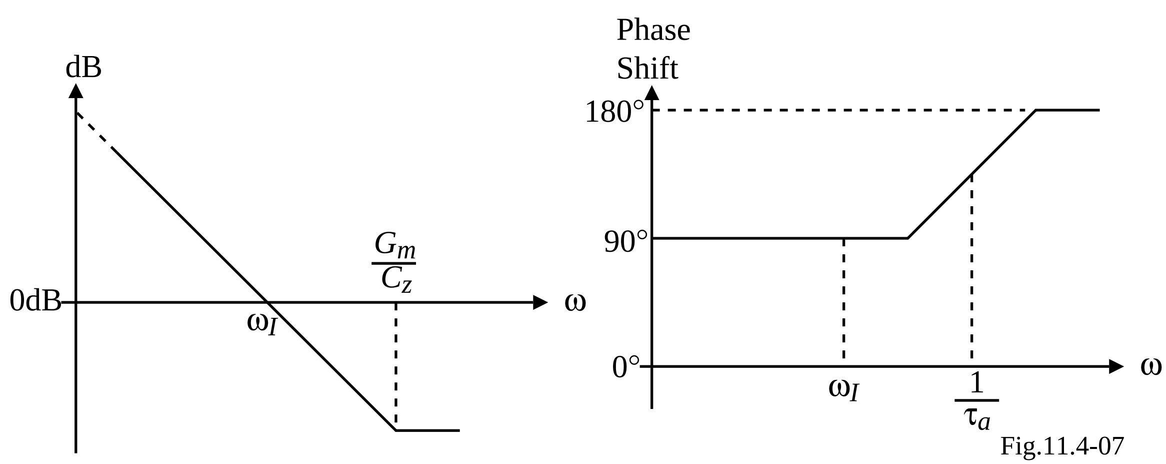


Fig.11.4-07

Solution:

Neutralization techniques

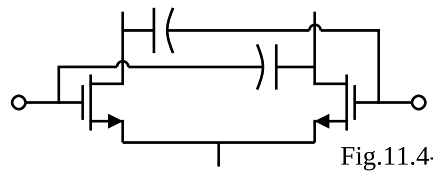


Fig.11.4-08

OTA-C Integrators

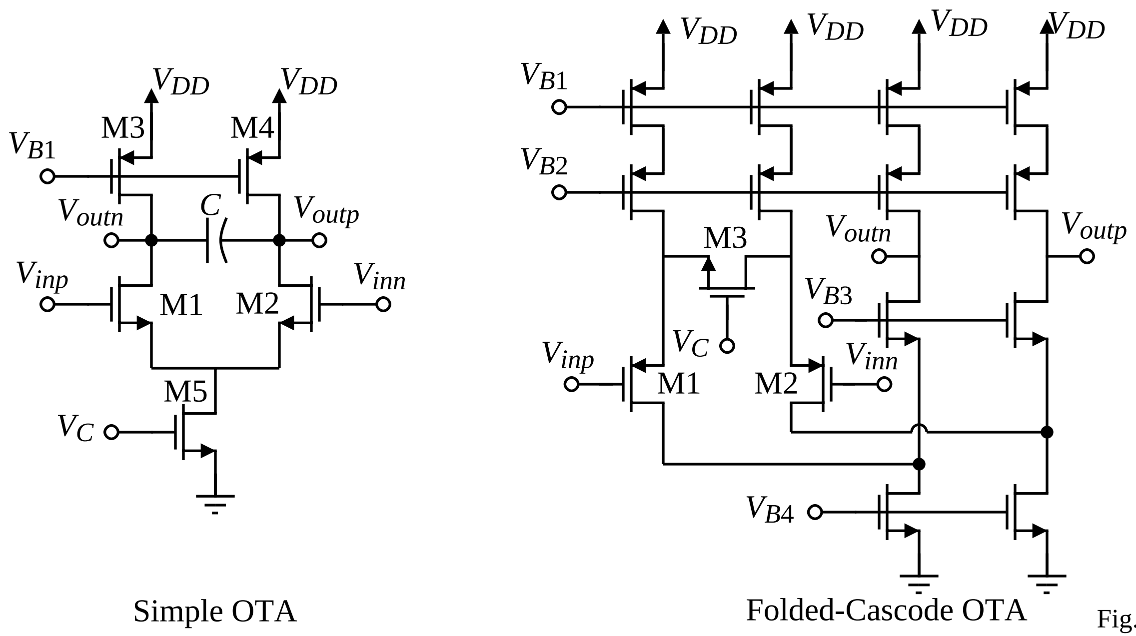


Fig.11.4-10

Simple OTA:

Suffers from poor linearity, signal-to-noise limitations, and low output resistance

Folded-Cascode OTA (M3 in active region):

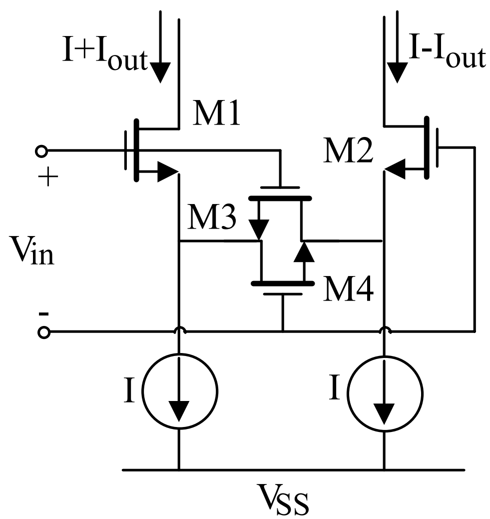
$$G_m = \frac{g_{m1}}{1+g_{m1}R_{M3}} \approx \frac{1}{R_{M3}} = K_n'\frac{W_3}{L_3}(V_C - V_{S3} - V_{Tn})$$

Suffers from linearity and excessive power dissipation†

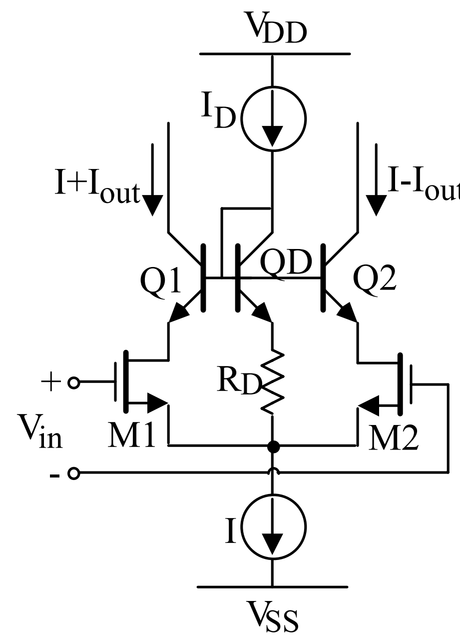
† F. Krummenacher, and G.V. Ruymbeke, “Integrated Selectrivity for Narrow-Band FM IF Systems,” *IEEE J. os Solid-State Circuits*, vol. SC-25, no. 3, pp. 757-760, June 1990.

OTA-C Integrators - Continued

Linearization Schemes:



(a.)

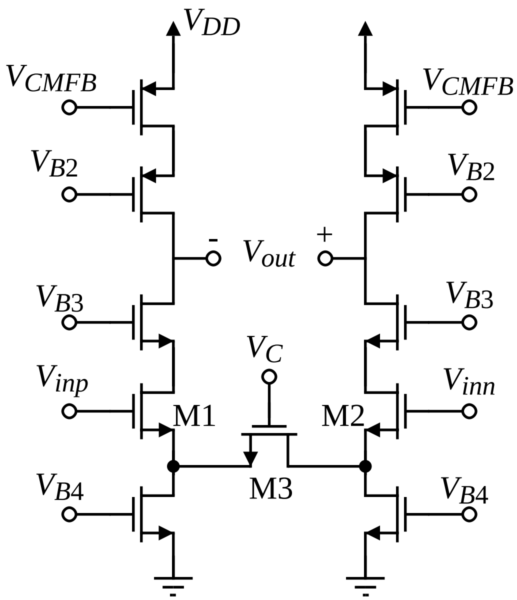


(b.)

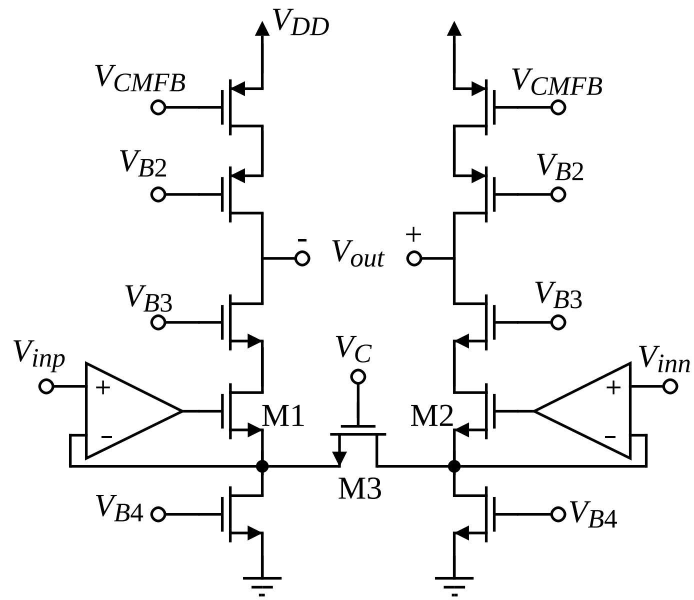
(a.) M3-M4 work in a saturation-active mode for positive V_{in} and in an active-saturation mode for negative V_{in}. Can result in a linear operation. I varies G_M. (b.) M1 and M2 are in active region. Current is proportional to V_{DS1} (V_{DS2}). I_D varies G_M.

OTA-C Integrators - Continued

Tradeoff between power and linearity:



Low-power consumption OTA

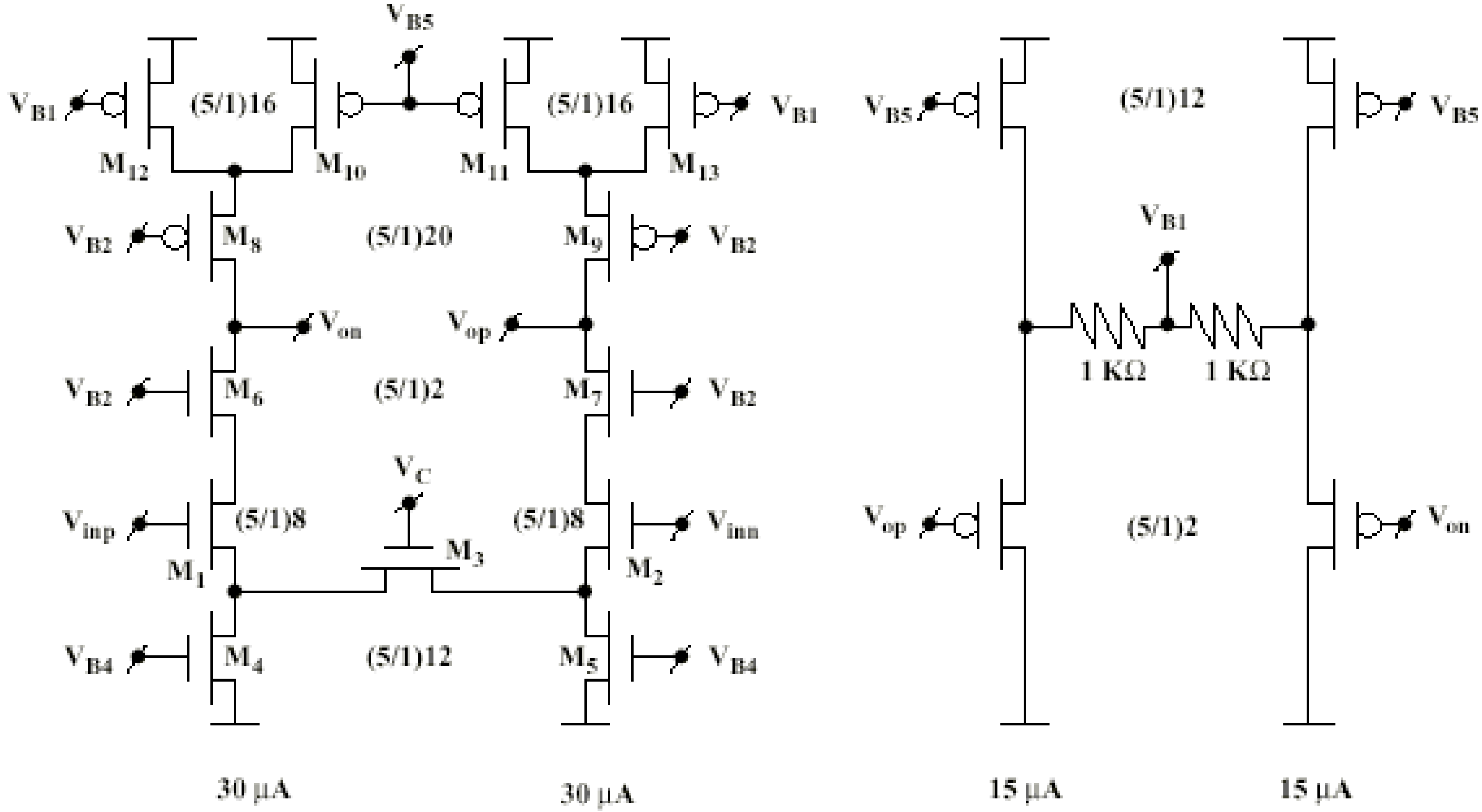


Linearized OTA

Fig.11.4-11

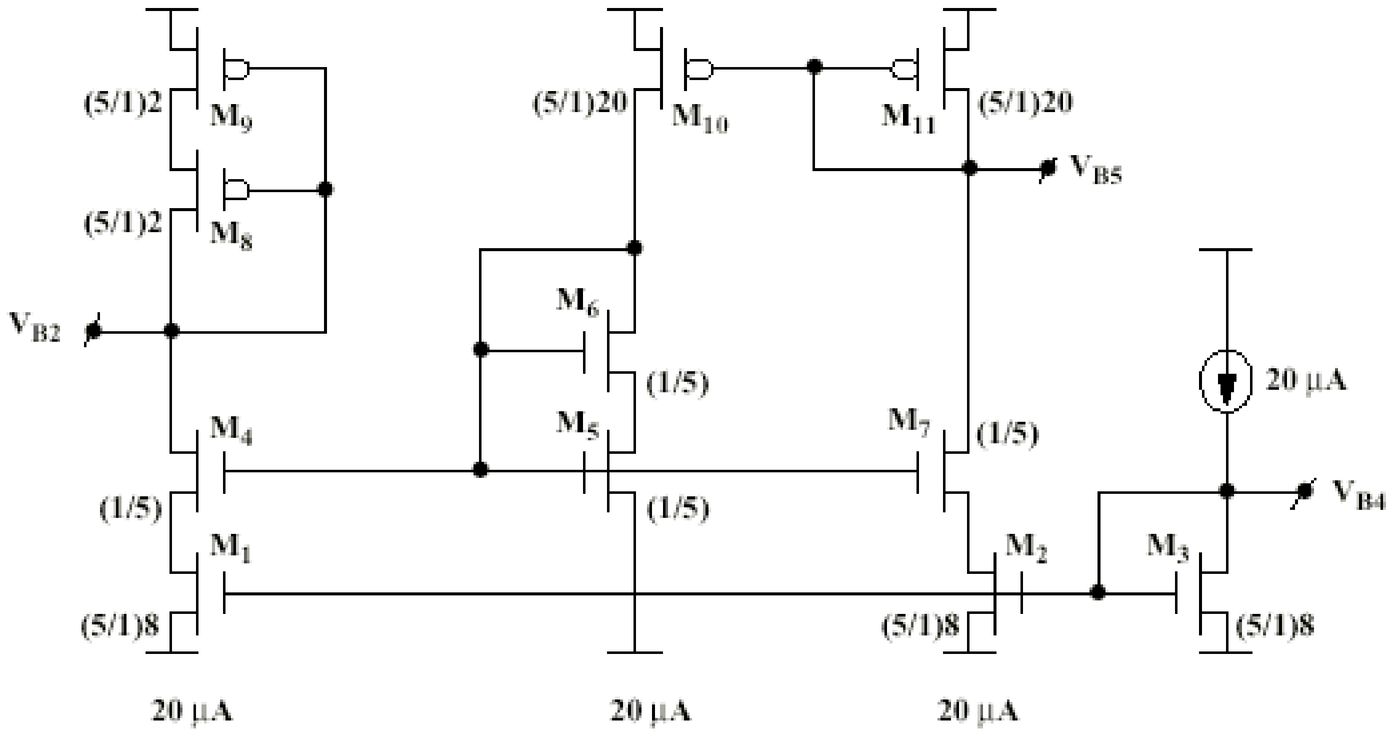
OTA-C Integrators - Continued

Practical implementation of an OTA with 90µA of power supply current.



OTA-C Integrators - Continued

Biasing Circuit for the previous OTA:



Linearized OTA-C Integrator

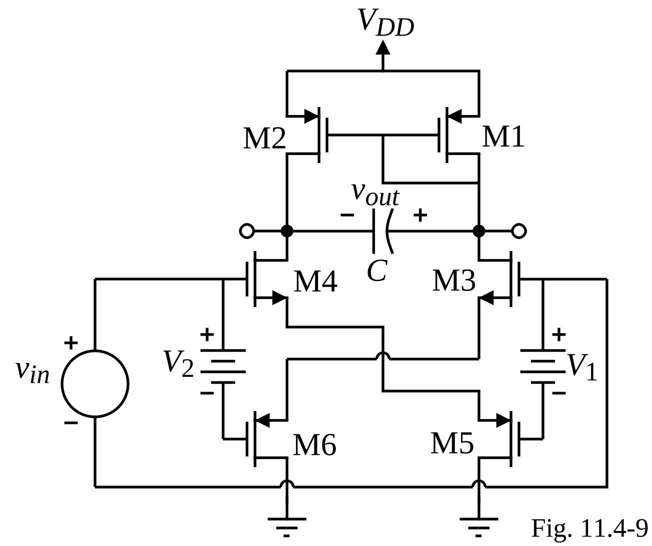


Fig. 11.4-9

This circuit provides a linear transconductance relationship as follows:

Let $K_N'(W/L)_N = A\, K_P'(W/L)_P$

$\therefore V_{GSP} - |V_{TP}| = \sqrt{A}\,(V_{GSN} - V_{TN})$ and $V_{GSN} + V_{GSP} = V_{in} + V$ where $V_1 = V_2 = V$

Combining these two equations gives

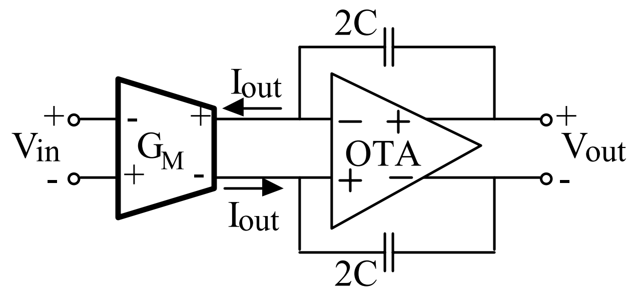
$$V_{GSN} - V_{TN} = \frac{V_{in} + V - V_{TN} - |V_{TP}|}{\sqrt{A} + 1} \quad \Rightarrow \quad i_D = \frac{1}{2} K_N \frac{W}{L} \frac{(V_{in} + V - V_{TN} - |V_{TP}|)^2}{A + 1 + 2\sqrt{A}}$$

$$\therefore \quad G_m = \frac{i_{D1} - i_{D2}}{V_{in}} = 2 \cdot \frac{K_N \frac{W}{L}}{A + 1 + 2\sqrt{A}} (V - V_{TN} - |V_{TP}|)$$

Tuning?

G_M-C-OTA Integrator

Integrator:



$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{I_{out}}{V_{in}} \cdot \frac{V_{out}}{I_{out}} = G_M \frac{1}{sC} = \frac{G_M}{sC}$$

Advantages:

Avoids influence of parasitic capacitances.

High frequency (20MHz, 6th-order filter for disk-drive read channels)

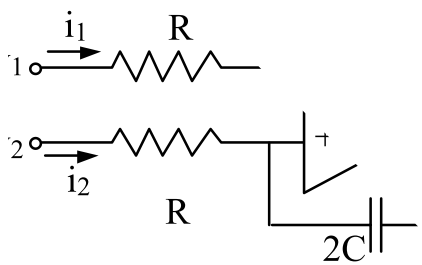
Tuning can be done by current multiplication of I_{out}

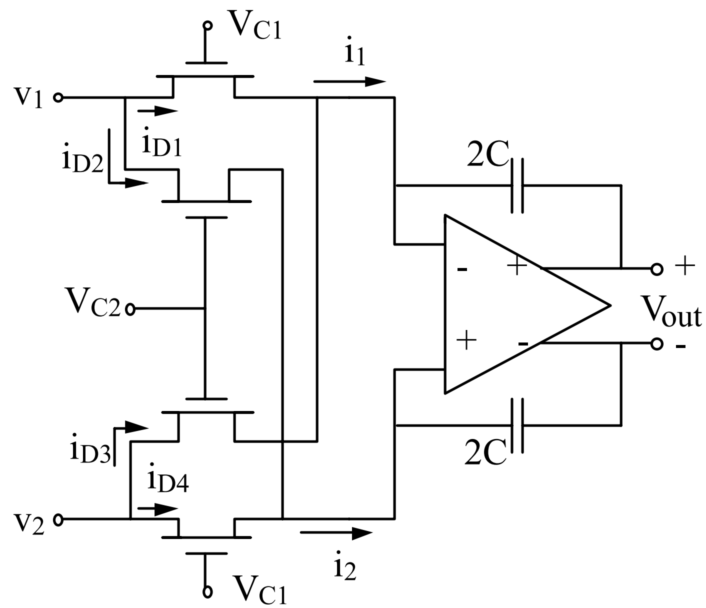
Disadvantages:

More noise - two active elements (G_M and OTA)

MOSFET-C-Op Amp Integrator

Integrator:





Differential input resistance is (see Sec. 4.2):

$$R_{in} = 2R = \frac{v_1\text{-}v_2}{i_1\text{-}i_2} = \frac{v_1\text{-}v_2}{\beta\,(V_{C1}\text{-}V_{C2})(v_1\text{-}v_2)} = \frac{1}{\beta\,(V_{C1}\text{-}V_{C2})}\,, \quad v_1,v_2 \le \min\{(V_{C1}\text{-}V_T),(V_{C2}\text{-}V_T)\}$$

Advantages:

Straight-forward implementation of RC active filters
Insensitive to parasitics

Disadvantages:

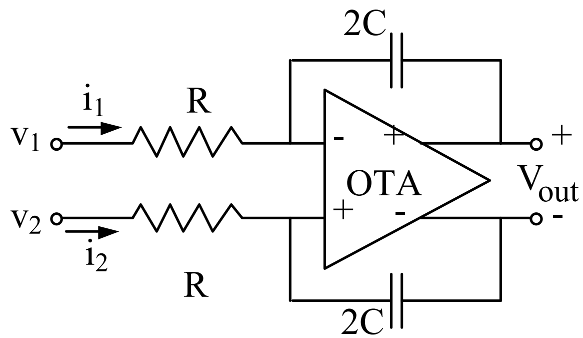
Requires input currents

Amplifiers must be able to drive the resistors

MOSFET-C-OTA Filters

Integrator:

Replace the op amp of MOSFET-C-OP AMP filters with a high-gain OTA. At high frequencies, if the G_M is large, the circuit behaves as an op amp.



