Comparative Efficiency Analysis of Dynamically Supplied Power Amplifiers (PA)

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***Abstract*—Data throughput, reliability, bit-error rates (BER), and backward compatibility complicate modern protocols so much that power amplifiers (PA) drive and demand most of the power mobile devices consume. As a result, PA power efficiency often limits battery life, which is why dynamically adaptive supplies are popular today. Justifying one supply scheme over another in terms of efficiency, however, is not straightforward. This paper analyzes the fundamental loss mechanisms in PAs to compare how power- and envelope-tracking (PT and ET) supplies improve efficiency. Results show that, even when ET supply efficiencies are 20% worse than PT's, ET schemes are 4% – 29% more efficient than PT systems for CDMA IS95 (and UMTS), CDMA 2000, and 802.11 a/g protocols across input power, and better in newer protocols where peak-average ratios (PAR) are higher. In fact, envelope elimination and restoration (EER) with a dynamic supply and feedback is optimal with 29% – 35% more efficiency than Class-A ET because a nonlinear PA dissipates little conduction power, the smart supply loses less power than the nonlinear PA saves, and feedback corrects nonlinear errors.**

***Index Terms*—Envelope Elimination and Restoration (EER), Envelope Tracking, Power Tracking, Power Amplifier (PA).**

I. POWER-AMPLIFIER EFFICIENCY

SMALL and large-scale systems save energy, time, cost, and lives by sharing information across a networked space. In this context, mobile and portable wireless devices play a pivotal role in gathering and disseminating intelligence. Unfortunately, replacing or recharging batteries across a wide-area network demands personnel costs and disrupts the network's connectivity, ultimately curbing the benefits of communication. Battery life, as a result, is a critical parameter.

Prolonging the single-charge life of a small (and easily exhaustible) state-of-the-art battery amounts to decreasing losses in the system. Regrettably, radiating electromagnetic waves is fundamentally lossy because distant receivers only absorb a small fraction of what a transmitter delivers. As a result, the radio-frequency (RF) power amplifier (PA) demands most of the power a system dissipates [1]–[2], which means that extending life hinges on increasing PA efficiency.

II. SUPPLY-MODULATED SCHEMES

PAs fall under two categories: linear and nonlinear. While linear PAs amplify their inputs linearly to produce outputs with proportionately higher power, nonlinear PAs generate constant output power, irrespective of the input. Power in nonlinear outputs, however, increases with supply voltage so adjusting the supply modulates the nonlinear PA's power gain.

A. *Linear Power Amplifiers*

In linear PAs, maximum output power dictates the physical size and conduction time of the lossy power switch for a particular supply voltage V_{DD}. As such, conventional designs favor peak-load efficiency η_{PK} over its backed-off counterpart η_{BO} because the voltage (power) dropped across the switch is high during back-off conditions, as the lossy region in Fig. 1a illustrates. Unfortunately, the impact of this tradeoff on battery life is profound because wireless devices mostly idle and operate in backed-off mode, which means a higher η_{BO} saves more energy than a higher η_{PK} in mobile applications.

To alleviate the severity of this tradeoff, [3] and [4] slowly adjust the PA's supply v_{DD} and corresponding bias current according to the worst-case power needs of the system when subjected to a particular transmitter-receiver distance. Tracking the power-control envelope this way reduces the average voltage dropped across the switch substantially, as Fig. 1b shows. Nevertheless, power tracking (PT) still subjects the switch to considerable voltages during instantaneous back-off events, which is where the (signal) envelope-tracking (ET) supply of Fig. 1c offers further efficiency gains [3].