Typically requires bipolar devices to get high-enough G_M.

Has been used to build filters at video frequencies.

MOSFET-C-Op Amp Integrator Performance

Ideal Performance:

Uses FETs in the triode region to achieve a linear four-quadrant multiplier.

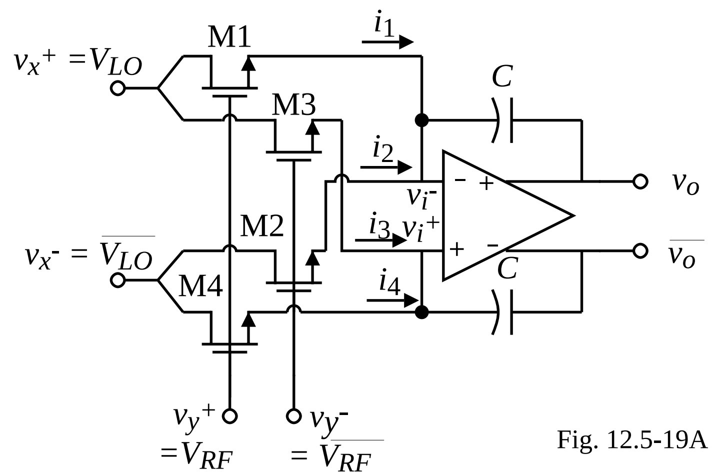


Fig. 12.5-19A

Ideal Operation (Op Amp Gain = ∞):

$$I_1 = K'\left[\left(V_{GS} + \frac{V_y}{2} - V_T\right)\frac{V_x}{2} - \frac{1}{2}\left(\frac{V_x}{2}\right)^2\right]$$

$$I_2 = K'\left[\left(V_{GS} - \frac{V_y}{2} - V_T\right)\left(\frac{-V_x}{2}\right) - \frac{1}{2}\left(\frac{-V_x}{2}\right)^2\right]$$

$$I_3 = K'\left[\left(V_{GS} - \frac{V_y}{2} - V_T\right)\frac{V_x}{2} - \frac{1}{2}\left(\frac{V_x}{2}\right)^2\right]$$

$$I_4 = K'\left[\left(V_{GS} + \frac{V_y}{2} - V_T\right)\left(\frac{-V_x}{2}\right) - \frac{1}{2}\left(\frac{-V_x}{2}\right)^2\right]$$

$$V_{out} = \frac{K'}{sC}(V_o^+ - V_o^-) = \frac{K'}{sC}\ (+I_4 - I_3 - I_1 + I_2) = \frac{K'}{sC}\left(\frac{V_xV_y}{2} + \frac{V_xV_y}{2}\right) = \frac{K'}{sC}V_xV_y$$

$$\therefore \quad \boxed{V_{out} = \frac{K'}{sC}V_{in}V_C}$$

As long as the following conditions are satisfied, the integrator will be linear.

$$V_{inp}, V_{inn} \le \min\{(V_{y+} - V_T), (V_{y-} - V_T)\}$$

MOSFET-C-Op Amp Integrator Performance

Previous circuit with a finite op amp gain (A):

$$V_{out} = V_o^{+} - V_o^{-} = A\,(V_i^{+} - V_i^{-}) = AV_{in} \quad \Rightarrow \quad V_i^{+} = \frac{V_o}{2A} \quad \text{and} \quad V_i^{-} = \frac{-V_o}{2A}$$

$$I_1 = K'\left[\left(V_{GS} + \frac{V_y}{2} - \frac{V_{out}}{2A} - V_T\right)\left(\frac{V_x}{2} - \frac{V_{out}}{2A}\right) - \frac{1}{2}\left(\frac{V_x}{2} - \frac{V_{out}}{2A}\right)^2\right]$$

$$I_2 = K'\left[\left(V_{GS} - \frac{V_y}{2} - \frac{V_{out}}{2A} - V_T\right)\left(-\frac{V_x}{2} - \frac{V_{out}}{2A}\right) - \frac{1}{2}\left(-\frac{V_x}{2} + \frac{V_{out}}{2A}\right)^2\right]$$

$$I_3 = K'\left[\left(V_{GS} - \frac{V_y}{2} + \frac{V_{out}}{2A} - V_T\right)\left(\frac{V_x}{2} + \frac{V_{out}}{2A}\right) - \frac{1}{2}\left(\frac{V_x}{2} + \frac{V_{out}}{2A}\right)^2\right]$$

$$I_4 = K'\left[\left(V_{GS} + \frac{V_y}{2} + \frac{V_{out}}{2A} - V_T\right)\left(-\frac{V_x}{2} + \frac{V_{out}}{2A}\right) - \frac{1}{2}\left(-\frac{V_x}{2} + \frac{V_{out}}{2A}\right)^2\right]$$

$$V_{out} = \frac{K'/sC}{1 + \frac{1}{A}}\,(+I_4 - I_3 - I_1 + I_2)$$

$$= \frac{K'/sC}{1 + \frac{1}{A}}\left[\frac{V_xV_y}{2} + \frac{V_TV_{out}}{A} - \frac{V_{GS}V_{out}}{A} + \frac{V_xV_y}{2} - \frac{V_{GS}V_{out}}{A} + \frac{V_TV_{out}}{A}\right]$$

$$V_{out}\left[\left(1 + \frac{1}{A}\right)\frac{2K'/sC}{A}\,(V_{GS} - V_T)\right] = \frac{K'}{sC}V_xV_y$$

$$\therefore \quad V_{out} = \frac{K'/sC}{1 + \frac{1}{A} + \frac{2K'/sC}{A}\left(V_{GS} - V_T\right)} V_{in} V_C$$

MOSFET-C-Op Amp Integrator Performance

Previous circuit with a finite op amp gain (A) and a threshold variation (ΔV_T^+ and ΔV_T^-):

$$I_1 = K'\left[\left(V_{GS} + \frac{V_y}{2} - \frac{V_{out}}{2A} - V_T - \Delta V_T^+\right)\left(\frac{V_x}{2} - \frac{V_{out}}{2A}\right) - \frac{1}{2}\left(\frac{V_x}{2} - \frac{V_{out}}{2A}\right)^2\right]$$

$$I_2 = K'\left[\left(V_{GS} - \frac{V_y}{2} - \frac{V_{out}}{2A} - V_T - \Delta V_T^+\right)\left(-\frac{V_x}{2} - \frac{V_{out}}{2A}\right) - \frac{1}{2}\left(-\frac{V_x}{2} + \frac{V_{out}}{2A}\right)^2\right]$$

$$I_3 = K'\left[\left(V_{GS} - \frac{V_y}{2} + \frac{V_{out}}{2A} - V_T - \Delta V_T^-\right)\left(\frac{V_x}{2} + \frac{V_{out}}{2A}\right) - \frac{1}{2}\left(\frac{V_x}{2} + \frac{V_{out}}{2A}\right)^2\right]$$

$$I_4 = K'\left[\left(V_{GS} + \frac{V_y}{2} + \frac{V_{out}}{2A} - V_T - \Delta V_T^-\right)\left(-\frac{V_x}{2} + \frac{V_{out}}{2A}\right) - \frac{1}{2}\left(-\frac{V_x}{2} + \frac{V_{out}}{2A}\right)^2\right]$$

$$V_{out} = \frac{K'/sC}{1 + \frac{1}{A}}(+I_4 - I_3 - I_1 + I_2)$$

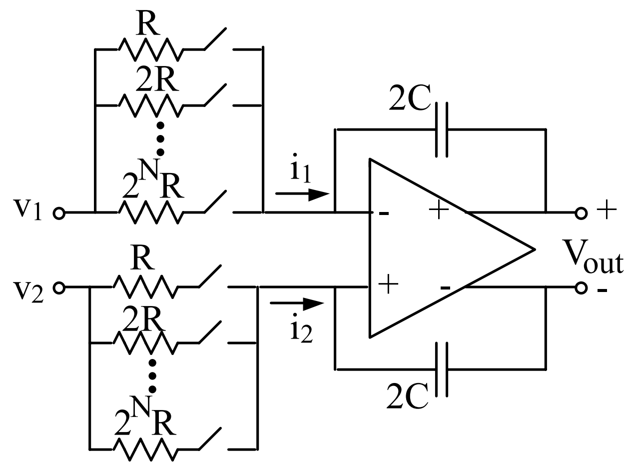
$$= \frac{K'/sC}{1+\frac{1}{A}}\left[\frac{V_xV_y}{2} + \frac{\Delta V_T^+ V_o}{A} + \frac{V_TV_o}{A} + \frac{V_o^2}{2A^2} - \frac{V_{GS}V_o}{A} + \frac{V_xV_y}{2} + \frac{\Delta V_T^- V_o}{A} + \frac{V_TV_o}{A} - \frac{V_o^2}{2A^2} - \frac{V_{GS}V_o}{A}\right]$$

$$V_o = V_{out} = \frac{K'/sC}{1 + \frac{1}{A}}\left[V_xV_y - \frac{2V_{out}}{A}(V_{GS} - V_T) + \frac{V_{out}}{A}(\Delta V_T^+ + \Delta V_T^-)\right]$$

$$\therefore \quad \boxed{V_{out} = \frac{K'/sC}{\left(1 + \frac{1}{A}\right)\left[1 + \frac{2K'/sC}{1+A}(V_{GS} - V_T) - \frac{K'/sC}{1+A}(\Delta V_T^+ + \Delta V_T^-)\right]} V_{in} V_C}$$

True Active RC Filters

Integrator:



Advantages:

Good linearity

Wide dynamic range

Disadvantages:

Op amp must be able to drive resistances

Requires large area

Log Domain Integrator

Log domain filters are essentially g_m-C filters with an exponential relationship for g_m.

A log domain lowpass filter:

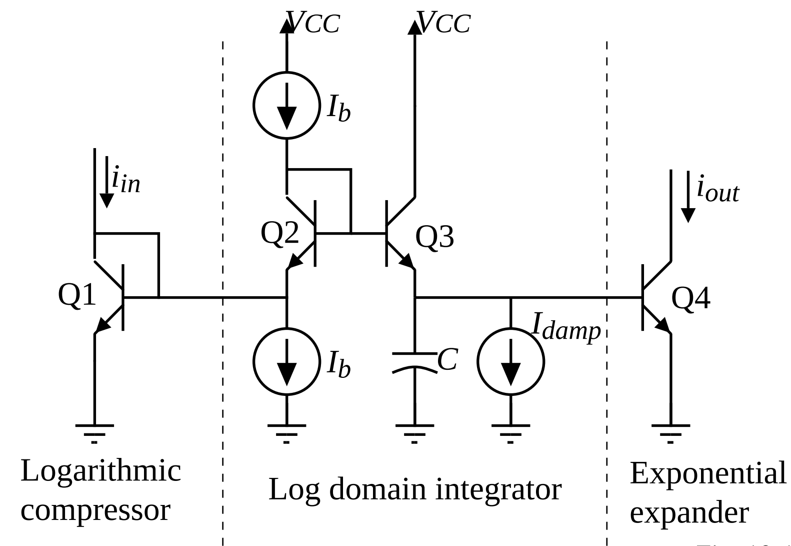


Fig. 10-15

Analysis:

Note that $V_{BE1} + V_{BE2} = V_{BE3} + V_{BE4}$

$$\Rightarrow \quad i_1 i_2 = i_3 i_4 \quad \Rightarrow \quad i_{in} I_b = i_3 i_{out}$$

But, $i_3 = i_C + I_{damp} = C\frac{dv_{BE4}}{dt} + I_{damp}$

and

$$C\frac{dv_{BE4}}{dt} = CV_T\frac{d}{dt}ln\left(\frac{i_{out}}{I_s}\right) = \frac{CV_T}{i_{out}}\frac{di_{out}}{dt} = \frac{CV_T}{i_{out}}s \cdot i_{out}$$

$$\therefore\ i_{in} I_b = (sCV_T + I_{damp})i_{out}$$

$$\Rightarrow \frac{i_{out}}{i_{in}} = \frac{I_b}{sCV_T + I_{damp}} = \frac{\frac{I_b}{CV_T}}{s + \frac{I_{damp}}{CV_T}}$$

(set $I_{damp} = 0$ for true integrator)

Comments:

- The corner frequency of the filter depends on temperature (tuning is required)

- Lack of pure logarithmic VI characteristics and finite β of the BJTs lead to linearity degradation
- Good for high frequency (100MHz)

Log Domain Filters - Continued

Differential log-domain integrator:

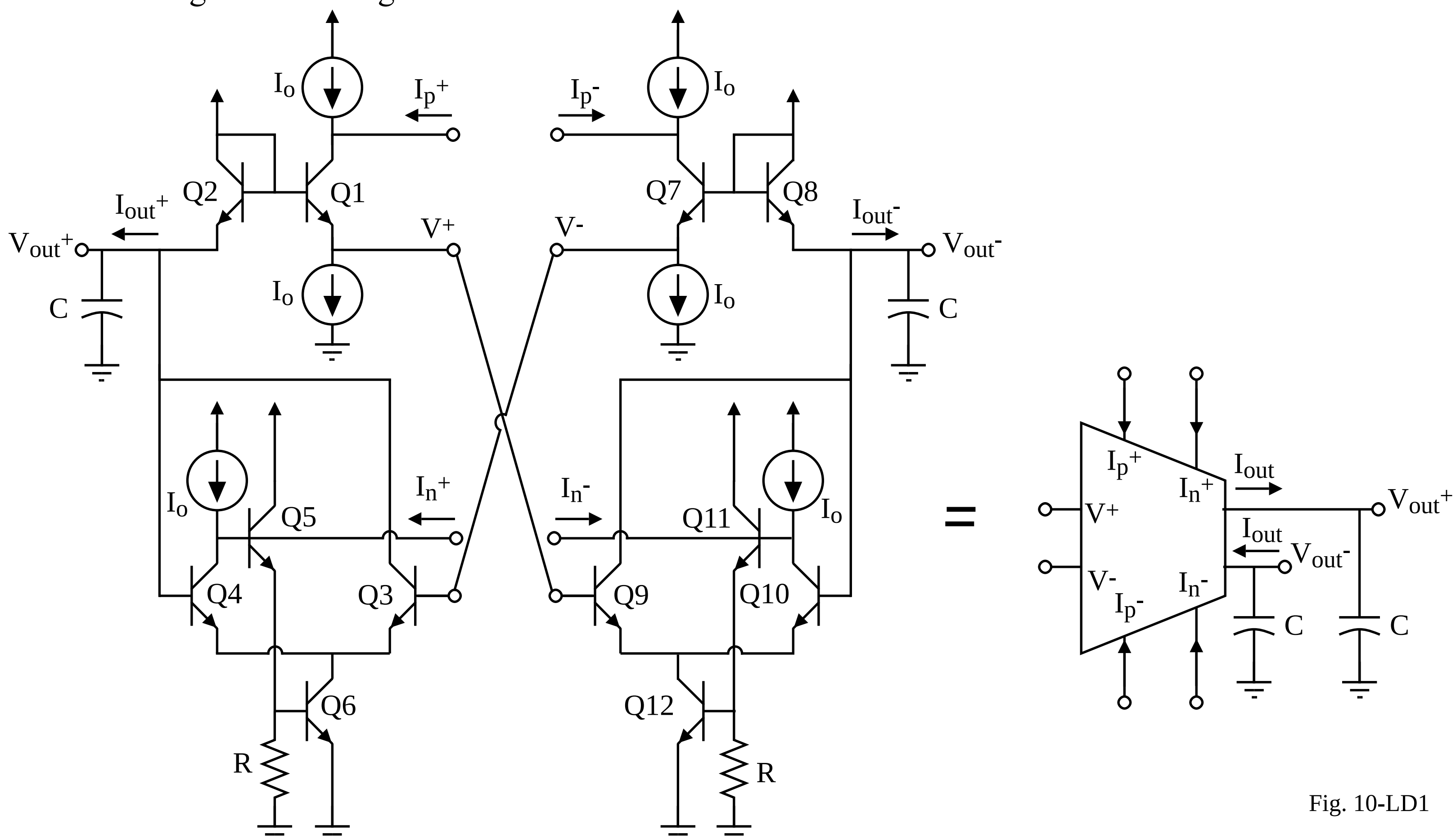
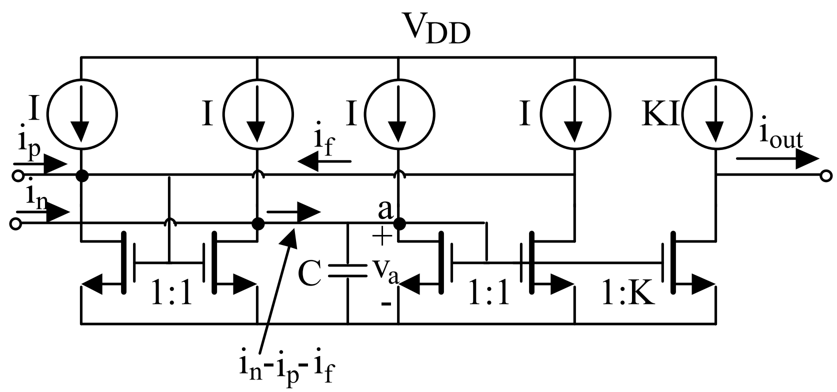


Fig. 10-LD1

Current Mode Integrator

Circuit:



Summation of currents at node a gives:

$$i_n - i_p - i_f = sCv_a + g_m v_a$$

Also,

$$-i_f = g_m v_a$$

Combining,

$$i_n - i_p - i_f = \frac{-sCi_f}{g_m} - i_f \rightarrow i_f = \frac{g_m}{sC}(i_p - i_n)$$

Thus,

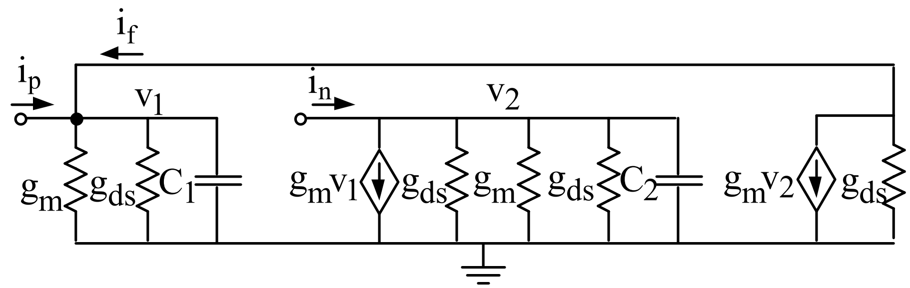
$$i_{out} = Ki_f = (i_p - i_n)$$

Use the dc current sources (I and KI) to tune the transconductances.

Current Mode Integrators - Continued

Current-Mode Integrator Nonidealities:

Small signal model-



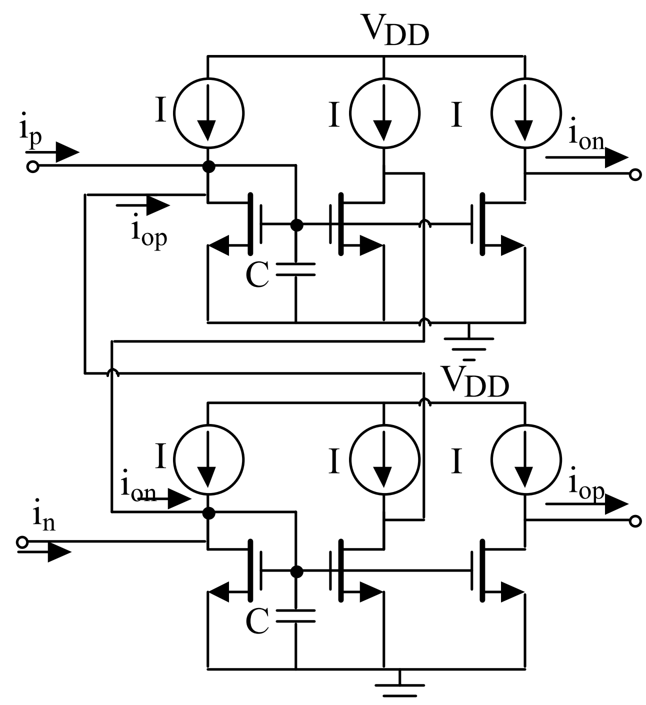
Analysis gives,

$$i_f = \frac{\frac{g_m}{4g_{ds}}}{\left(1 + \frac{sC_2}{4g_{ds}}\right)\left(1 + \frac{sC_1}{g_m}\right)}\left[\left(1 + \frac{sg_{ds}C_2}{g_m{}^2}\right)i_p - \left(1 - \frac{sC_1}{g_m}\right)i_n\right]$$

- Dominant pole moves from the origin to $-4g_{ds}/C_2$
- Unity gain frequency is g_m/C_2
- The undesired capacitance, C_1, inversely affects the frequency of the non-dominant pole (g_m/C_1) causing an undesirable excess phase shift at the integrator unity gain frequency
- If $C_2 >> C_1$, and one uses cascode techniques to reduce the output conductances, Q factors exceeding 20 can be achieved at high frequencies

Current Mode Filters - Continued

Fully-Balanced Current Mode Integrator:



Ideally,

$$i_{op} - i_{on} = \frac{g_m}{sC}(i_p - i_n)$$

In reality,

$$i_{op} - i_{on} = A\frac{\left(1 - \frac{s}{z_1}\right)}{\left(1 + \frac{s}{p_1}\right)}(i_p - i_n)$$

where

$$A = \frac{g_m - g_{ds}}{g_{ds}}$$

$$z_1 = \frac{g_m - g_{ds}}{2C_{gd}}$$

and

$$p_1 = \frac{g_{ds}}{C + 4C_{gd}}$$

To achieve high Q factor, the RHP zero should be as large as possible.

SECTION 4 - BIQUADS

The Biquad

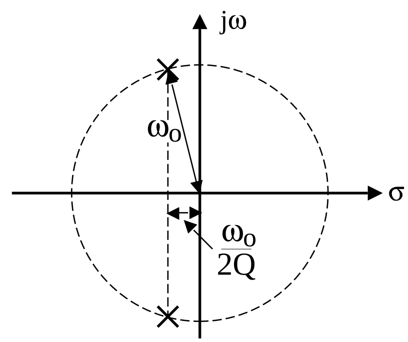
A biquad has two poles and two zeros.

Poles are complex and always in the LHP.

The zeros may or may not be complex and may be in the LHP or the RHP.

Transfer function:

$$H_a(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{-(K_2s^2 + K_1s + K_0)}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2} = K\left(\frac{(s-z_1)(s-z_2)}{(s-p_1)(s-p_2)}\right)$$



Low pass: zeros at ∞

High pass: zeros at 0

Bandpass: One zero at 0 and the other at ∞

Bandstop: zeros at $\pm j\omega_o$

Allpass: Poles and zeros are complex conjugates

Development of a Biquad Realization

Rewrite H_a(s) as:

$$s^2V_{out}(s) + \frac{\omega_o s}{Q}V_{out}(s) + \omega_o{}^2V_{out}(s) = -(K_2s^2 + K_1s + K_0)V_{in}(s)$$

Dividing through by s^2 and solving for $V_{out}(s)$, gives

$$V_{out}(s) = \frac{-1}{s}\left[(K_1 + K_2s)V_{in}(s) + \frac{\omega_o}{Q}V_{out}(s) + \frac{1}{s}(K_0V_{in}(s) + \omega_o{}^2V_{out}(s))\right]$$

If we define the voltage V_1(s) as

$$V_1(s) = \frac{-1}{s}\left[\frac{K_0}{\omega_o}V_{in}(s) + \omega_oV_{out}(s)\right]$$

then $V_{out}(s)$ can be expressed as

$$V_{out}(s) = \frac{-1}{s}\left[(K_1 + K_2s)\,V_{in}(s) + \frac{\omega_o}{Q}V_{out}(s) - \omega_oV_1(s)\right]$$

Synthesizing the voltages $V_1(s)$ and $V_{out}(s)$, gives (assuming inverting integrators)

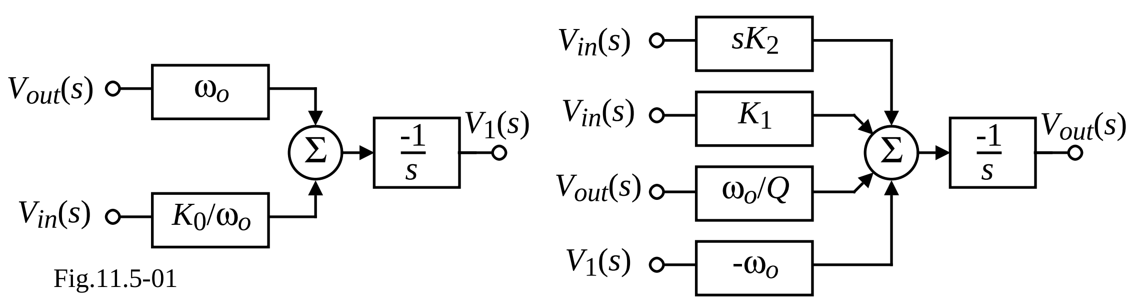


Fig.11.5-01

Example of a Biquad Realization

Connecting the two previous circuits gives:

For negative integrators-

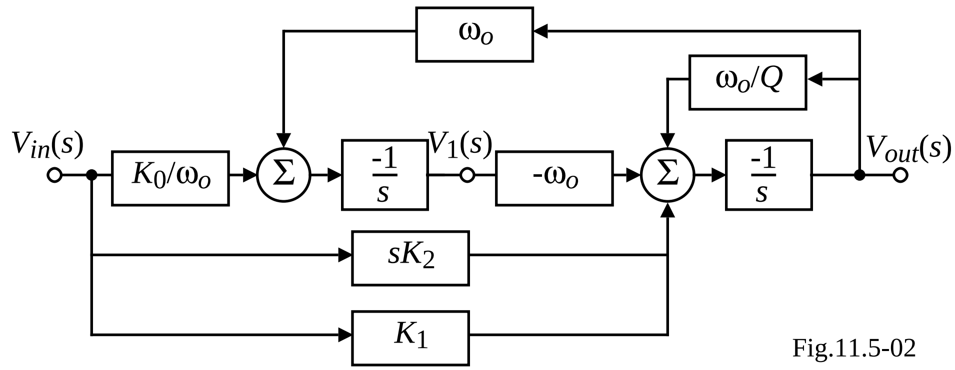


Fig.11.5-02

For positive integrators-

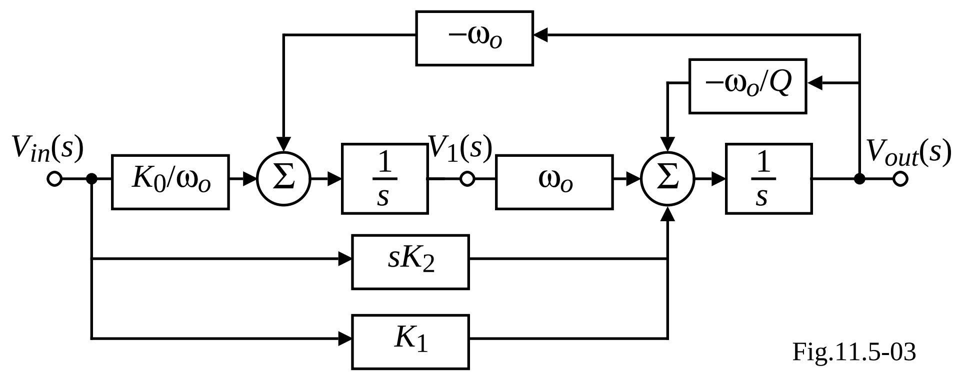


Fig.11.5-03

More Biquads

KHN:

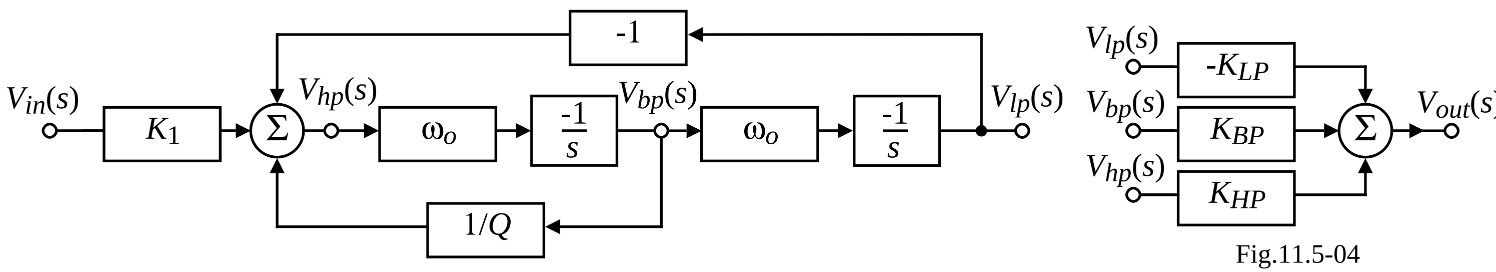


Fig.11.5-04

Tow-Thomas Biquad:

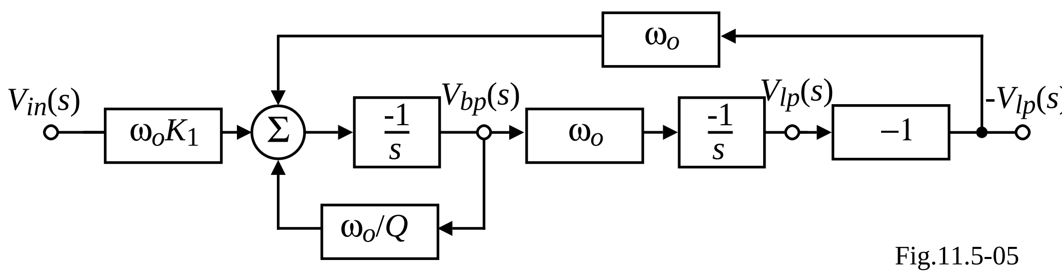


Fig.11.5-05

Needs some additions to make it a universal biquad.

OTA-C BIQUADS

Building Blocks for OTA-C Biquads

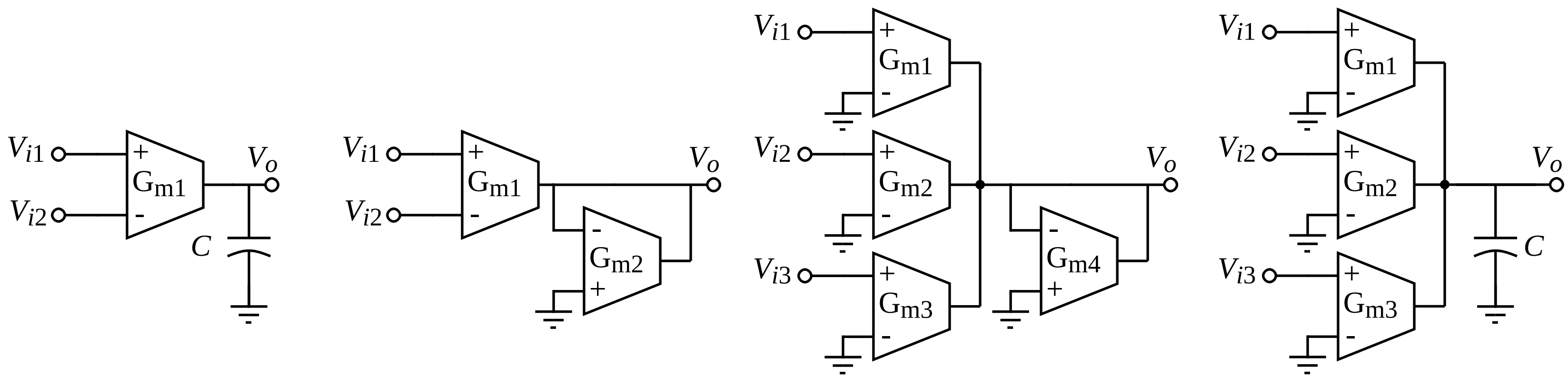


Fig.11.5-06

Integrator

$$V_o = \frac{V_{i1}-V_{i2}}{sC} = \frac{G_{m1}V_{in}}{sC}$$

Amplifier

$$V_o = \frac{G_{m1}(V_{i1}-V_{i2})}{G_{m2}} = \frac{G_{m1}}{G_{m2}}V_{in}$$

Summing Amplifier

$$V_o = \frac{G_{m1}V_{i1}}{G_{m4}} + \frac{G_{m2}V_{i2}}{G_{m4}} + \frac{G_{m3}V_{i3}}{G_{m4}}$$

Summing Integrator

$$V_o = \frac{G_{m1}V_{i1}}{sC} + \frac{G_{m2}V_{i2}}{sC} + \frac{G_{m3}V_{i3}}{sC}$$

A Single-Ended Bandpass OTA-C Biquad

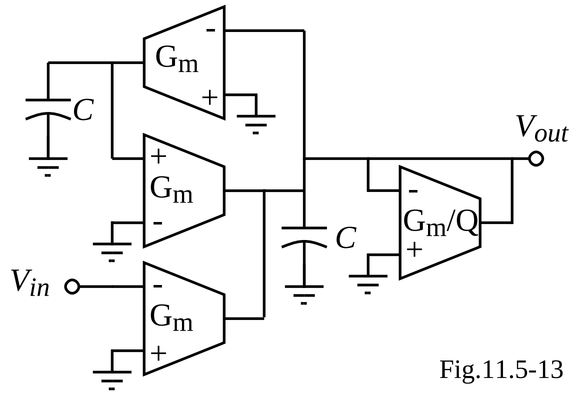
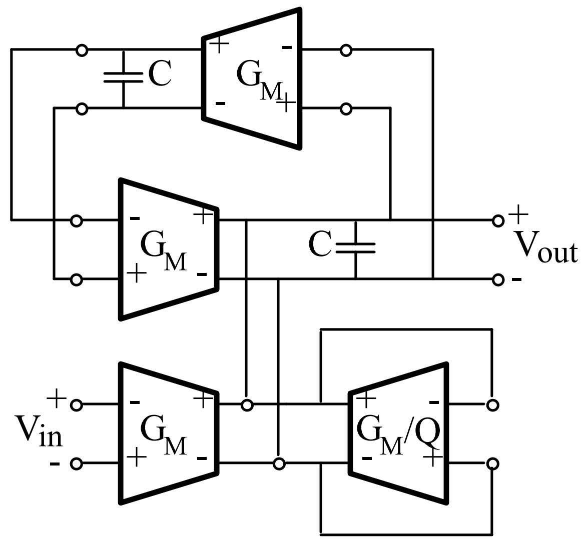


Fig.11.5-13

Transfer function:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{\frac{G_M}{C}s}{s^2 + \frac{G_M}{QC}s + \left(\frac{G_M}{C}\right)^2}$$

A Differential Bandpass OTA-C Biquad



Transfer function:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{\frac{G_M}{C}s}{s^2 + \frac{G_M}{QC}s + \left(\frac{G_M}{C}\right)^2}$$

Single-Ended OTA-C Biquad

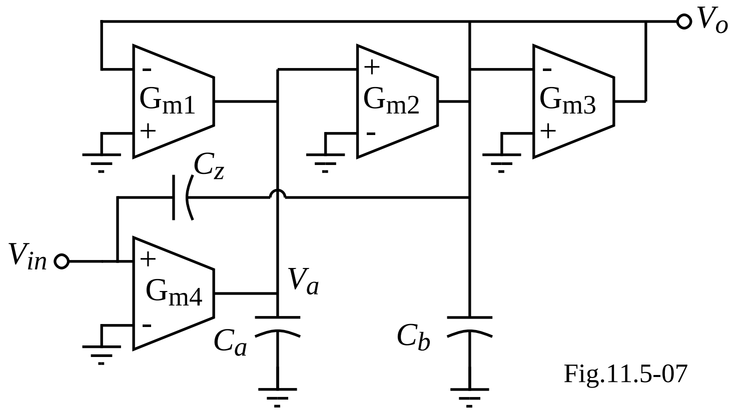


Fig.11.5-07

Analysis:

Σi=0 at V_a $\Rightarrow$ $sC_aV_a - G_{m4}V_{in} + G_{m1}V_o = 0$

Solving for V_a, $V_a = -\dfrac{-G_{m4}V_{in}+G_{m1}V_o}{sC_a}$

Σi=0 at V_o $\Rightarrow$ $sC_bV_o + sC_z(V_o - V_{in}) - G_{m2}V_a + G_{m3}V_o = 0$

Combining equations gives,

$$sC_bV_o + sC_z(V_o - V_{in}) - G_{m2}\left(-\frac{-G_{m4}V_{in}+G_{m1}V_o}{sC_a}\right) + G_{m3}V_o = 0$$

Which gives,

$$\left(sC_b + sC_z + \frac{G_{m2}G_{m1}}{sC_a} + G_{m3}\right)V_o = \left(sC_z + \frac{G_{m2}G_{m4}}{sC_a}\right)V_{in}$$

$$\therefore \quad \frac{V_o}{V_{in}} = \frac{s^2C_zC_a + G_{m2}G_{m4}}{s^2C_a(C_z+C_b) + sG_{m3}C_a + G_{m1}G_{m2}} = \frac{s^2\dfrac{C_z}{C_b+C_z} + \dfrac{G_{m2}G_{m4}}{C_a(C_b+C_z)}}{s^2 + \dfrac{sG_{m3}}{C_z+C_b} + \dfrac{G_{m1}G_{m2}}{C_a(C_z+C_b)}}$$

A biquad with complex zeros on the $j\omega$ axis.

Differential OTA-C Biquad

Differential version of the previous biquad:

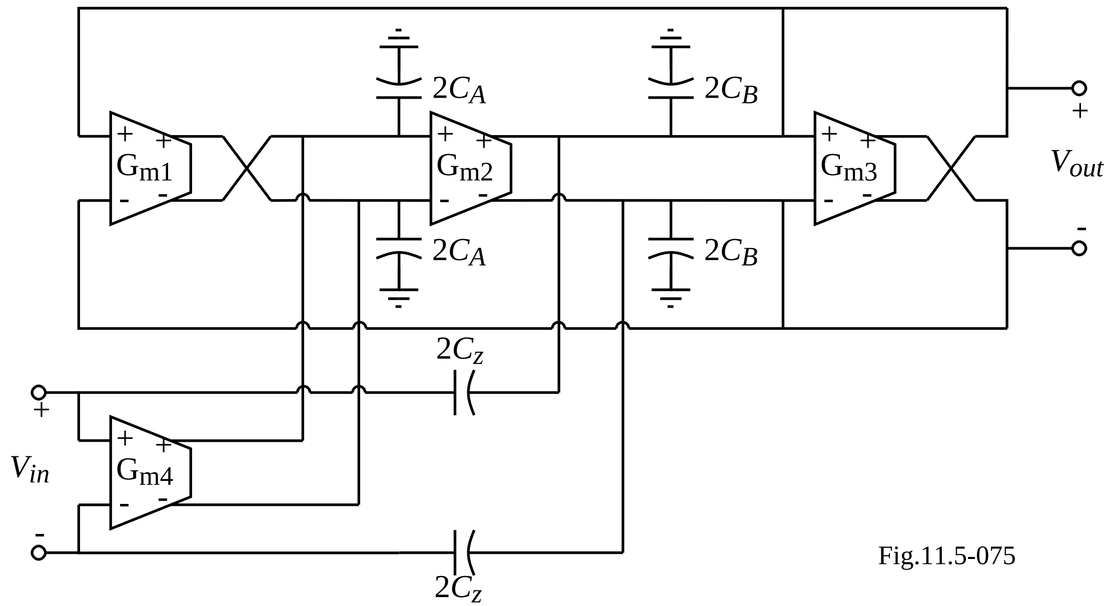


Fig.11.5-075

Example 4-1

Use the biquad of the previous page to design the following transfer function where K_2=4.49434x10^{-1}, K_0 = 4.580164x10^{12}, ω_o = 2.826590x10^6 and Q = 6.93765.

$$H_a(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{K_2 s^2 + K_0}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2}$$

Find the values of all transconductances and capacitors and let C_a =2pF, C_b = 35pF and G_{m1}=G_{m2}.

Solution

Equating $H_a(s)$ to the previous transfer function gives

$$K_2 = \frac{C_z}{C_b+C_z}, \qquad K_0 = \frac{G_{m2}G_{m4}}{C_a(C_b+C_z)}, \qquad \frac{\omega_o}{Q} = \frac{G_{m3}}{C_b+C_z}, \qquad \text{and} \qquad \omega_o^2 = \frac{G_{m1}G_{m2}}{C_a(C_b+C_z)}$$

Thus, $C_z = \frac{K_2 C_b}{1-K_2} = \frac{0.44943 \cdot 35\text{pF}}{1-0.44943} = \underline{\underline{28.2264\text{pF}}}$

$$G_{m1}=G_{m2}= \omega_o\sqrt{C_a(C_b+C_z)} = 2.826590\text{x}10^6\sqrt{2(35+28.2264)} = \underline{\underline{31.785\mu\text{S}}}$$

$$G_{m3}= \frac{\omega_o}{Q}(C_b+C_z) = \frac{2.826590\text{x}10^6}{6.93765}(35+28.2264)\text{x}10^{-12} = \underline{\underline{25.760\mu\text{S}}}$$

Note that $\frac{K_0}{\omega_o^2} = \frac{G_{m4}}{G_{m1}}$ which gives $G_{m4} = \frac{K_0}{\omega_o^2}G_{m1} = \frac{4.580164\text{x}10^{12}}{(2.826590\text{x}10^6)^2}31.785\mu\text{S} = \underline{\underline{18.221\mu\text{S}}}$

A Fully Differential, OTA-C General Biquad

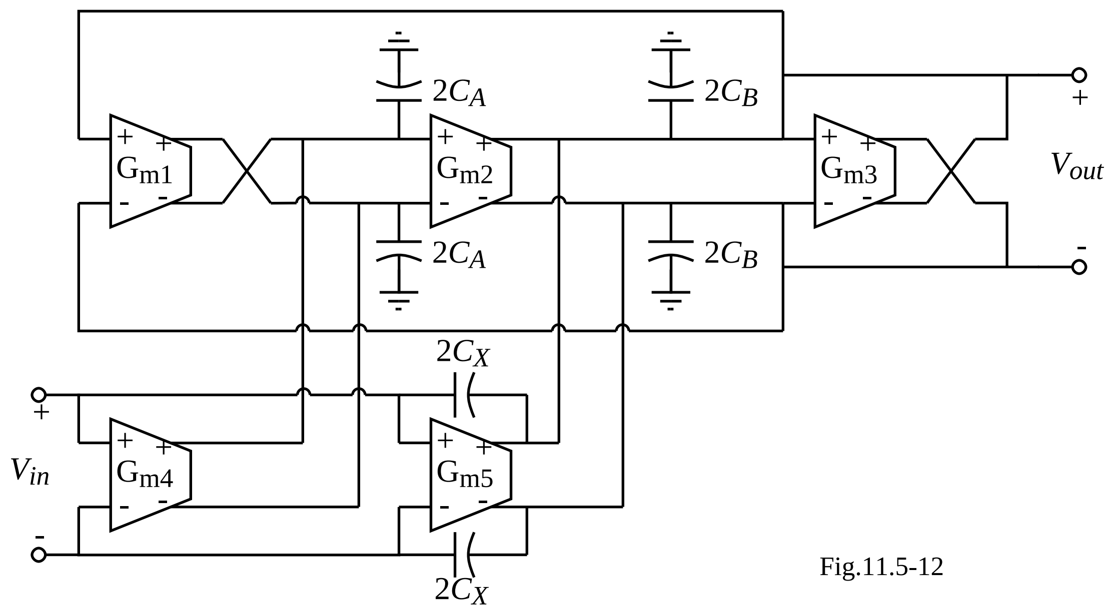


Fig.11.5-12

Transfer function:

$$\frac{V_{out}}{V_{in}} = \frac{\left(\frac{C_x}{C_x+C_B}\right)s^2 + \left(\frac{G_{m5}}{C_x+C_B}\right)s + \left(\frac{G_{m2}G_{m4}}{C_A(C_x+C_B)}\right)}{s^2 + \left(\frac{G_{m3}}{C_x+C_B}\right)s + \left(\frac{G_{m1}G_{m2}}{C_A(C_x+C_B)}\right)}$$

Two-Thomas Lowpass Biquad

We shall use this circuit later in a filter example.