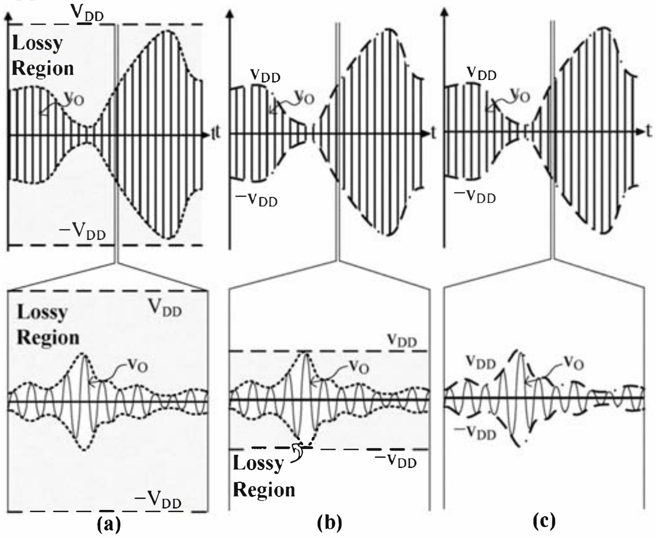

Fig 1. Linear PA outputs with (a) fixed, (b) power-tracking, and (c) envelope-tracking supplies across time.

B. *Nonlinear Power Amplifiers*

The fundamental advantage of a nonlinear PA is that the power switch conducts current only when subjected to small

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voltages. In other words, nonlinear PAs dissipate considerably lower conduction losses and therefore achieve substantially higher efficiency levels than linear PAs can. Modern communication protocols, however, as Fig. 1 demonstrates, conveys information in the signal's envelope, which a nonlinear PA alone cannot reproduce. As such, the system in Fig. 2 strips the signal's envelope from the nonlinear PA's input and rebuilds it in the output by dynamically adjusting the supply of the PA [5]. Ultimately, the cost of envelope elimination and restoration (EER) is supply complexity.

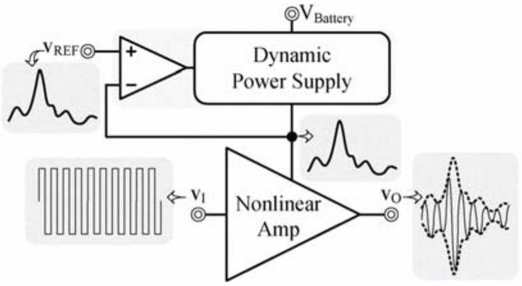

Fig 2. Systematic view of envelope elimination and restoration (EER).

III. LINEAR POWER-AMPLIFIER EFFICIENCY

Using a relatively simple, but representative circuit to compare supply schemes is important in drawing meaningful and fundamental conclusions. Accordingly, the schematic of Fig. 3, while elemental, embodies the principle loss mechanism in all linear PAs: conduction power lost across switch M_P. In this case, M_P closes and opens to energize and de-energize choke inductor L_C from v_{DD} into the load and M_P.

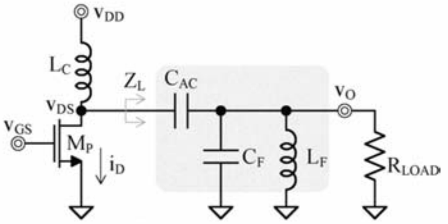

Fig. 3. Basic (Class A-C) linear PA schematic.

A. *Fixed Supply*

Peak-Load Efficiency: When fixed, the power sourced by supply V_{DD} is the product of V_{DD} and L_C's averaged (dc) current I_L or $<i_L>$. Considering M_P partially conducts i_L's sinusoid, as Fig. 4 shows, peak current I_{MAX} is the maximum current that M_P can sink under optimal design conditions. Here, M_P's drain current i_D is non-zero for a fraction d_C of period T_C, which is to say M_P conducts i_L for d_CT_C of T_C and

$$i_D = \begin{cases} I_Q + I_A\cos(2\pi f_C t) & -\frac{T_C d_C}{2} < t + NT_C < \frac{T_C d_C}{2} \\ 0 & \text{else} \end{cases}, \quad (1)$$

where N is any integer, I_Q is quiescent current, and I_A is i_L's amplitude, so I_{MAX} is $I_Q + I_A$. As such, i_D decomposes into a (Fourier) series of harmonics:

$$i_D = I_D + \sum_{i=1}^{\infty} i_{d(i)}\cos[2\pi(i \cdot f_C)t], \quad (2)$$

where M_P conducts through conduction angle (fraction) α or $2\pi d_C$, which means averaged drain current I_D or $<i_D>$ is

$$I_D = <i_D> = \frac{I_{MAX}}{2\pi}\left[\frac{2\tan(\alpha/2) - \alpha}{\sec(\alpha/2) - 1}\right] = I_L = <i_L> \quad (3)$$

where I_D equals I_L because coupling capacitor C_{AC} blocks dc current from flowing into the filter so input power P_{IN} is

$$P_{IN} = V_{DD}I_D. \quad (4)$$

Notice that only α or, equivalently, d_C sets P_{IN} through I_D.