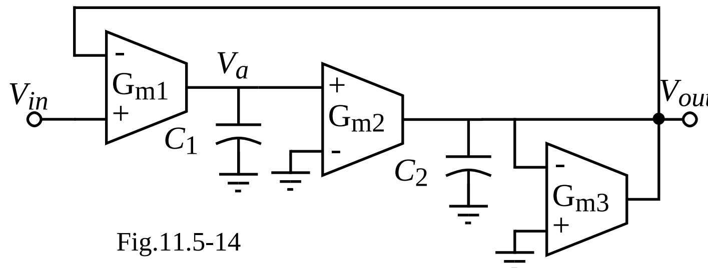


Fig.11.5-14

Analysis:

$\Sigma i=0$ at V_a $\Rightarrow$ $sC_1V_a - G_{m1}(V_{in}-V_{out}) = 0 \rightarrow$ $sC_1V_a = G_{m1}(V_{in}-V_{out})$

$\Sigma i=0$ at V_{out} $\Rightarrow$ $sC_2V_{o2} + G_{m3}V_{out} - G_{m2}V_a = 0$

Combining equations gives,

$$sC_2V_{o2} + G_{m3}V_{out} - G_{m2}\left(\frac{G_{m1}(V_{in}-V_{out})}{sC_1}\right) = 0 \;\rightarrow\; (s^2C_1C_2 + sC_1G_{m3} + G_{m1}G_{m2})V_{out} = G_{m1}G_{m2}V_{in}$$

$$\therefore \quad \frac{V_{out}}{V_{in}} = \frac{G_{m1}G_{m2}}{s^2C_1C_2 + sC_1G_{m3} + G_{m1}G_{m2}} = \frac{\frac{G_{m1}G_{m2}}{C_1C_2}}{s^2 + s\frac{G_{m3}}{C_2} + \frac{G_{m1}G_{m2}}{C_1C_2}}$$

Nonideal Effects in OTA Biquads

1.) Finite OTA output resistance.

Use structures that can absorb the output resistance.

The admittances, $Y1$ and $Y2$, consist of a capacitor in parallel with a resistor. The resistor can "absorb" the finite output resistances of the OTAs.

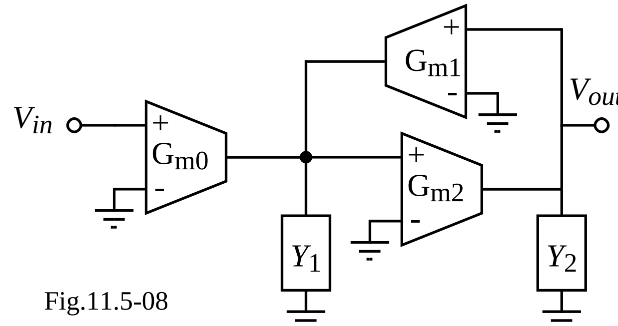


Fig.11.5-08

2.) Finite OTA bandwidth.

$$\frac{V_{out}}{V_{in}} = G_m(0)\frac{s_{RC}+1}{sC(s\tau_a,+1)}$$

If $RC = \tau_a$, then the dominant OTA pole is canceled

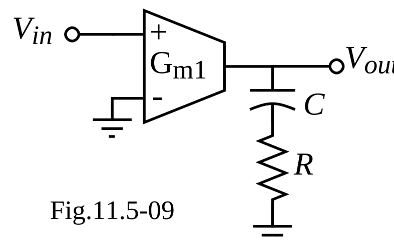


Fig.11.5-09

An alternate approach:

$$G_{meq} = (G_{m1}(0)\text{-}G_{m2}(0))\left[1 - s\,\frac{G_{m1}(0)\tau_{a1}\text{-}\ G_{m2}(0)\tau_{a2}}{G_{m1}(0)\text{-}G_{m2}(0)}\right]$$

$$G_{meq} = G_{m1}(0)\text{-}G_{m2}(0) \quad \text{if} \quad G_{m1}(0)\tau_{a1} = G_{m2}(0)\tau_{a2}$$

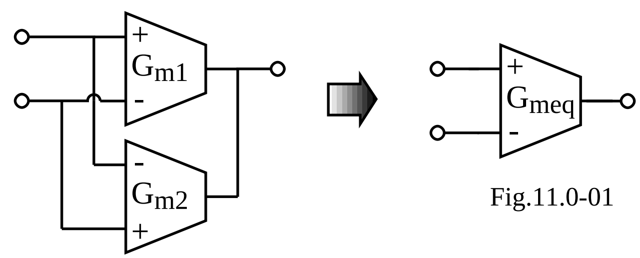


Fig.11.0-01

MOSFET-C BIQUADS

General Second-Order Biquad

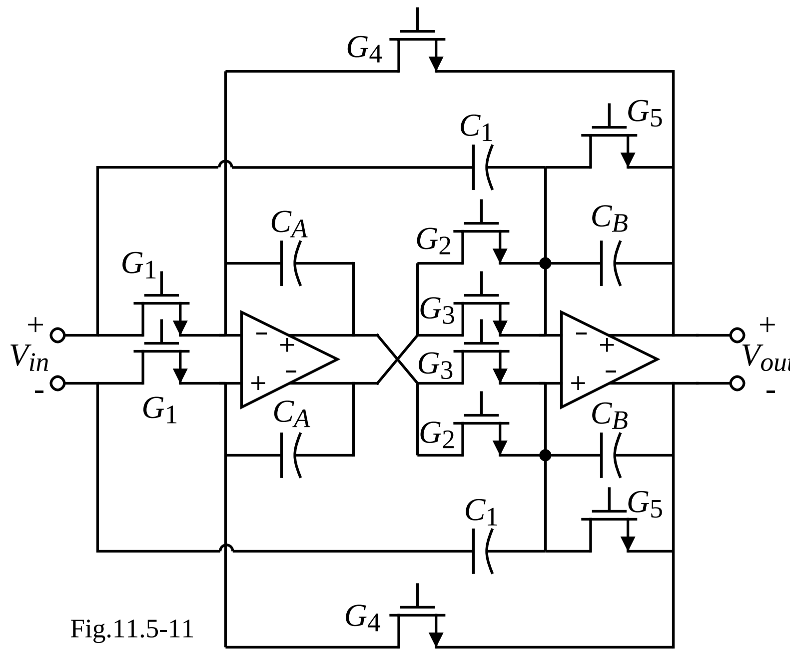


Fig.11.5-11

Transfer function:

$$\frac{V_{out}}{V_{in}} = -\frac{\left[\left(\frac{C_1}{C_B}\right)s^2 + \left(\frac{G_2}{C_B}\right)s + \left(\frac{G_1 G_3}{C_A C_B}\right)\right]}{\left[s^2 + \left(\frac{G_5}{C_B}\right)s + \left(\frac{G_3 G_4}{C_A C_B}\right)\right]}$$

SECTION 5 - IC FILTER DESIGN

A Design Approach for Lowpass Cascaded Filters

1.) From T_{PB}, T_{SB}, and Ω_n (or A_{PB}, A_{SB}, and Ω_n) determine the required order of the filter approximation, N.

2.) From the appropriate tables find the normalized poles of the approximation.

3.) Group the complex-conjugate poles into second-order realizations. For odd-order realizations there will be one first-order term.

4.) Realize each of the terms using first- and second-order blocks.

5.) Cascade the realizations in the order from input to output of the lowest-Q stage first (first-order stages generally should be first).

6.) Denormalize the filter if necessary.

More information can be found elsewhere[1,2,3,4].

---

[1] K.R. Laker and W.M.C. Sansen, *Design of Analog Integrated Circuits and Systems*, McGraw Hill, New York, 1994.

[2] P.E. Allen and E. Sanchez-Sinencio, *Switched Capacitor Circuits*, Van Nostrand Reinhold, New York, 1984.

[3] R. Gregorian and G.C. Temes, *Analog MOS Integrated Circuits for Signal Processing*, John Wiley & Sons, New York, 1987.

[4] L.P. Huelsman and P.E. Allen, *Introduction to the Theory and Design of Active Filters*, McGraw Hill Book Company, New York, 1980.

Example 5-1 - Fifth-order, Low Pass, Continuous Time Filter using the Cascade Approach

Design a cascade, switched capacitor realization for a Chebyshev filter approximation to the filter specifications of T_{PB} = -1dB, T_{SB} = -25dB, f_{PB} = 100kHz and f_{SB} = 150kHz. Give a schematic and component value for the realization. Also simulate the realization and compare to an ideal realization.

Solution

First we see that $\Omega_n = 1.5$. Next, recall that when $T_{PB} = -1dB$ that this corresponds to $\varepsilon = 0.5088$. We find that $N = 5$ satisfies the specifications ($T_{SB} = -29.9dB$).

Find the roots for the Chebyshev approximation with ε =0.5088 for $N = 5$. Therefore we can express the normalized lowpass transfer function as,

$$T_{LPn}(s_n) = T_1(s_n)T_2(s_n)T_3(s_n) = \left(\frac{0.2895}{s_n+0.2895}\right)\left(\frac{0.9883}{s_n^2+0.1789s_n+0.9883}\right)\left(\frac{0.4293}{s_n^2+0.4684s_n+0.4293}\right).$$

Next, we design each of the three stages individually.

Example 5-1 - Continued

Stage 1 - First-order Stage

Let us select OTA-C shown to realize the first-order stage. It is easy to show that the transfer function is given as

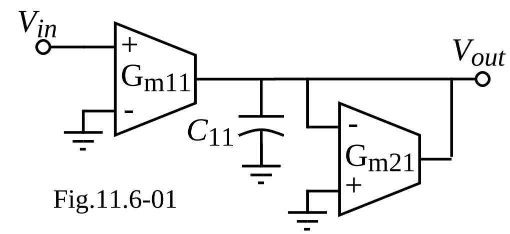


Fig.11.6-01

$$T_1(s_n) = \frac{G_{m11n}}{sC_{11n} + G_{m21n}}$$

Equating to $T_1(s_n)$ gives $G_{m11n} = G_{m21n} = 0.2895$S and C_{11n} =1F.

Next, we unnormalize these values using a value of $\Omega_n = 10^5 \cdot 2\pi$ and an arbitrary impedance scaling of $z_o = 10^5$. Thus, we get the following denormalizes values of

$$G_{m11} = G_{m21} = \frac{0.2895\text{S}}{10^5} = \underline{\underline{2.895\mu\text{S}}} \quad \text{and} \quad C_{11} = \frac{1\text{F}}{10^5 \cdot 2\text{x}10^5\pi} = \underline{\underline{15.9\text{pF}}}$$

Stage 2 - Second-order, High-Q Stage

The next product of $T_{LPn}(s_n)$ is

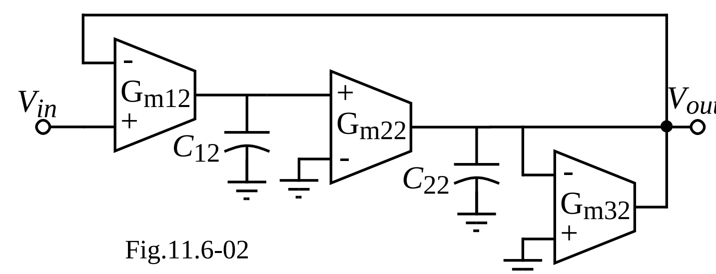


Fig.11.6-02

$$T_2(s_n) = \frac{0.9883}{s_n^2 + 0.1789s_n + 0.9883} = \frac{\dfrac{G_{m12}G_{m22}}{C_{12}C_{22}}}{s^2 + s\dfrac{G_{m32}}{C_{22}} + \dfrac{G_{m12}G_{m22}}{C_{12}C_{22}}}$$

Assume that $G_{m12n} = G_{m22n}$ and $C_{12n} = C_{22n} = 1$F. Equating coefficients gives, $G_{m12n} = G_{m22n} =$ 0.99413S and G_{m32n} =0.1789S,

Denormalizing with $\Omega_n = 10^5 \cdot 2\pi$ and $z_o = 10^5$ gives $G_{m12} = G_{m22} = \underline{\underline{9.94133\mu\text{S}}}$, $G_{m32} = \underline{\underline{1.789\mu\text{S}}}$

and $C_{12} = C_{22} = \underline{\underline{15.9\text{pF}}}$.

Example 5-1 - Continued

Stage 3 - Second-order, Low-Q Stage

The next product of $T_{LPn}(s_n)$ is

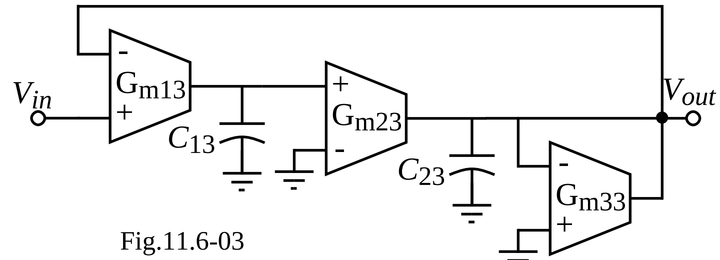


Fig.11.6-03

$$T_3(s_n) = \frac{0.4293}{s_n^2+0.4684s_n+0.4293} = \frac{\frac{G_{m13}G_{m23}}{C_{13}C_{23}}}{s^2+s\frac{G_{m33}}{C_{23}}+\frac{G_{m13}G_{m23}}{C_{13}C_{23}}}$$

Assume that $G_{m13n} = G_{m23n}$ and $C_{13n} = C_{23n} = 1$F. Equating coefficients gives, $G_{m13n} = G_{m23n} = 0.6552$S and $G_{m32n} = 0.4684$S,

Denormalizing with $\Omega_n = 10^5 \cdot 2\pi$ and $z_o = 10^5$ gives $G_{m13} = G_{m23} =$ 6.552µS, $G_{m33} =$ 4.684µS

and $C_{13} = C_{23} =$ 15.9pF.

Overall realization:

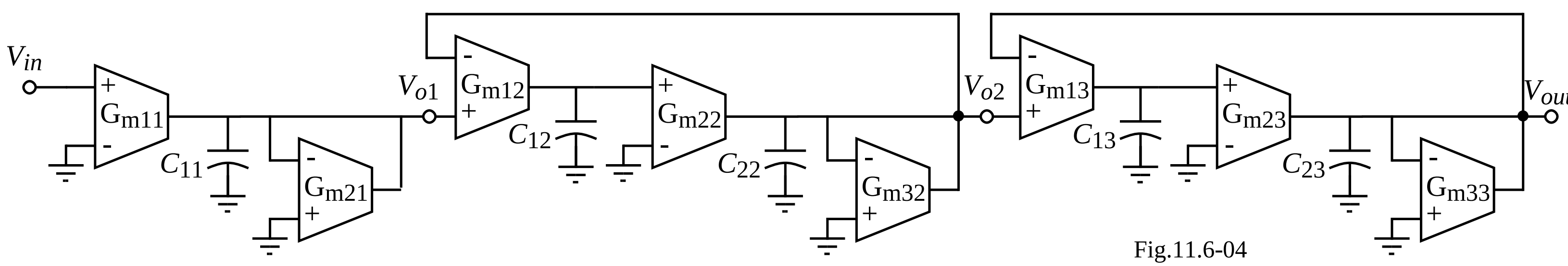


Fig.11.6-04

All capacitors are 15.9pF and $G_{m11} = G_{m21} = 2.895$µS, $G_{m12} = G_{m22} = 9.94133$µS, $G_{m32} = 1.789$µS, $G_{m13} = G_{m23} = 6.552$µS, and $G_{m33} = 4.684$µS.

Example 5-1 - Continued

Simulation results:

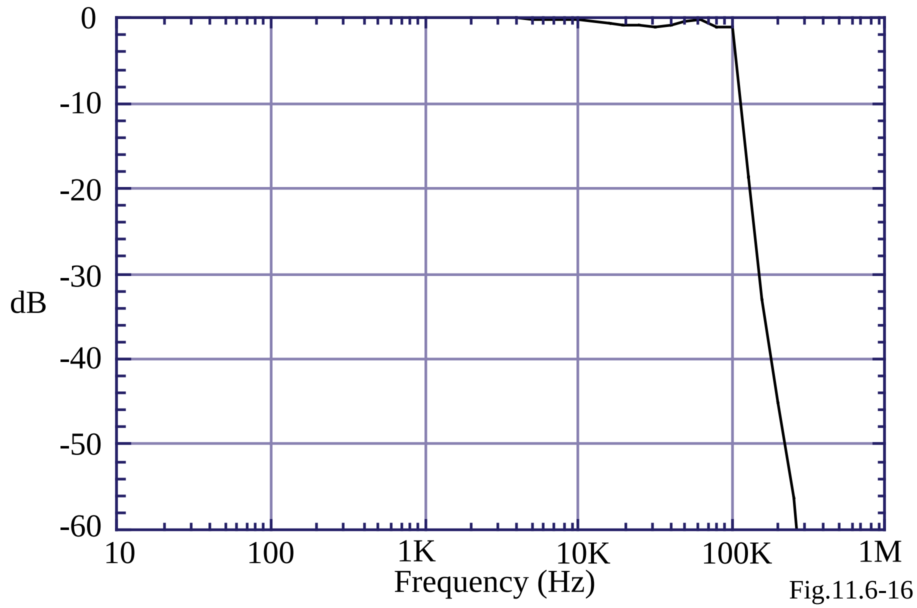


Fig.11.6-16

```
EXAMPLE 5-1 - OTA-C Cascade Filter
.OPTION LIMPTS=1000
VIN 1 0 DC 0 AC 1.0
R11 1 0 10000MEG
G11 0 2 1 0 2.895U
C11 2 0 15.9P
R2 2 0 10000MEG
G21 2 0 2 0 2.895U
G12 0 3 2 4 9.94133U
C12 3 0 15.9P
R3 3 0 10000MEG
R4 4 0 10000MEG
G32 4 0 4 0 1.789U
G13 0 5 4 6 6.552U
C13 5 0 15.9P
R5 5 0 10000MEG
G23 0 6 5 0 6.552U
C23 6 0 15.9P
R6 6 0 10000MEG
G33 6 0 6 0 4.684U
.AC DEC 10 1 10MEG
.PRINT AC VDB(6) VP(6) VDB(4) VP(4) VDB(2) VP(2)
```

```
G22 0 4 3 0 9.94133U
C22 4 0 15.9P
.PROBE
.END
```

Low-Power, High Accuracy Continuous Time 450 KHz Bandpass Filter

450 KHz Bandpass filter specifications:

Specification	Value
Lower Stopband	400 KHz
Lower Passband	439 KHz
Upper Passband	461 KHz
Upper Stopband	506 KHz
Passband Ripple	<0.5dB
Stopband Attenuation	>55dB
Tuning Resolution	<1%
Power Consumption	≈2mA
Total Inband Noise	$<340\mu V_{rms}$

Cascaded-biquad BFP structure:

To compromise between the group delay and complexity, a Chebyshev approximation of 12th order is used.

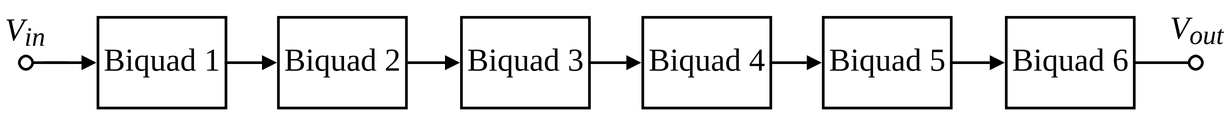


Fig.11.6-06

455 KHz Filter - Continued

Table 3: Values of the parameters of a biquad

Biquad	K_2	K_0	ω_0	Q
1	4.494340×10^{-1}	4.580164×10^{12}	2.826590×10^{6}	6.93765
2	7.782568×10^{-1}	4.875459×10^{12}	2.826590×10^{6}	6.937844
3	3.926305×10^{-1}	2.251889×10^{12}	2.669664×10^{6}	17.80233
4	3.723485×10^{-1}	4.144164×10^{12}	2.992737×10^{6}	17.80241
5	6.946693×10^{-1}	1.340316×10^{12}	2.992739×10^{6}	17.80286
6	2.790028×10^{-1}	9.230595×10^{12}	2.669660×10^{6}	17.80292

455 KHz Filter - Continued

Biquad Design:

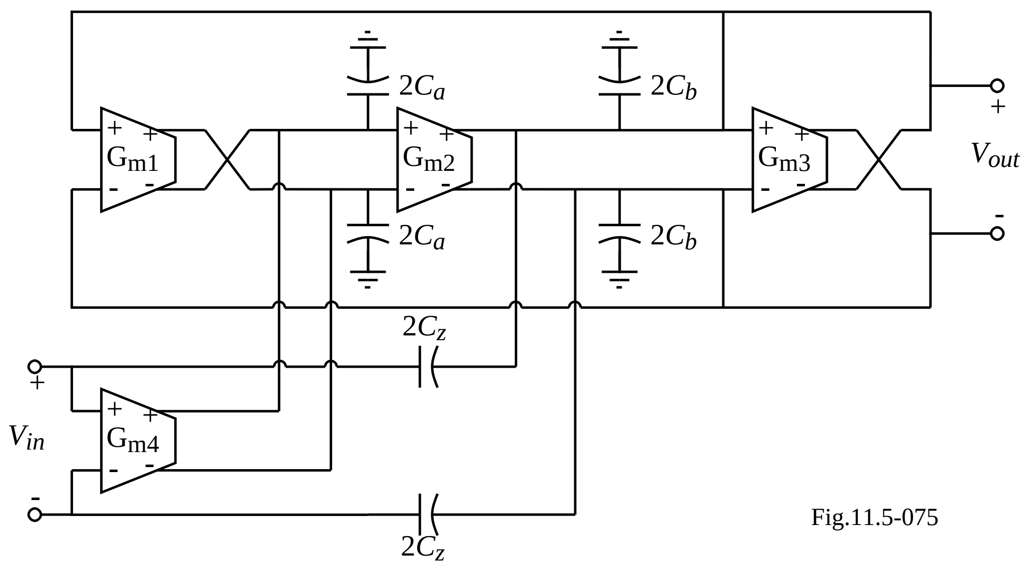


Fig.11.5-075

Biquad	G_{m1}(S)	G_{m2}(S)	G_{m3}(S)	G_{m4}(S)	C_a(pF)	C_a(pF)	C_z(pF)
1	3.06493×10^{-5}	3.06493×10^{-5}	1.384522×10^{-5}	1.58119×10^{-5}	4.0	70	30
2	2.82619×10^{-5}	2.82619×10^{-5}	1.179439×10^{-5}	1.553489×10^{-5}	4.8	30	70
3	3.636792×10^{-5}	3.636792×10^{-5}	1.376188×10^{-5}	1.034177×10^{-5}	3.8	70	50
4	3.481898×10^{-5}	3.481898×10^{-5}	1.125281×10^{-5}	1.449968×10^{-5}	7.8	60	20
5	3.909740×10^{-5}	3.909740×10^{-5}	1.064108×10^{-5}	5.265746×10^{-5}	10.4	60	8

6	3.902157×10^{-5}	3.902157×10^{-5}	1.188262×10^{-5}	4.548467×10^{-5}	6.8	60	40

455 KHz Filter - Continued

Noise minimization of the OTA:

The equivalent input-referred noise of the OTA is,

$$\overline{v_n^2} = 2\sum_{i=1,4,10,12} \overline{v_{ni}^2} = 2\sum_{i=1,4,10,12}\left[\left(\frac{g_{mi}}{g_{m1}}\right)^2 \overline{e_{ni}^2}\right]$$

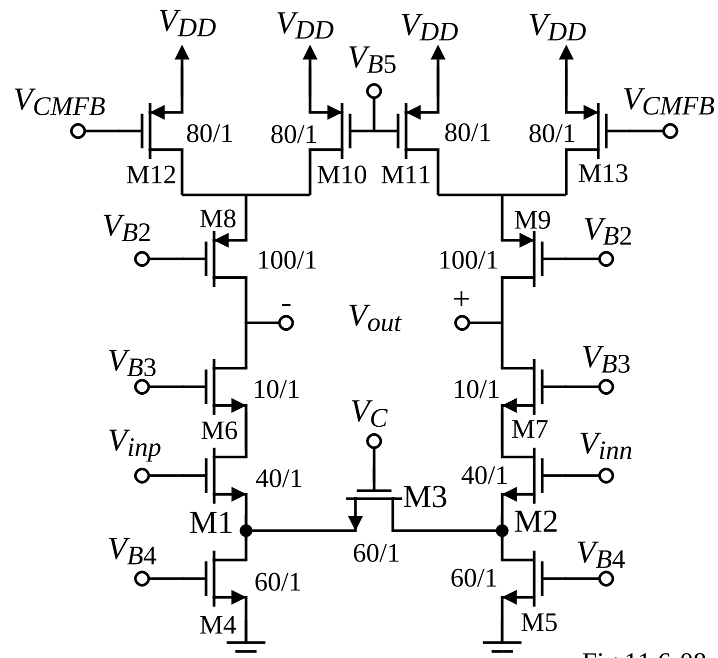


Fig.11.6-08

where

$\overline{v_{ni}^2}$ = Noise power of transistors M1, M4, M10, and M12

Using the definition of flicker-noise spectral density we get

$$\overline{e_{nif}^2} = S(f) = \frac{K_F}{C_{ox}WL}\frac{\Delta f}{f}$$

Substituting into the above equation gives

$$\overline{v_n^2} = 2\sum_{i=1,4,10,12}\left\{\left(\frac{g_{mi}}{g_{m1}}\right)^2\left[\frac{K_{Fi}}{C_{ox}W_iL_i}\frac{\Delta f}{f} + 4kT\left(\frac{2}{3g_{mi}}\Delta f\right)\right]\right\}$$

It can be shown that the equivalent input-referred noise is minimized in the bandwidth of interest if

$$L_1 \approx \sqrt{\left[\frac{1}{L_4{}^2} + \frac{K_{Fp}}{KFn} \cdot \frac{K_P'}{K_N'} \cdot \frac{2}{L_{10}{}^2}\right]^{-1}} \approx \sqrt{\left[1 + \frac{1.3\text{x}10\text{-}24}{3.6\text{x}10\text{-}24} \cdot \frac{50\mu}{140\mu} \cdot 2\right]^{-1}} \approx 1\mu\text{m}$$

455KHz Filter - Continued

Nominal Frequency Response:

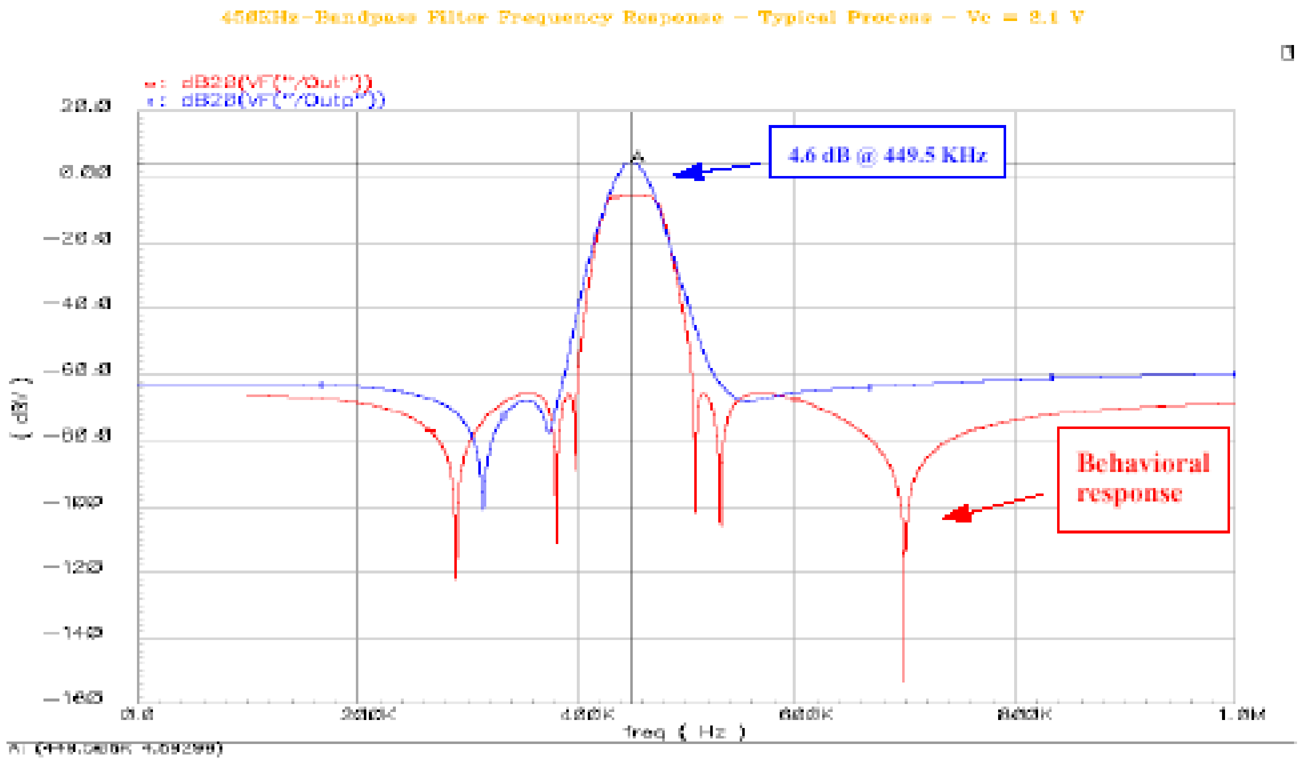


Fig. 8: Frequency-responses of the 450-KHz BPF - $V_c = 2.1$ V

455KHz Filter - Continued

Temperature dependence of the filter:

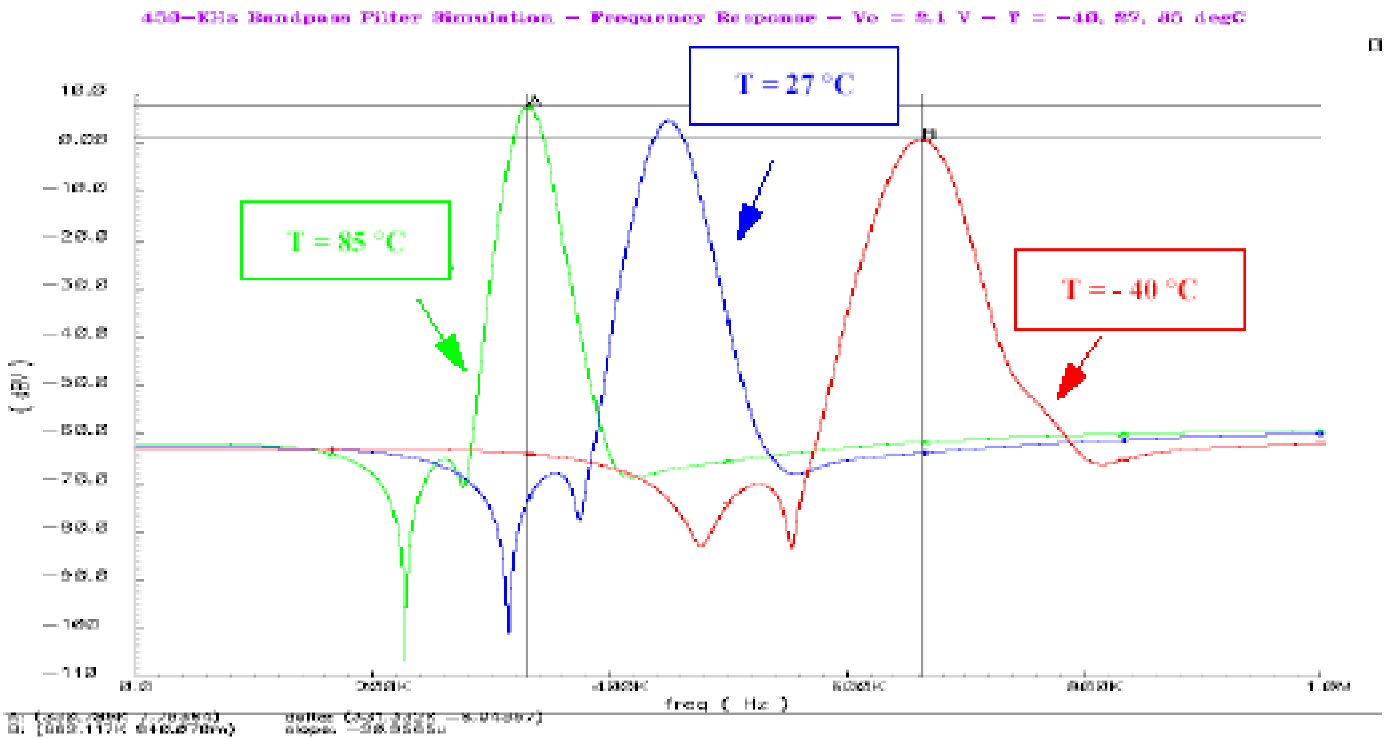


Fig. 10: Frequency-response of the 450-KHz BPF - $V_c = 2.1$ V - T $\in$ [- 40 °C, 27 °C, 85 °C] - Typical

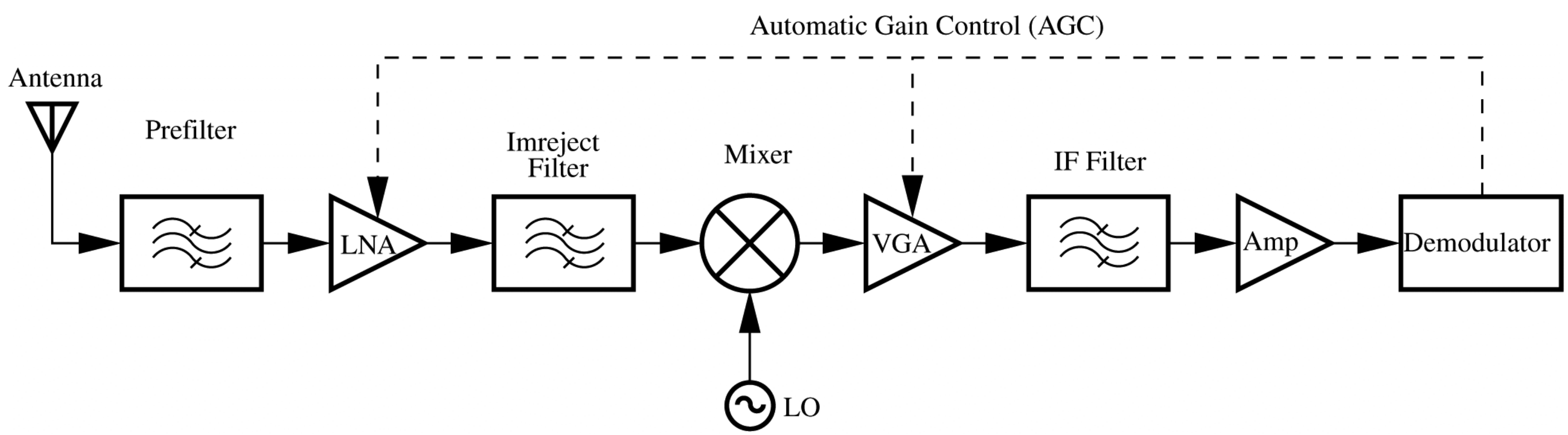
We will consider the tuning of this filter in the next section.

RF Image Reject Filter

In many RF and IF applications, bandpass filters are very difficult because large Q exacerbates the difficulties in achieving the desired performance.

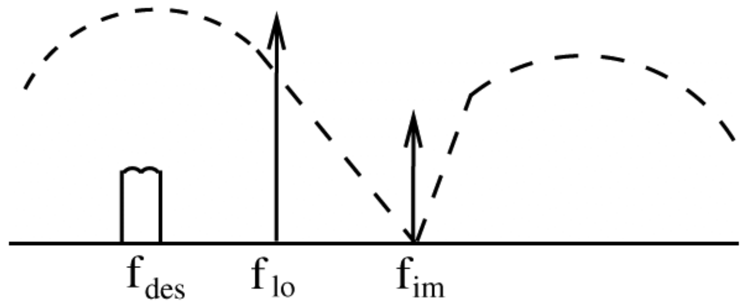
A possible solution is to use notch filtering in place of bandpass filtering. The following is an example of a image reject filter suitable for GSM applications.

Typical heterodyne receiver:



RF Image Reject Filter

Principle:



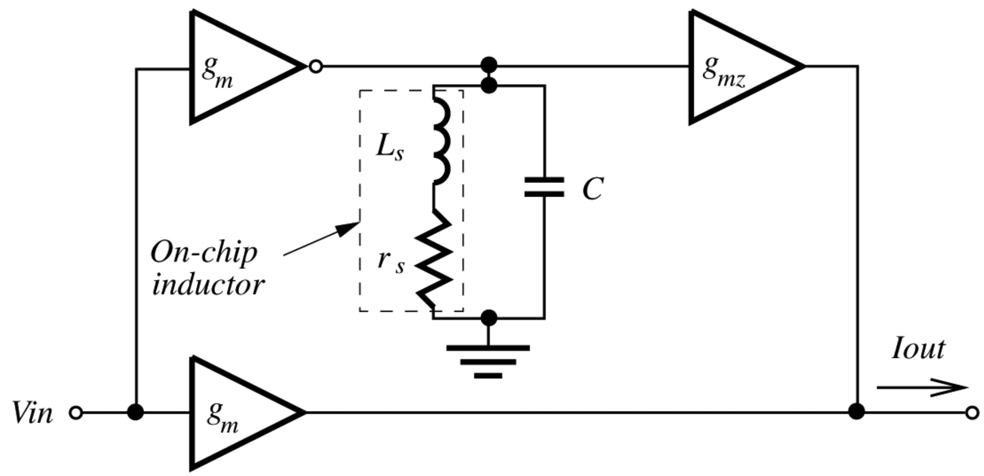
Does not require high-Q inductors

No stability problems

No excessive noise and linearity degradation

RF Image Reject Filter

Filter Architecture:



Comments:

- Uses low-Q, on-chip spiral inductors
- Linearity and noise is determined by the transconductors and the impedance level

Transfer function:

$$G_m(s) = \frac{I_{out}}{V_{in}}(s) = g_m \frac{s^2 + s\left(\frac{r_s}{L_s} - \frac{g_{mz}}{C}\right) + \left(\frac{1 - g_{mz} r}{L_s C}\right)}{s^2 + s\frac{r_s}{L_s} + \frac{1}{L_s C}} = g_m \frac{s^2 + \frac{\omega_z}{Q_z} s + \omega_z{}^2}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p{}^2}$$

where

$$\omega_z = \sqrt{\frac{1 - g_{mz} r_s}{L_s C}} \qquad Q_z = \frac{\omega_z}{\frac{r_s}{L_s} - \frac{g_{mz}}{C}}$$

RF Image Reject Filter

Transconductors:

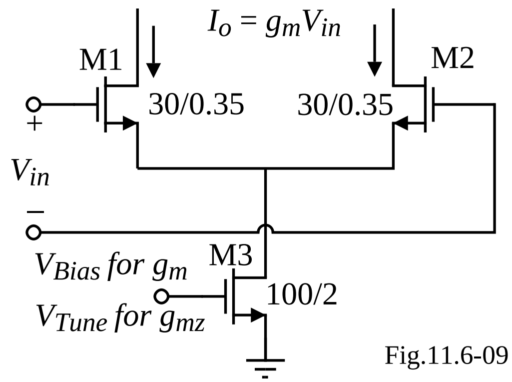


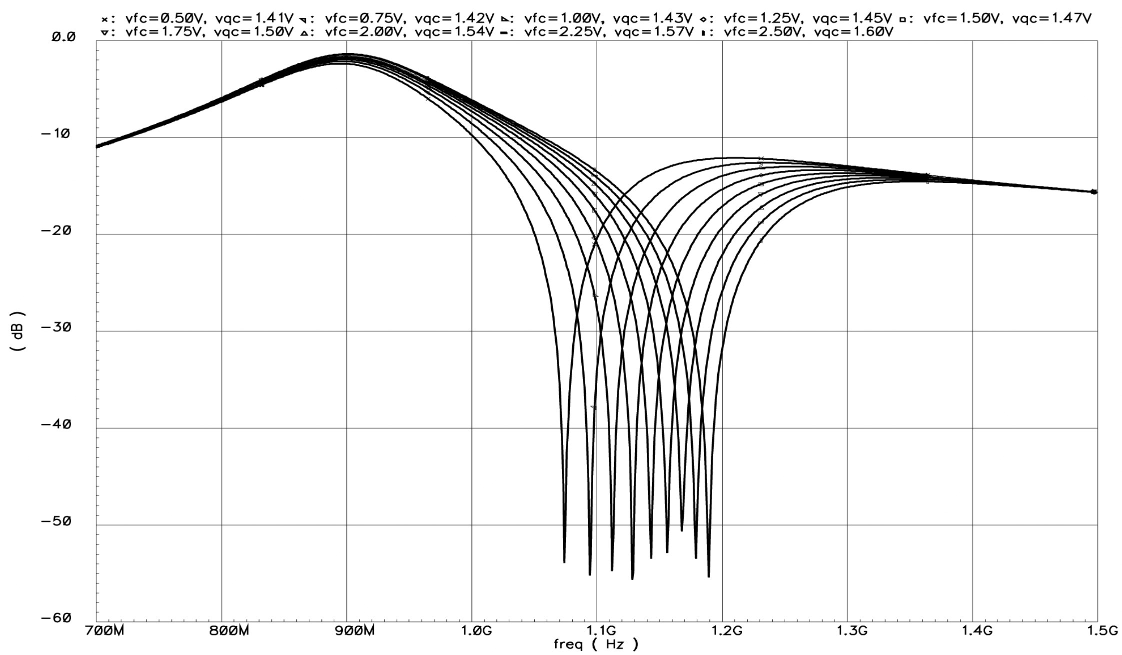
Fig.11.6-09

Comments:

- Simple source-coupled differential pair is used for less noise
- Minimum size transistors is selected to minimize parasitic capacitance
- Linearity is a function of the gate-source overdrive voltage and hence the tail current

RF Image Reject Filter

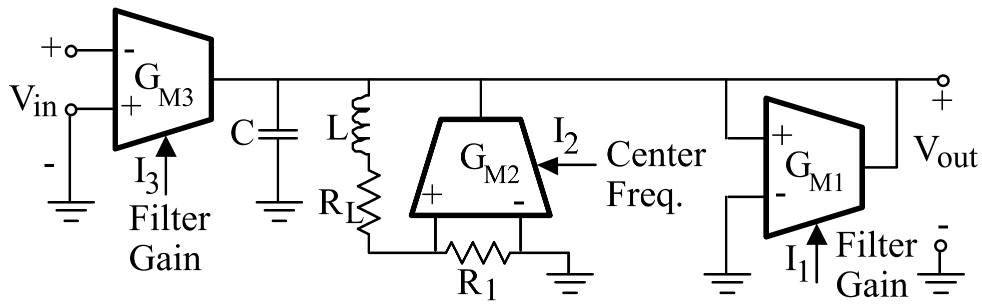
Frequency Response:



Tuning to be considered later.

Q-Enhanced LC Filters

Second-order, Bandpass Filter:



1GHz capability in bipolar.

Q-Enhanced LC Filters - Continued

CMOS Q-enhanced Filter (Kuhn[†]):

Second-order circuit-

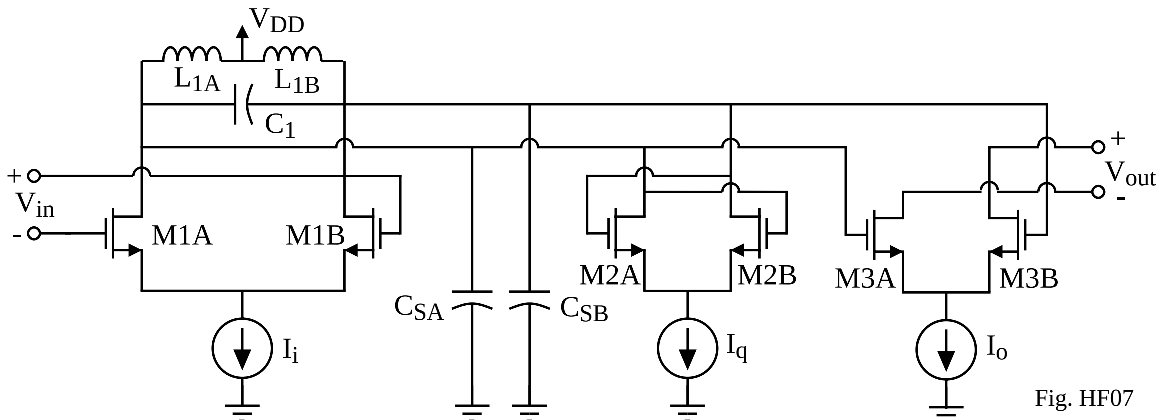


Fig. HF07

Differential architecture allows:

- Negative resistances implemented by positive feedback (M2A & M2B)
- Reduces power supply noise
- Second-order nonlinearities are cancelled
- Increases the signal swing

Results:

Q's of up to 10,000 at 100MHz

† W.B. Kuhn, "Design of Integrated, Low Power, Radio Receivers in BiCMOS Technologies, PhD Dissertation, Virginia Polytechnic Institute and State University, Dec. 1995.

Q's of up to 100 at 200MHz

Ladder Filter Design (Low Pass)

1.) From T_{BP}, T_{SB}, and Ω_n (or A_{PB}, A_{SB}, and Ω_n) determine the required order of the filter approximation.

2.) From tables similar to Table 9.7-3 and 9.7-2 find the *RLC* prototype filter approximation.

3.) Write the state equations and rearrange them so each state variable is equal to the integrator of various inputs.

4.) Realize each of rearranged state equations by continuous time integrators or switched capacitor integrators.

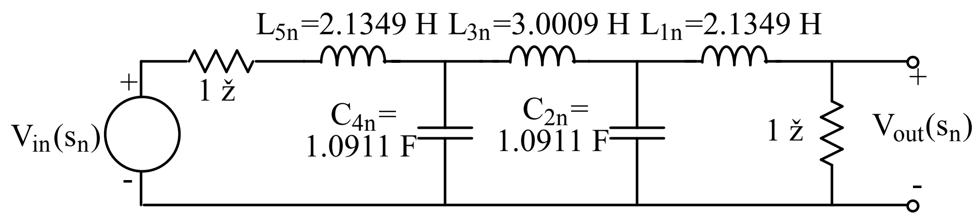
5.) Denomalize the filter if necessary.