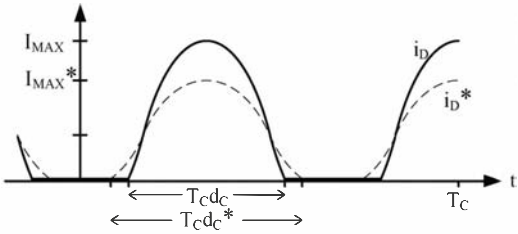

Fig. 4. Maximum (solid) and backed-off (dashed) drain current i_D.

Output power P_O is the root-mean-squared (RMS) power load R_{LOAD} dissipates in conducting whatever fraction of i_L reaches it. To determine P_O, consider that all of i_D's high-frequency current flows through C_{AC} into filtered load impedance Z_L because L_C's impedance at and above carrier frequency f_C is considerably higher than Z_L. Since L_F-C_F filter shunts all harmonics of i_D to ground, only i_D's fundamental carrier component $i_{d(fc)}\cos(2\pi f_C t)$ reaches R_{LOAD} as i_R, where

$$i_R = i_{d(fc)} = \frac{I_{MAX}}{2\pi}\left[\frac{\alpha - \sin(\alpha)}{1 - \cos(\alpha/2)}\right] \quad (5)$$

and

$$P_O = i_{R(RMS)}{}^2 R_{LOAD} = \left(\frac{i_{d(fc)}}{\sqrt{2}}\right)^2 R_{LOAD}. \quad (6)$$

An optimal design, while keeping M_P away from triode, minimizes the power lost across M_P (and maximizes efficiency) by allowing its drain voltage v_D to fall just above one drain-source-saturation voltage $v_{DS(SAT)}$ from ground:

$$v_D > v_{DS(SAT)} = v_{GS} - V_T, \quad (7)$$

where v_{GS} and V_T are M_P's gate-source and threshold voltages, respectively. Because L_C and C_{AC} are, correspondingly, dc and ac shorts, v_D is the juxtaposition of V_{DD} and R_{LOAD}'s output voltage v_O, the latter of which carries no dc component:

$$v_D = V_{DD} + v_O = V_{DD} + \left[i_{d(fc)}\cos(2\pi f_C t)\right]R_{LOAD}. \quad (8)$$

As a result, to maximize P_O (through v_O) when conducting I_{MAX}, while keeping v_D above $v_{DS(SAT)}$, R_{LOAD} should be

$$R_{LOAD} \equiv \frac{v_{O(MAX)}}{i_{R(MAX)}} = \frac{V_{DD} - v_{DS(SAT)}}{i_{d(fc)}} \approx \frac{V_{DD}}{i_{d(fc)}}, \quad (9)$$

where $v_{DS(SAT)}$ is usually negligible relative to V_{DD}. Designed this way, peak (and maximum-load) efficiency $\eta_{PA(PK)}$ is

$$\eta_{PA(PK)} \equiv \frac{P_O}{P_{IN}} = \frac{\left(\frac{i_{d(fc)}}{\sqrt{2}}\right)^2 R_{LOAD}}{V_{DD}I_D} = \frac{i_{d(fc)}}{2I_D}. \quad (10)$$

Note that $\eta_{PA(PK)}$ depends on d_C (or α_C) via $i_{d(fc)}$ and I_D.

Backed-Off Efficiency: When backed off, the PA sources less power than its rated limit. To do so, the fixed-supply system decreases only the ac component of M_P's v_{GS}, which means M_P's biasing (quiescent) current I_Q remains unchanged. As a result, backed-off current i_D* falls below maximum drain current i_D in Fig. 4 and M_P's conduction time $T_C d_C^*$ increases (because backed-off i_L^* and peak-load i_L both swing about I_Q).