Example 5-2 - Fifth-order, Low Pass, OTA-C Filter using the Ladder Approach

Design a ladder, OTA-C realization for a Chebyshev filter approximation to the filter specifications of T_{BP} = -1dB, T_{SB} = -25dB, f_{PB} = 100kHz and f_{SB} = 150 kHz. Give a schematic and component value for the realization. Also simulate the realization and compare to an ideal realization. Adjust your design so that it does not suffer the -6dB loss in the pass band. (Note that this example should be identical with Ex. 5-1.)

Solution

From Ex. 5-1, we know that a 5th-order, Chebyshev approximation will satisfy the specification. The corresponding low pass, *RLC* prototype filter is



Next, we must find the state equations and express them in the form of an integrator. Fortunately, the above results can be directly used in this example.

Example 5-2 - Continued

$$L_{1n}:\quad V_1'(s_n) = \frac{R'}{s_n L_{1n}}\left[V_{in}(s_n) - V_2(s_n) - \left(\frac{R_{0n}}{R'}\right)V_1'(s_n)\right] \qquad (1)$$

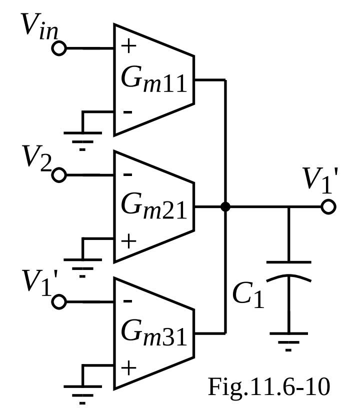


Fig.11.6-10

This equation can be realized by the OTA-C integrator shown which has one noninverting input and two inverting inputs. The transfer function for this integrator is

$$V_1'(s_n) = \frac{1}{C_{1n}s_n}\left[G_{m11n}V_{in}(s_n) - G_{m21n}V_2(s_n) - G_{m31n}V_1'(s_n)\right] \quad (2)$$

Choosing $L_{1n} = C_{1n} = 2.1349\text{F}$ gives $G_{m11n} = G_{m21n} = G_{m31n} = 1\text{S}$

assuming that $R_{0n} = R' = 1\Omega$. Also, double the value of G_{m11n} ($G_{m11n} = 2\text{S}$) in order to gain 6*dB* and remove the -6*dB* of the *RLC* prototype.

$$C_{2n}:\qquad V_2(s_n) = \frac{1}{s_n R' C_{2n}}\left[V_1'(s_n) - V_3'(s_n)\right] \qquad (3)$$

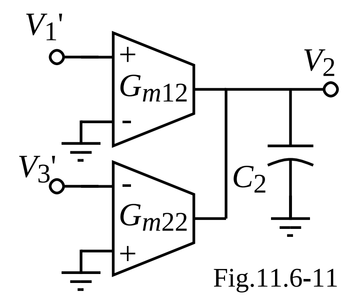


Fig.11.6-11

This equation can be realized by the OTA-C integrator shown which has one noninverting input and one inverting input. As before we write that

$$V_2(s_n) = \frac{1}{s_n C_{2n}}\left[G_{m12n}V_1'(s_n) - G_{m22n}V_3'(s_n)\right] \qquad (4)$$

Choosing $C_{2n} = 1.0911\text{F}$ gives $G_{m12n} = G_{m22n} = 1\text{S}$

Example 5-2 - Continued

L_{3n}: $\quad V'_{3n}(s) = \frac{R'}{sL_{3n}} [V_2(s_n) - V_4(s_n)]$ (5)

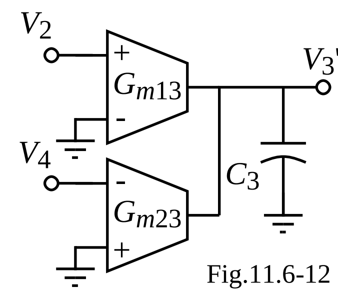


Fig.11.6-12

Eq. (5) can be realized by the OTA-C integrator shown which has one noninverting input and one inverting input. For this circuit we get

$$V'_3(s_n) = \frac{1}{s_n C_{3n}}\left[G_{m13n}V_2(s_n) - G_{m23n}V_4(s_n)\right] \quad (6)$$

Choosing $L_{3n} = C_{3n} = 3.0009$F gives $G_{m13n} = G_{m23n} = 1$S

C_{4n}: $\quad V_4(s_n) = \frac{1}{sR'C_{4n}} [V'_3(s_n) - \left(\frac{R'}{R_{6n}}\right) V_{out}(s_n)]$ (7)

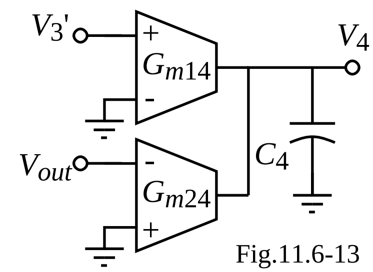


Fig.11.6-13

Eq. (7) can be realized by the OTA-C integrator shown with one noninverting and one inverting input. As before we write that

$$V_4(s_n) = \frac{1}{s_n C_{4n}}\left[G_{m14}V'_3(s_n) - G_{m24}V_{out}(s_n)\right] \quad (8)$$

Choosing $C_{4n} = 1.0911$F gives $G_{m14n} = G_{m24n} = 1$S

L_{5n}: $\quad V_{out}(s_n) = \frac{R_{6n}}{sL_{5n}} [V_4(s_n) - V_{out}(s_n)]$ (9)

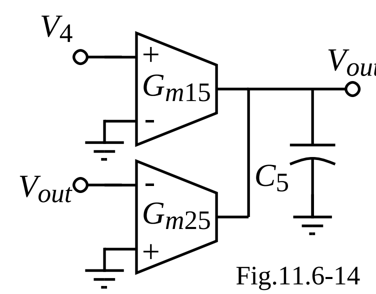


Fig.11.6-14

The last state equation, Eq. (9), can be realized by the OTA-C integrator shown which has one noninverting input and one inverting input. For this circuit we get

$$V_{out}(s_n) \approx \frac{1}{s_n C_{5n}}\left[G_{m15}V_4(s_n) - G_{m25}V_{out}(s_n)\right]. \quad (10)$$

Choosing $L_{5n} = C_{5n} = 2.1439\text{F}$ gives $G_{m14n} = G_{m24n} = 1\text{S}$

Example 5-2 - Continued

Realization:

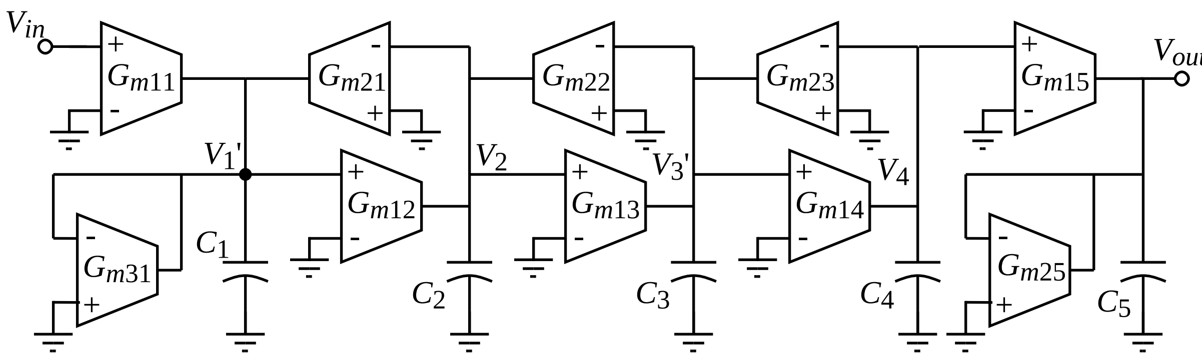


Fig.11.6-15

To denormalize, $\Omega_n = 200{,}000\pi$ and pick $z_o = 10^5$.

$\therefore$ C_1 = 33.9780pF, C_2 = 17.3654pF, C_3 = 47.7608pF, C_4 = 17.3654pF, and C_1 = 33.9780pF

All transconductances are G_{mi} = 10µS.

Example 5-2 - Continued

Simulation of Example 5-2:

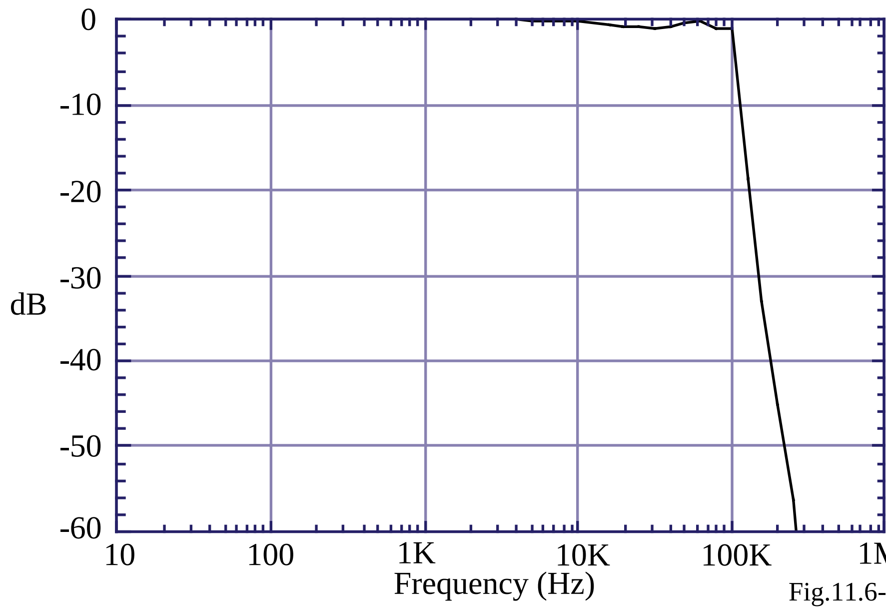
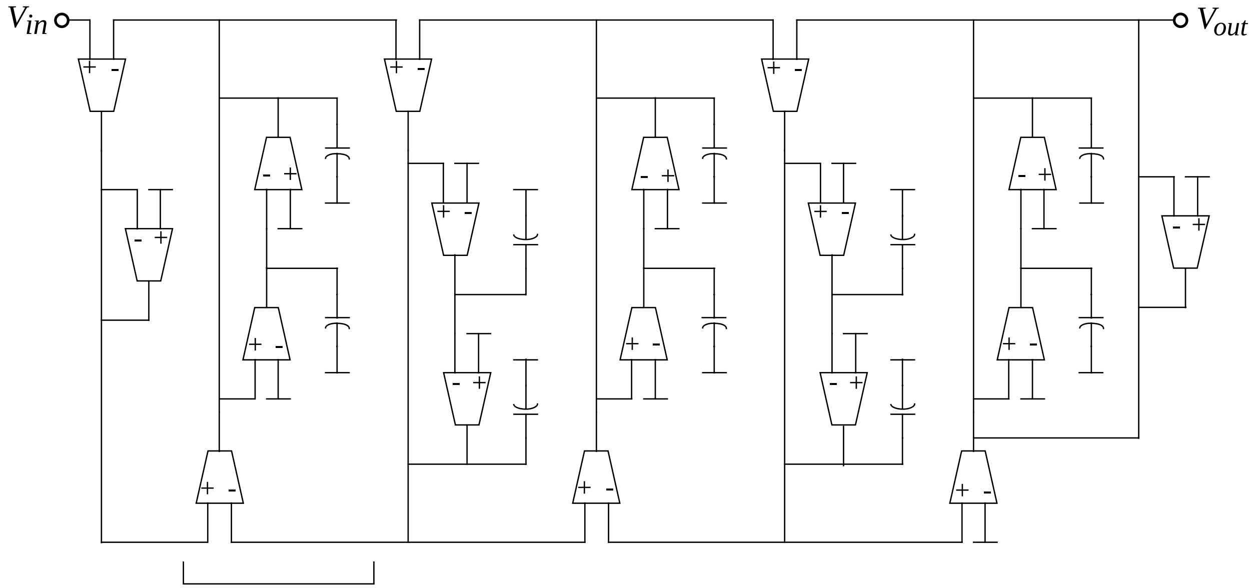


Fig.11.6-16

10th Order Bandpass Ladder OTA-C Filter



: An example of the leapfrog BPF structure - Order N = 10

Log Domain Filters

Fourth-order log-domain bandpass filter:

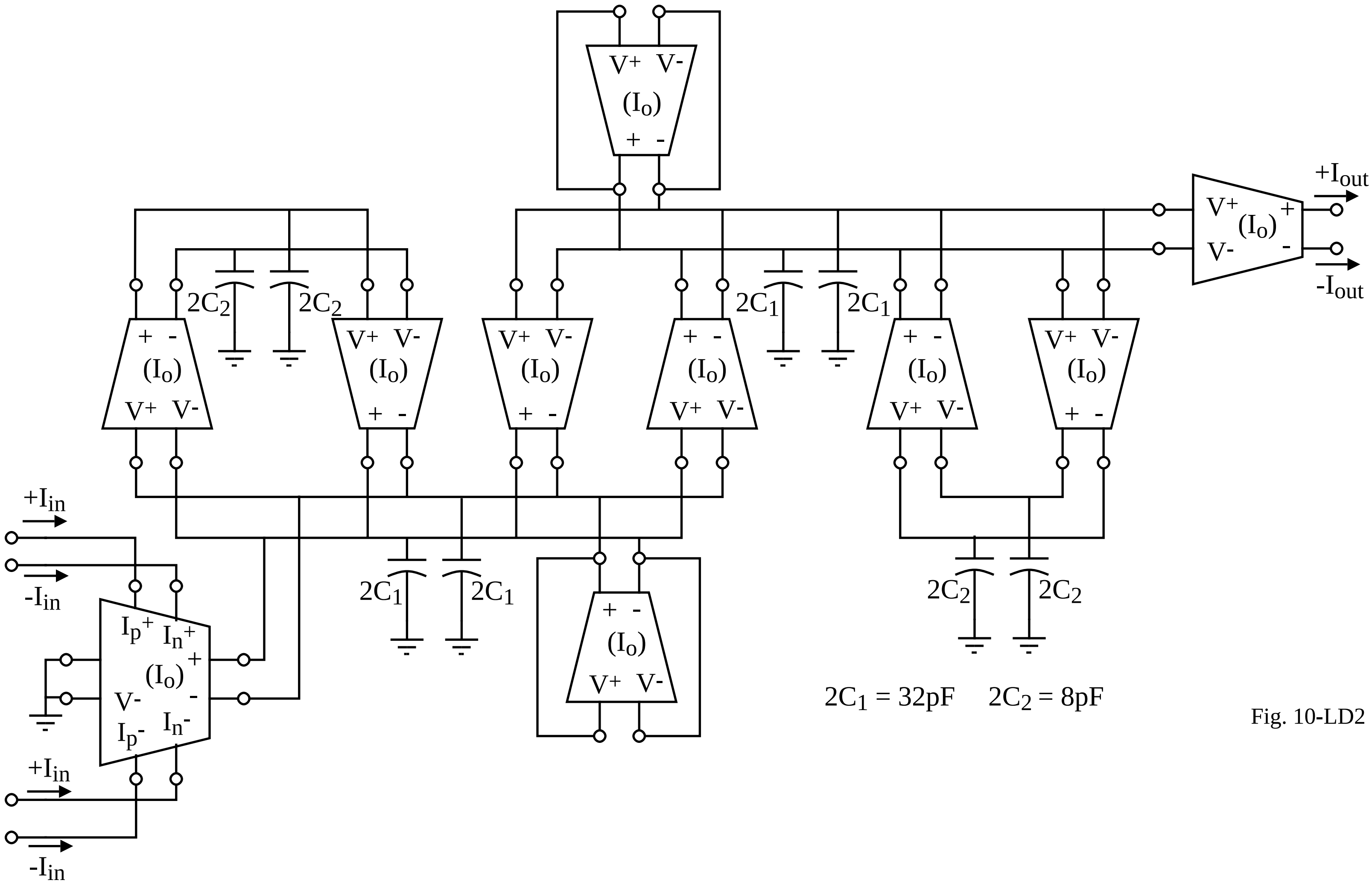


Fig. 10-LD2

Log Domain Filters - Continued

Measured results of the fourth-order, bandpass filter of previous page:

Characteristic	Value
Frequency tuning range	50MHz - 130MHz
Integrator bias current	220µA @ f_o = 130MHz
Integrating capacitors	8pF and 32pF
Power consumption with V_{CC} = 5V	233mW @ f_o = 130MHz
Quality factor	≈
3rd-order intermodulation distortion	-45.6dB @ f_o = 83MHz
Output current 3rd-order intercept point	-14.5dBm @ f_o = 83MHz
Output noise power density @ f_o = 83MHz	-152.4 dBm/Hz

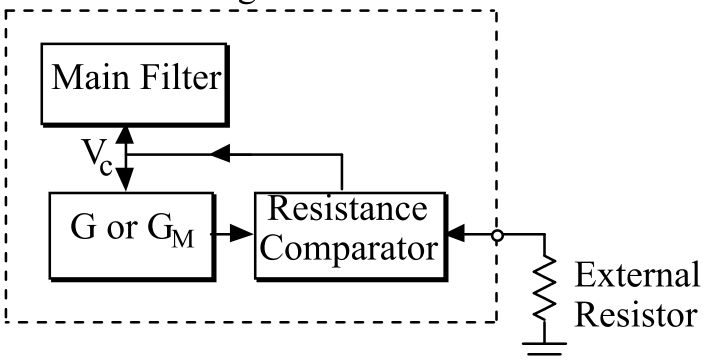
SECTION 6 - FILTER TUNING

Tuning Methods For Continuous Time Filters

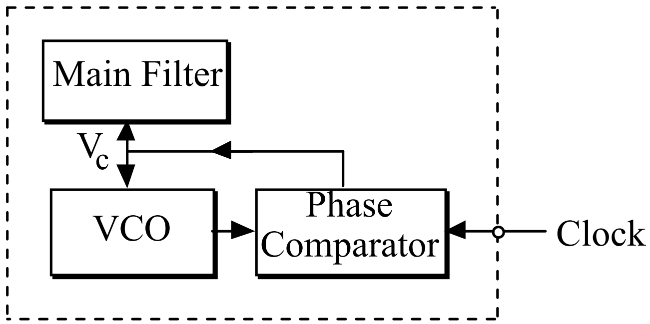
In all tuning methods, an on-chip reference circuit is monitored and tuned. The main filter becomes tuned by virtue of matching with the on-chip reference circuit.

Common Techniques for Automatic Filter Tuning:

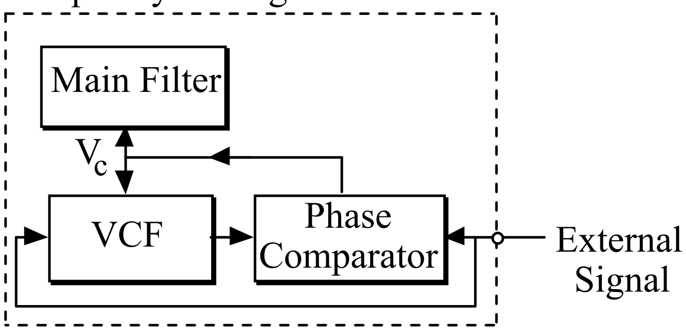
Resistive Tuning:



Phase Tuning:



Frequency Tuning:



Tuning Methods

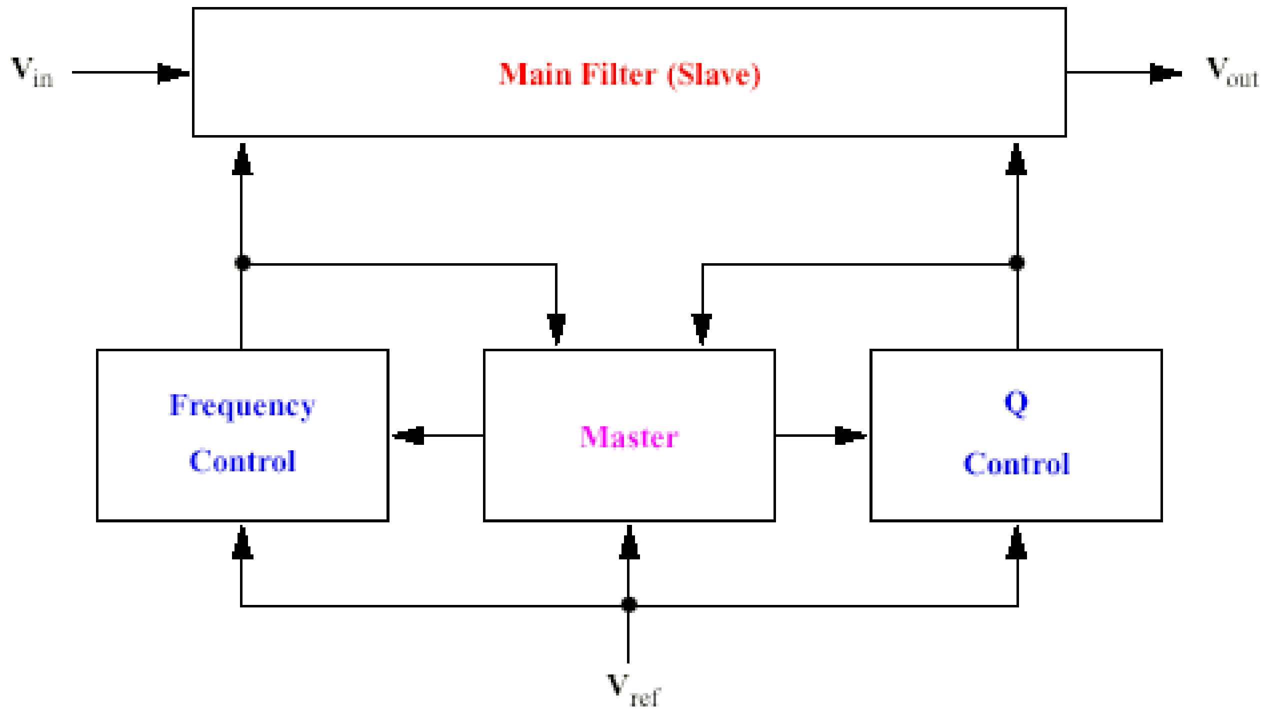
Indirect method:

- The filter is tuned in place ("in situ"). A master filter which is not in the system is tuned and the tuning signals are applied to the slave filter which is in the system.
- This can be done at a high rate so the filter is constantly being tuned or infrequently such as at power up or during some predetermined calibration period.

Direct method:

- The filter is taken out of the circuit and tuned. If another filter has been tuned it can be inserted in the circuit while the other filter is being tuned.

Master-Slave (Indirect) Tuning Scheme

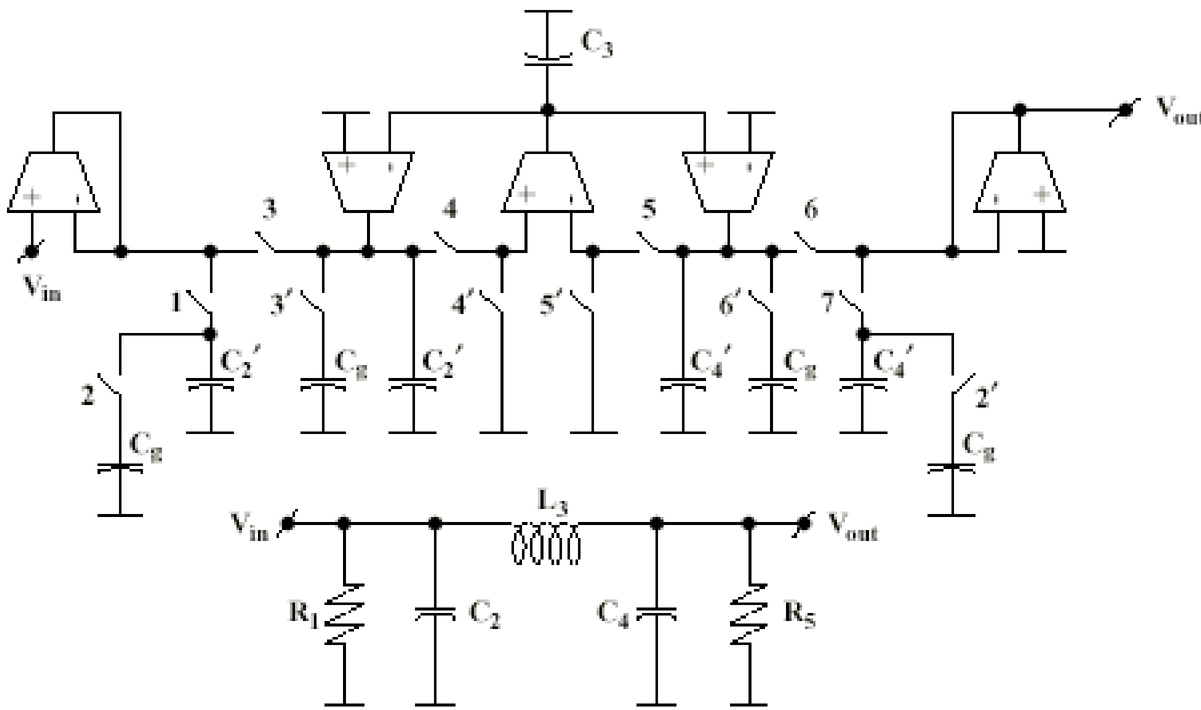


Comments:

- Filter (slave) does not need to be disconnected from the system
- Two filters are required

Direct Tuning

Example:



Tuning procedure:

1.) The filter is take apart into several first- or second-order sections.

2.) Each section is tuned to the center frequency.

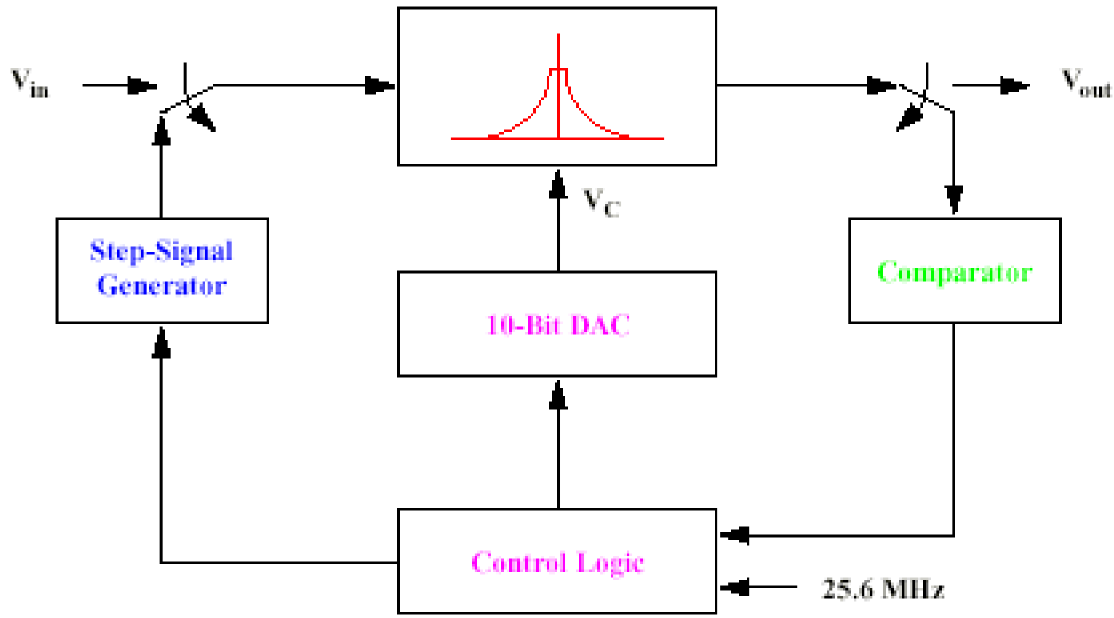
Comments:

- Necessary switches may influence the filter performance

- Need to remember the tuning voltages (memory)

Tuning a High-Q Bandpass Filter using Direct Tuning

Block diagram for the direct tuning of the previous 455KHz bandpass filter† .



Tuning procedure:

1.) The filter is removed from the circuit and a step voltage applied.

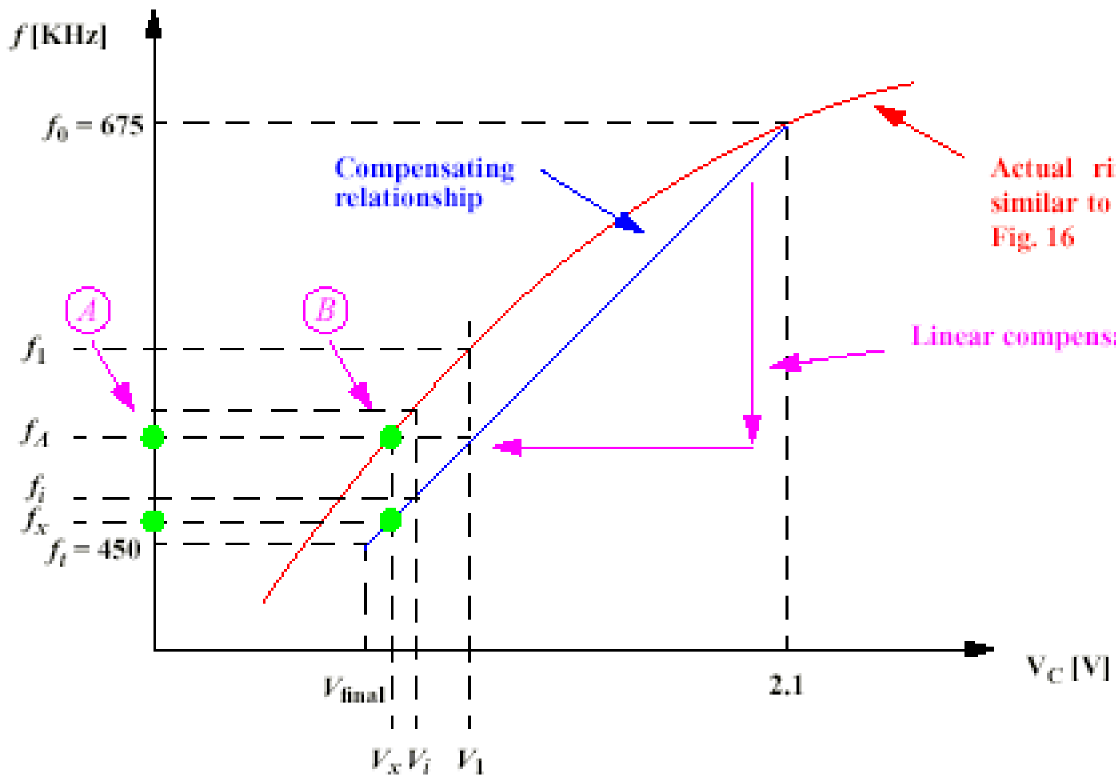
2.) The number of cycles in the ringing waveform and their period is used to tune the filter.

3.) Most tuning algorithms work better with a linear frequency tuning voltage relationship.

† H. Yamazki, K. Oishi,, and K. Gotoh, , "An Accurate Center Frequency Tuning Scheme for 450KHz CMOS Gm-C Bandpass Filter," *IEEE. J. of Solid-State Circuits*, vol. 34, no. 12, pp. 1691-1697, Dec. 1999.

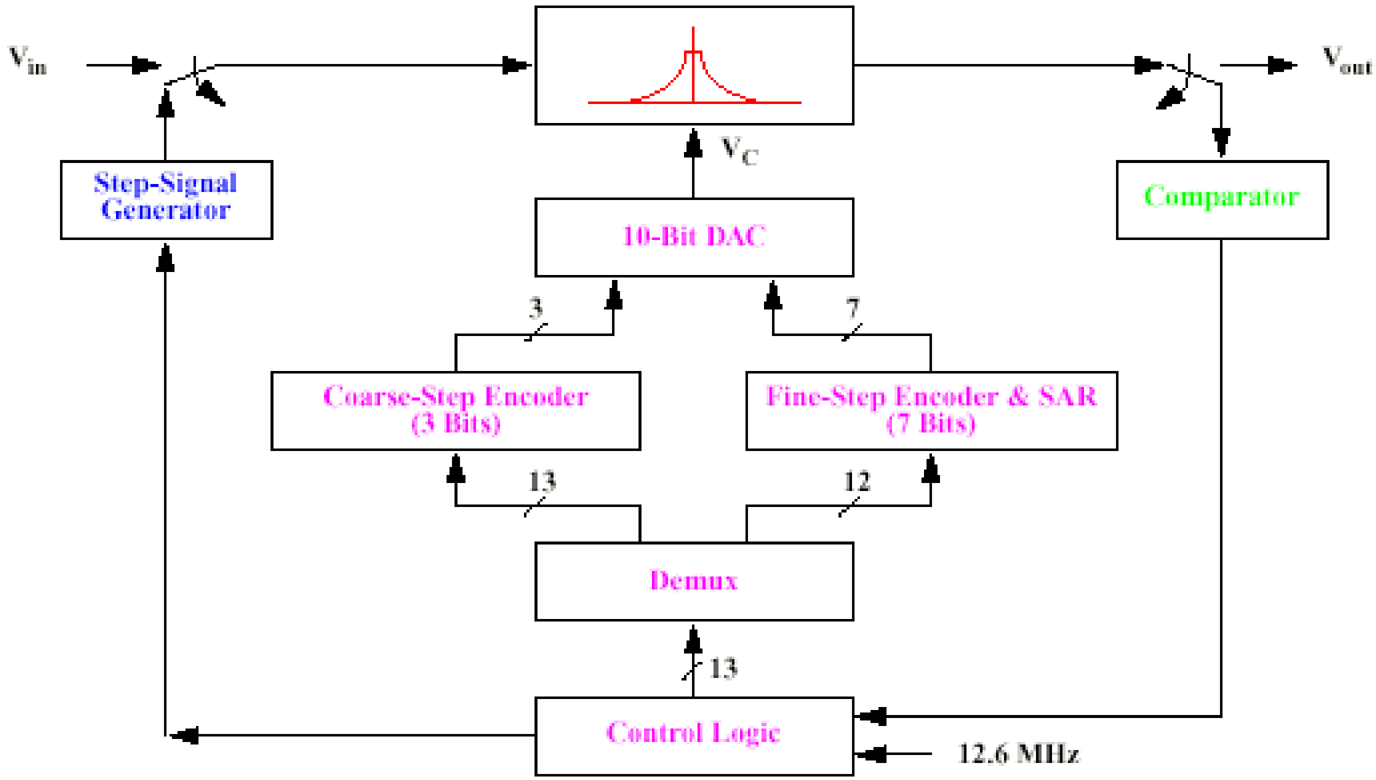
A Linear, Differential-Compensated Tuning Scheme

Avoids the problem of nonlinearity in the tuning scheme.



Oscillations between two states of tuning iteration can occur. This becomes a problem as the frequency closely approaches the desired frequency.

A Tuning Scheme that Avoids the Oscillation Problem



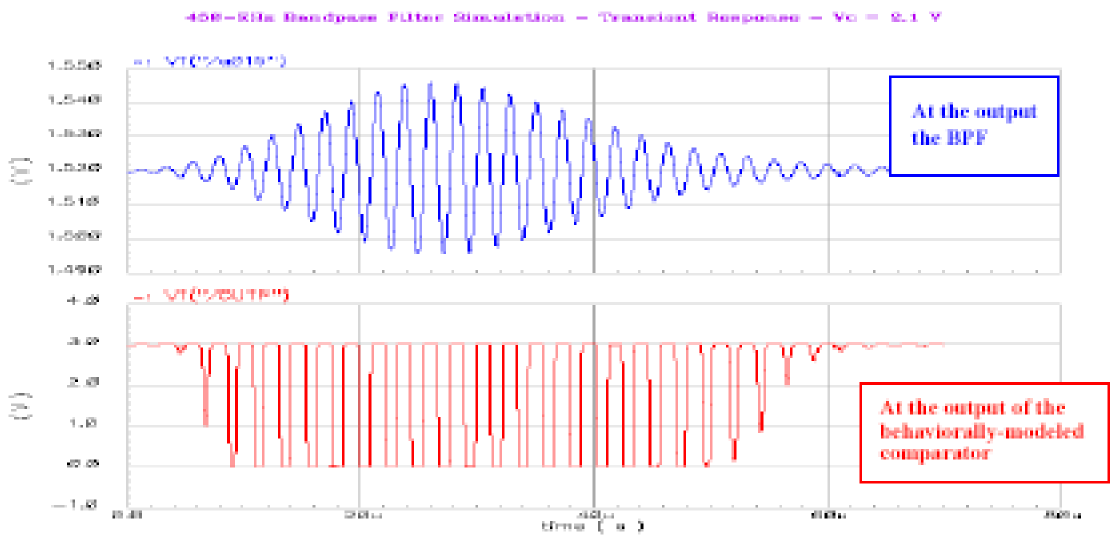
Operation:

1.) A coarse tuning cycle using the LDC algorithm is used to find the actual frequency.

2.) The difference between the actual frequency and desired frequency is used to begin a successive approximation cycle to fine-tune the filter.

Example of the High-Q Filter Step Response

Generation of a square wave from the step response:

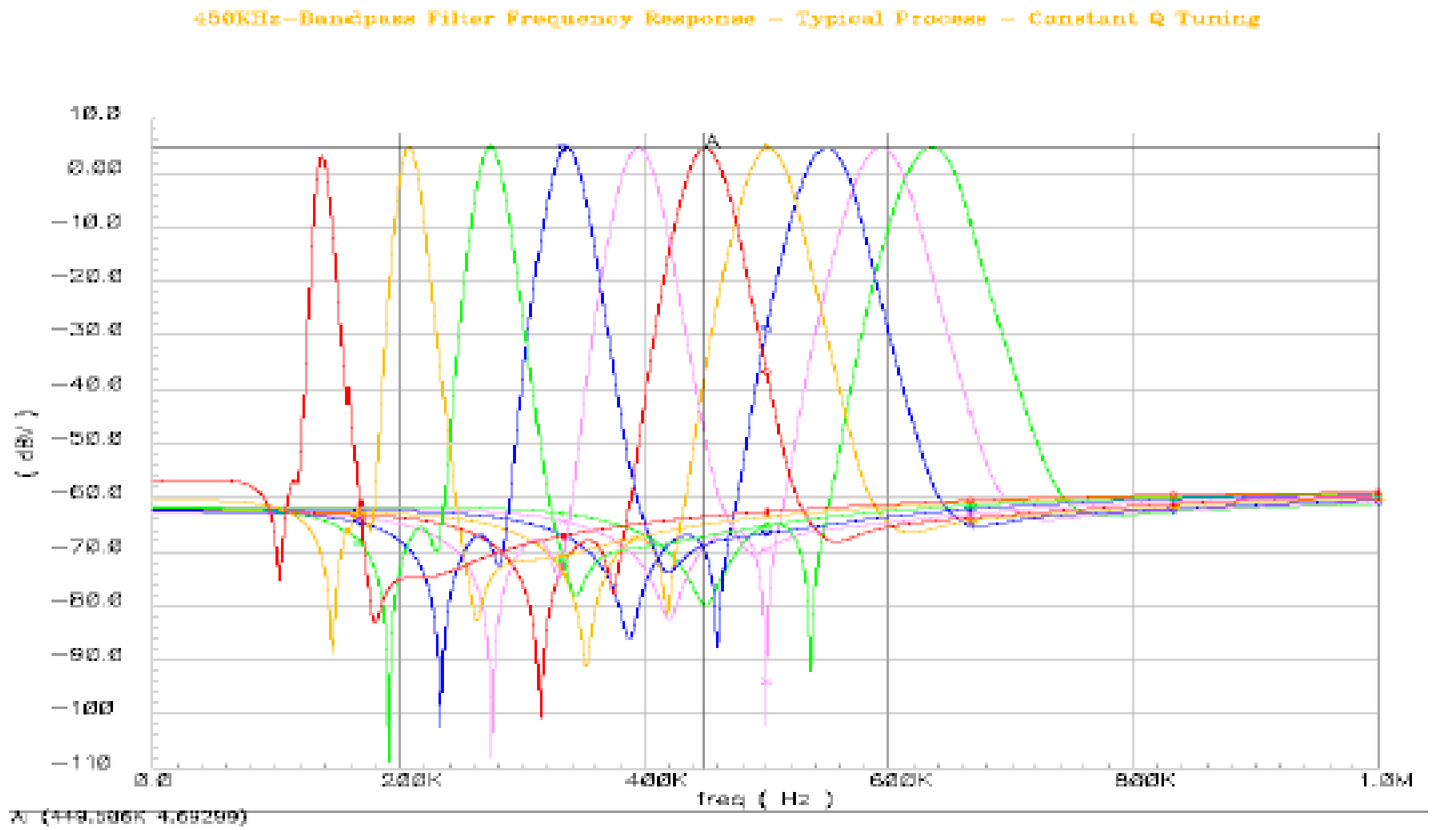


Tuning time is approximately 800µs to an accuracy within ±1%.

Note: Amplitude $= g_{m1}R_o \approx \dfrac{R_{upper}||R_{M3}(g_m r_{ds})^2}{R_{M3}} \approx (g_m r_{ds})^2$ if $R_{upper} >> R_{M3}(g_m r_{ds})^2$

Constant-Q Tuning

By taking the advantage of the OTA topology and proper sizing, the Q-factor of the filter will remain constant during tuning as illustrated below.

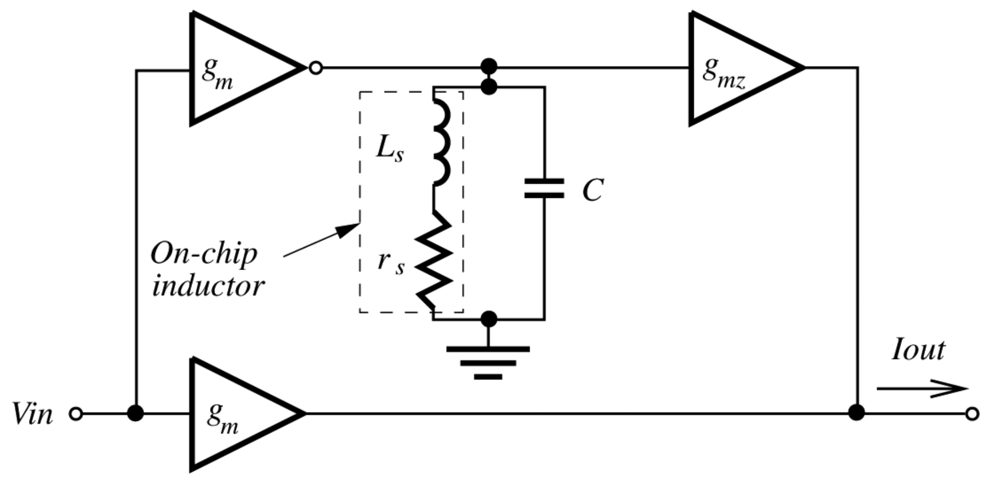


Simulation Results of the 450KHz Filter

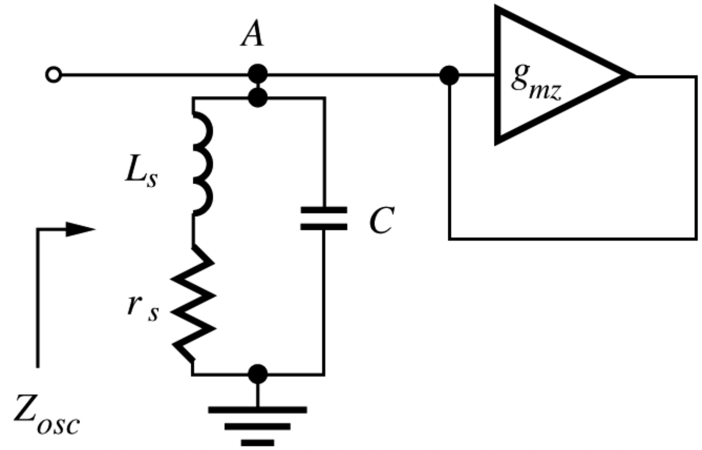
Parameters	Simulated Results
Center frequency	450±1.8KHz
Bandwidth (-3dB)	22 KHz
Gain at 450 KHz (Max.)	≈ 8dB
Tunable frequency	225KHz to 675KHz
Tuning time	800μs
Total in-band noise	$314\mu V_{rms}$
Maximum single-level	<100dBμV
Power dissipation	2.2mA from 3V
Power Supply	3V

Tuning of the RF Image Reject Filter

Filter:



Direct tuning method:



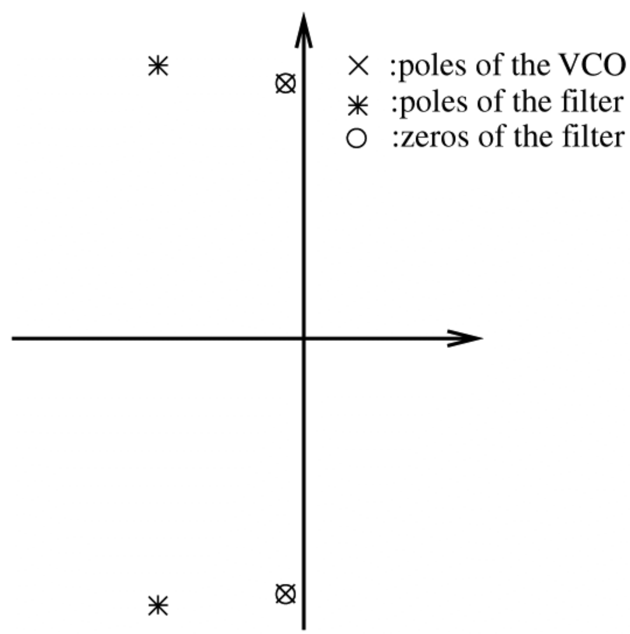
Filter becomes a voltage controlled oscillator (VCO).