Ultimately, like before, the fundamental component of i_D^* (i.e., $i_{d(fc)}^*$) reaches R_{LOAD}, which means (from Eq. (5)) that backed-off maximum drain current I_{MAX}^* and conduction fraction d_C^* or α^* set R_{LOAD}'s i_R^* and, by translation, P_O^*. Correspondingly, the average of i_D^* (i.e., I_D^*) determines the dc current pulled from V_{DD}, thus (from Eq. (3)) I_{MAX}^* and α^* also sets P_{IN}^*. Since i_D^* and i_D both cross I_Q at a quarter of the carrier period T_C, peak and backed-off conduction fractions d_C or α and d_C^* or α^* also relate peak I_{MAX} to backed-off I_{MAX}^*:

$$I_{MAX}^* = I_{MAX}\left[\frac{\cos(\alpha^*/2)-1}{\cos(\alpha/2)-1}\right]\left[\frac{\cos(\alpha/2)}{\cos(\alpha^*/2)}\right]. \tag{11}$$

Similarly, backed-off conduction angle α^* relates to α through backed-off and peak output voltages v_O and V_{DD}:

$$\frac{V_{DD}}{v_o} = \left[\frac{\sin(\alpha)-\alpha}{\sin(\alpha^*)-\alpha^*}\right]\left[\frac{\cos(\alpha^*/2)}{\cos(\alpha/2)}\right]. \tag{12}$$

That is, backed-off efficiency $\eta_{PA(BO)}$ relates through $i_{d(fc)}^*$ and I_D^* from backed-off I_{MAX}^* and α^* and peak I_{MAX} and α:

$$\eta_{PA(BO)} = \frac{P_O^*}{P_{IN}^*} = \frac{\left(\frac{i_{d(fc)}^*}{\sqrt{2}}\right)^2 R_{LOAD}}{V_{DD} I_D^*} = \frac{\left(\frac{v_O}{\sqrt{2}}\right)^2 \frac{1}{R_{LOAD}}}{V_{DD} I_D^*} \tag{13}$$

In other words, because high values of v_O mean M_P suffers from smaller drain-source voltages, PA efficiency η_{PA} increases with v_O, as Fig. 5 corroborates (from the above relationships). η_{PA} is highest at $\eta_{PA(PK)}$ when d_C is 50% (i.e., α is π) because, at that point, the linear PA delivers peak output power P_O at a v_O that is close to V_{DD}. As d_C or α increase to d_C^* or α^*, v_O and η_{PA} decrease to backed-off levels.

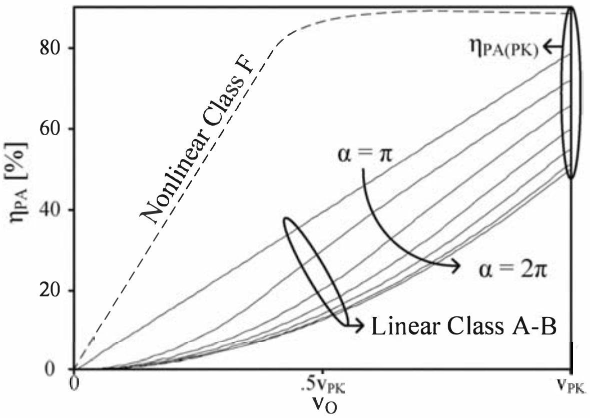

Fig. 5. Efficiency across output voltage v_O and conduction angle α (or fraction d_C) for linear (solid) and nonlinear (dashed) PAs.

B. Power-Tracking Supply

In power tracking (PT), the system adjusts supply v_{DD} (in Fig. 1b) and quiescent current i_Q dynamically to ensure peak output power P_O tracks the load's highest peak across a given (distance) setting. More specifically, a PT system ensures η_{PA} reaches $\eta_{PA(PK)}$ at least once (but not continually) by changing (*i*) M_P's i_Q so conduction fraction d_C (and α) reaches the peak's designed value *and* (*ii*) v_{DD} so v_O peaks just below v_{DD} at least once. The result is that PT efficiency $\eta_{PA(PT)}$ reaches $\eta_{PA(PK)}$ as often as the load peaks, which the probability density function (PDF) of the communication protocol (in Fig. 6) determines. That is, v_O's PDF $p(v_O)$ and backed-off relations P_{IN}^*, P_O^*, and $\eta_{PA(BO)}$ describe $\eta_{PA(PT)}$, as [2]–[3] show:

$$\eta_{PA(PT)} = \frac{\int_0^{v_{PK}} P_O^* \cdot p(v_O) dv_O}{\int_0^{v_{PK}} P_{IN}^* \cdot p(v_O) dv_O} = \frac{\int_0^{v_{PK}} P_{IN}^* \eta_{PA(BO)} \cdot p(v_O) dv_O}{\int_0^{v_{PK}} P_{IN}^* \cdot p(v_O) dv_O} \tag{14}$$

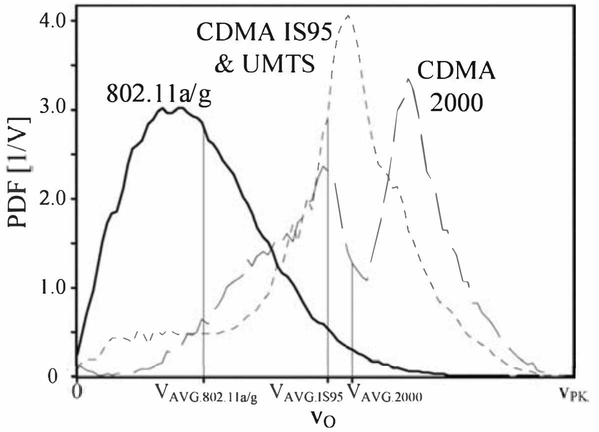

Fig. 6. The signal envelope's probability density functions (PDF) of modern telecommunication protocols across output voltage v_O.

C. Envelope-Tracking Supply

The difference between PT and envelope tracking (ET) is the latter adjusts v_{DD} (as in Fig. 1c) *but not* i_Q (to maintain linearity) *at speed* with the signal's envelope. That is, ET efficiency $\eta_{PA(ET)}$ approaches $\eta_{PA(PK)}$ and continually suppresses the voltage (power) across M_P (via v_O). As such, v_{DD} remains at v_O while backed-off relationships P_O^*, I_D^* describe $\eta_{PA(ET)}$, in accordance to v_O's PDF $p(v_O)$:

$$\eta_{PA(ET)} = \frac{\int_0^{v_{PK}} P_O^* \cdot p(v_O) dv_O}{\int_0^{v_{PK}} [I_D^* v_O] \cdot p(v_O) dv_O}. \tag{15}$$

D. Comparison

Comparing PT and ET PA efficiencies $\eta_{PA(PT)}$ and $\eta_{PA(ET)}$ directly is unfair because their respective supplies do not consume the same power, and the system's overall efficiency also depends on the power supply's efficiency η_{PS}:

$$\eta_{TOTAL} = \eta_{PS}\eta_{PA}. \tag{16}$$

To appreciate this, consider that, while ET supplies regulate their outputs at the speed of the signal's envelope at maybe 1–20 MHz, PT supplies vary at considerably lower frequencies at less than 10 KHz. Therefore, ET supplies dissipate more switching losses than their PT counterparts', which means the former are less efficient. The efficiency of PT supplies in [4], [6]–[8], for example, average at 90% and ETs' in [9]–[11] at 70%. Notwithstanding, even with 20% lower supply efficiency, ET is still more efficient than PT, as Fig. 7 shows, by at least 4% and up to 29% for CDMA IS95 (and UMTS), CDMA 2000, and OFDM 802.11 a/g across conduction fraction d_C (and angle α). Still more, the benefits of ET are more profound in newer protocols because their respective peak-average ratios (PAR) are higher, which means v_O, and by translation, PT efficiency η_{PT} peak to $\eta_{PA(PK)}$ less often.