Tuning Algorithm

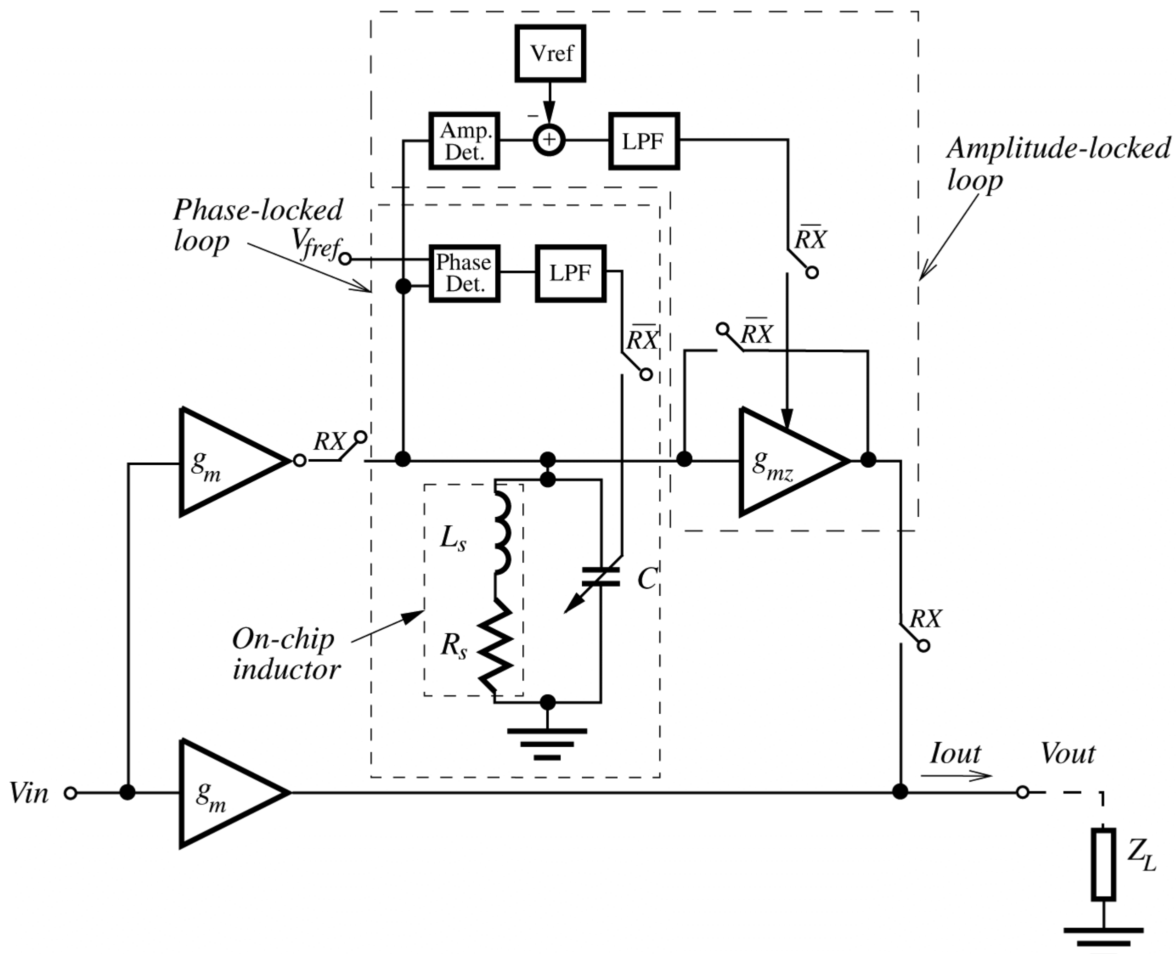
Since the poles of the VCO and zeros of the filter are identical, tuning the poles tunes the zeros.

Need to tune both the frequency and the notch depth.

Root locations:

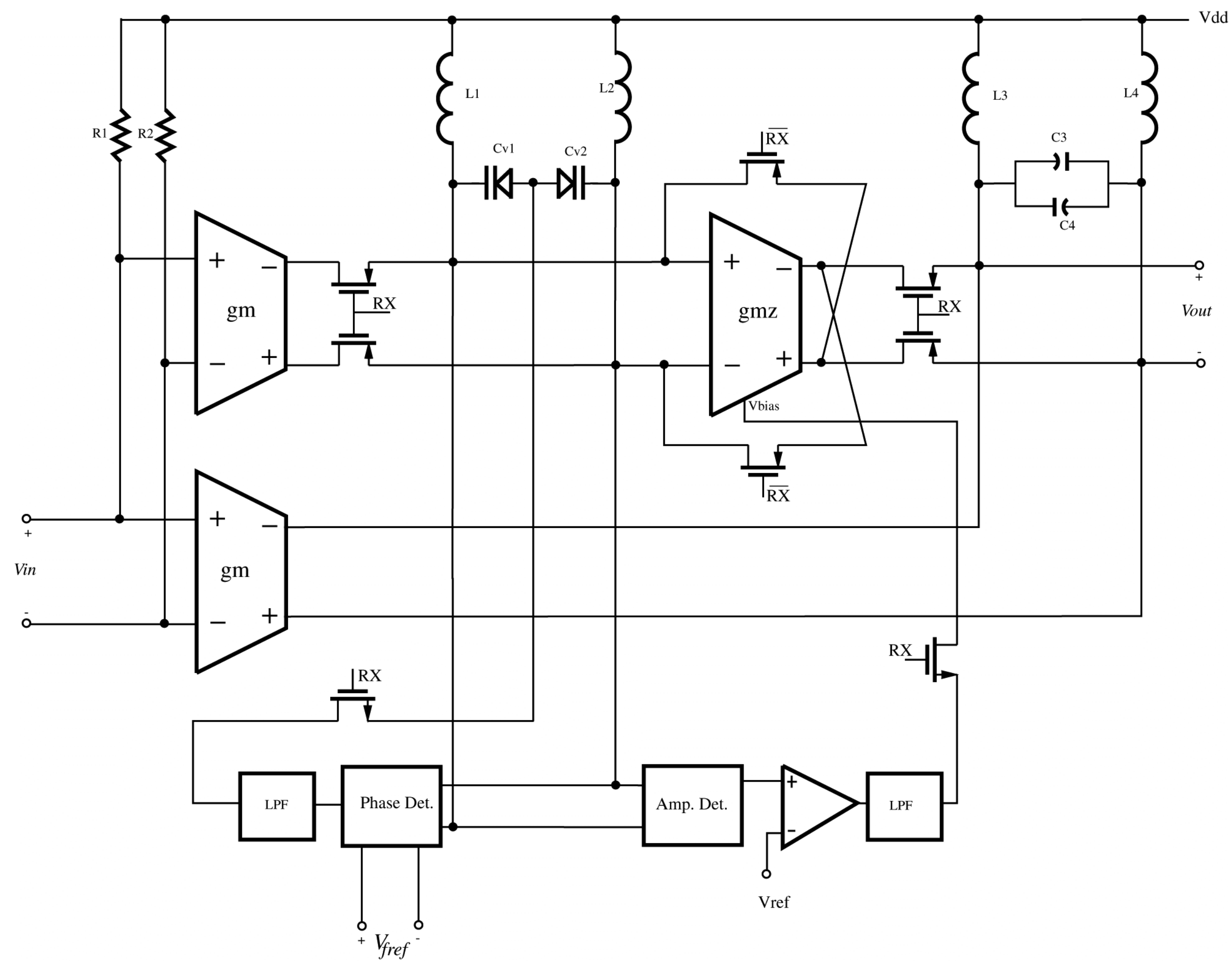


Notch Filter and Tuning Circuits

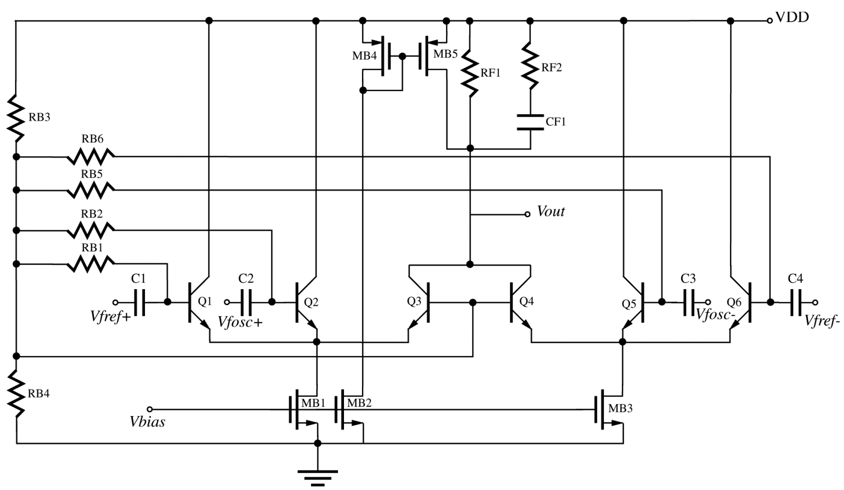


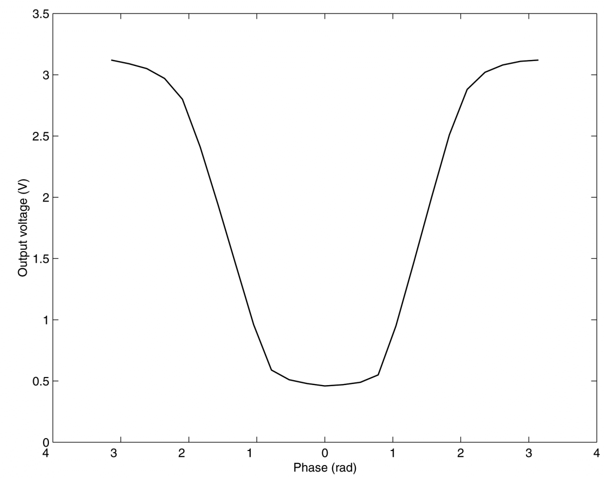
Notch Filter and Tuning Circuits - Continued

Entire circuit:

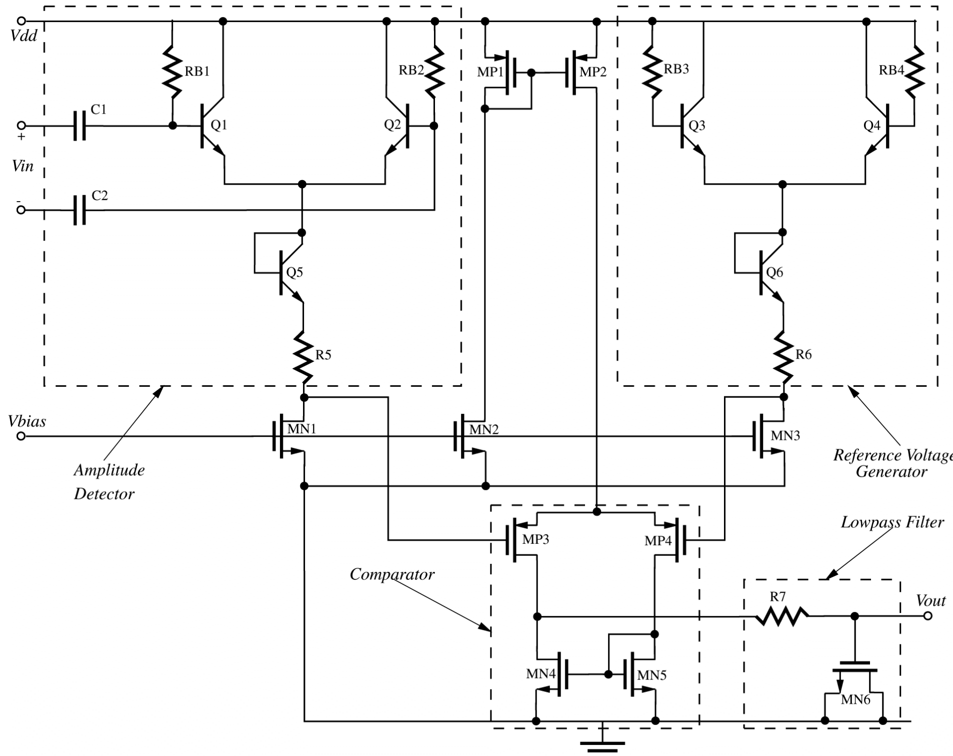


Phase Detector and Lowpass Filter





Amplitude-Locked Loop

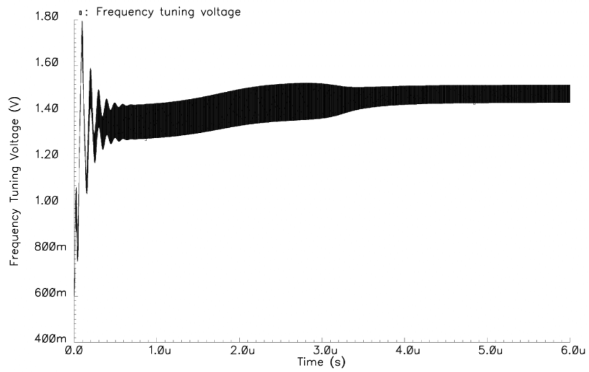


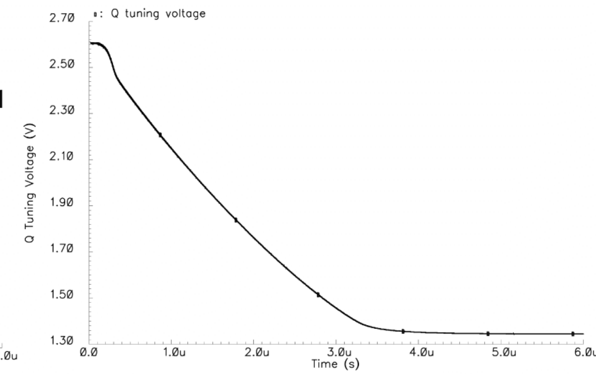
Input rectified at the emitters of Q1 and Q2.

Tuning Response

Frequency tuning voltage

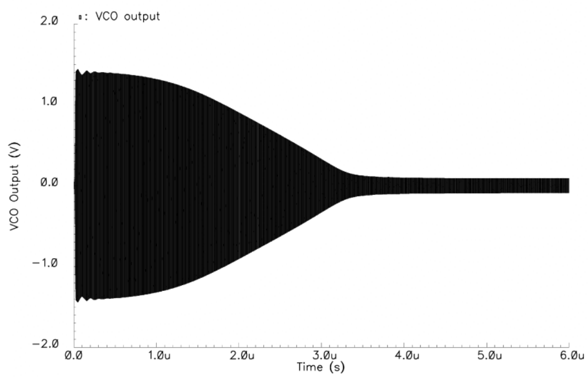
Q tuning voltage →

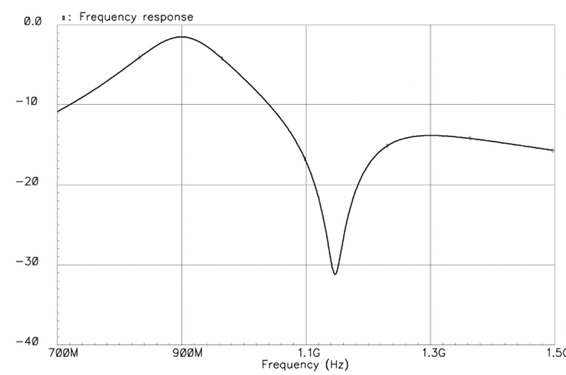




VCO output

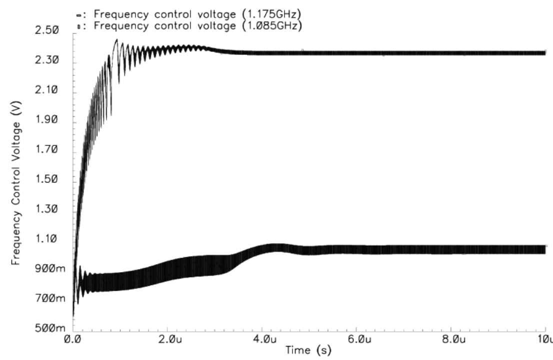
Frequency response →



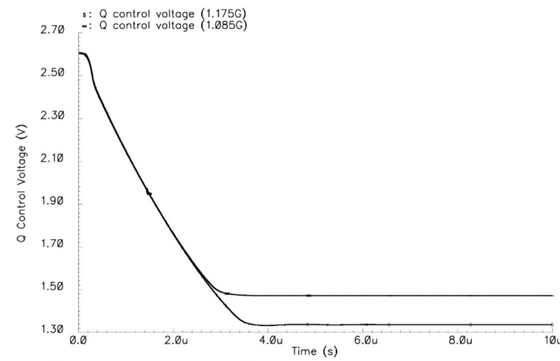


Tuning From One Frequency to Another

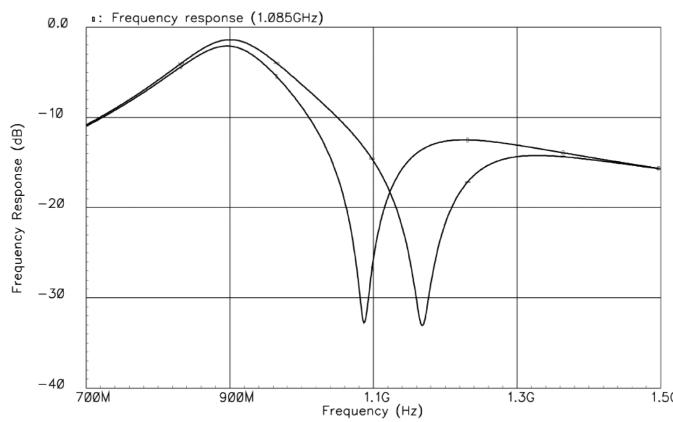
Frequency control voltages for 1.175 GHz and 1.085 GHz



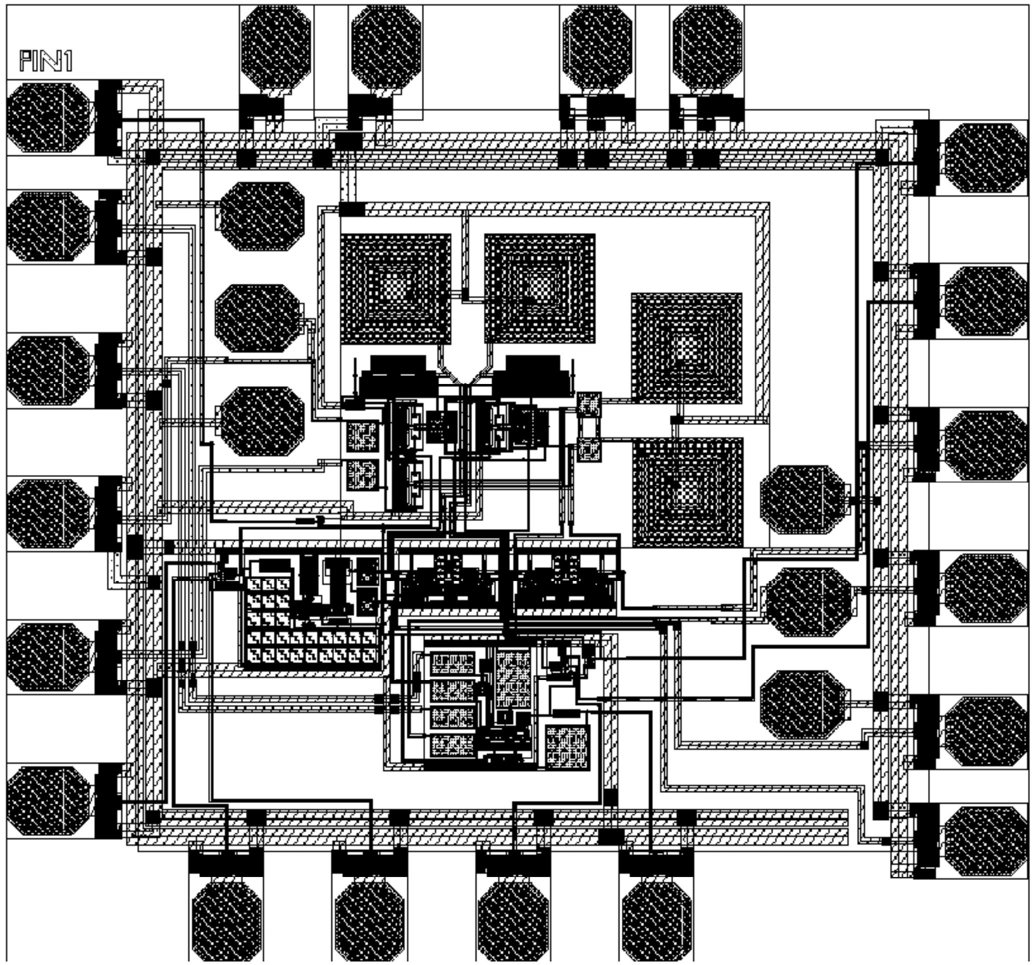
Q control voltage for 1.175 GHz and 1.085 GHz →



Frequency response for 1.175 GHz and 1.085 GHz



RF Image Reject Filter Layout



Simulation results: Notch attenuation over 200KHz ≈ 20-25dB, NF ≈ 3-4dB

SECTION 7 - SUMMARY

Comparison of Filter Types

Filter Type	Advantages	Disadvantages	Frequency	Applications
Digital	Precision, dynamic range, programmability	Power consumption, chip area, aliasing, ADC requirements, external clock requirement	<10MHz	Low IF filtering, baseband filtering and signal processing
Passive LC	Dynamic range, stability	Quality factor, chip area	>100MHz	Power amplifier harmonic suppression, low Q RF preselection
Electro Acoustic	Dynamic range, stability	Process modifications, chip area	>100MHz	RF preselection, IF filtering
Switched Capacitor	Precision	Dynamic range at high Q, aliasing, external clock requirement	<10MHz	Low frequency, moderate Q IF filtering, baseband filtering
G_m-C	Frequency of operation	Dynamic range at high Q, tuning requirement	<100MHz	Moderate Q IF filtering, baseband filtering
Q-enhancement LC	Dynamic range, stability	Chip area, tuning requirement	>100MHz	RF preselection, IF filtering
Current Mode Filters	Frequency of operation	Dynamic range at high Q, tuning requirement	<150MHz	Moderate Q IF filtering, baseband filtering

What is the Future of IC Filters?

- To be attractive, integrated circuit filters must be:
    - Low power
    - Accurate
    - Small area
    - Low noise
    - Large dynamic range (linear)
- Active filters will be limited to around 100MHz and will be implemented by OTA-C, log-domain, current mode techniques. The key is to reduce power, area and noise.
- The higher the Q in bandpass filters, the more difficult the filter is to implement
- RF filtering can be done using notch filters or filters with $j\omega$ axis zero
- Submicron CMOS (<0.25µm) will allow filters to work at higher frequencies but the success of these filters depends on clever circuit techniques.
- SiGe BiCMOS probably will allow integrated circuit filters up to 10GHz using clever circuit techniques.
- Switched capacitor filters continue to be the technique of choice up to several MHz.