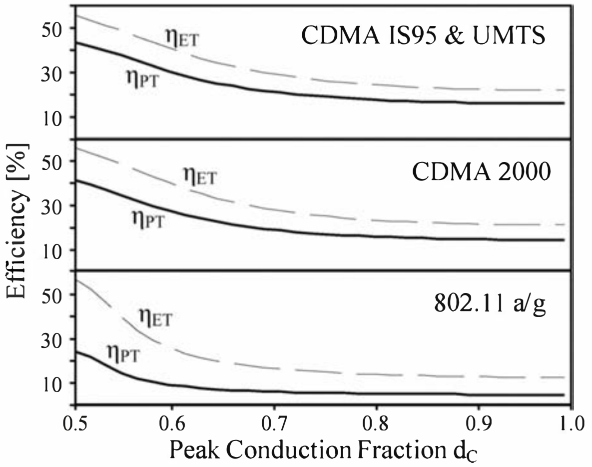

Fig. 7. Power- and envelope-tracking efficiencies η_{PT} and η_{ET} for CDMA IS95 (and UMTS), CDMA 2000, and OFDM 802.11 a/g protocols across conduction fraction d_C (and angle α and output power).

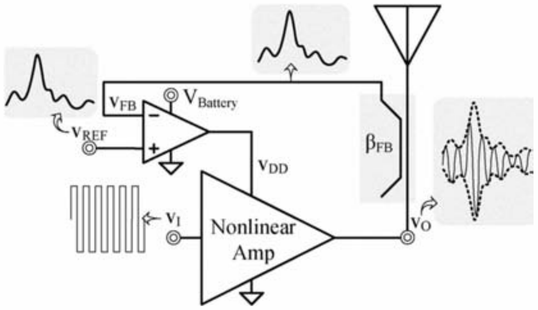

Fig. 8. Envelope elimination and restoration (EER) with feedback linearization.

IV. NONLINEAR POWER-AMPLIFIER EFFICIENCY

As noted earlier, a nonlinear PA (in Class-F mode) is more efficient than linear PAs are, as Fig. 5 shows (from [4]), because conduction losses across the power switch are substantially lower. Sadly, efficiency improves as drastically as linearity worsens, so the PA's supply must follow the signal's envelope, like in ET, but with sufficient accuracy to recover linearity. Synchronization (delay) errors between the envelope and the signal prevent modern implementations of envelope elimination and restoration (EER) from meeting modern linearity standards [12]–[13]. However, [14] meets linearity specifications by coupling the output of the PA from the antenna back in feedback fashion, as Fig. 8 shows, to the controlling input of the dynamic supply. In other words, [14] regulates the PA's output directly, rather than its supply. Because the coupler required does not degrade efficiency, EER supply efficiency η_{PS} is equivalent to that of ET's for linear PAs [4], which means EER can achieve (from Fig. 5, Eq. (14), and power-control envelopes [15]–[16]) 29.2% – 35.0% higher efficiency than Class-A ET. While feedback sets v_{FB} to v_{REF}, β_{FB} introduces delay, so v_O's envelope and v_{REF} exhibit delay mismatch, except the delay is negligible [14]. However, modulating v_{DD} affects the PA's phase response, the effect of which feedback does not correct.

V. CONCLUSIONS

With feedback, envelope elimination and restoration (EER) is the most efficient scheme because conduction losses are lower in nonlinear PAs than in linear PAs and the power the dynamic supply loses is less than what the nonlinear PA saves. EER is the nonlinear equivalent of envelope tracking (ET) in linear PAs, which is more efficient (and better in newer protocols where peak-average ratios PARs are higher) than power tracking (PT) because the dynamic supply of the former maximally suppresses the voltage across the power switch at speed with the signal's envelope while the latter's only on occasion. PT, however, is more efficient than a fixed supply because, when backed off, unlike a fixed supply does, PT ensures efficiency peaks at least once. Ultimately, the value of the presented analysis is that it demonstrates fundamentally that EER with feedback, while meeting linearity standards, maximally extends the battery life of modern wireless devices.

REFERENCES

[1] S. Cripps, *RF Power Amplifiers for Wireless Communications*, Artech House, Inc., Norwood, MA, 1999.

[2] J. Groe and L. Larson, *CDMA Mobile Radio Design*, Artech House, Inc., Boston, MA, 2000.

[3] G. Hanington et al., "High-efficiency power amplifier using dynamic power-supply voltage for CDMA applications," *IEEE Trans. Microw. Theory Tech.*, vol. 47, no. 8, pp. 1471–1476, Aug. 1999.

[4] B. Minnis et al., "System-efficiency analysis of power amplifiers supply-tracking regimes in mobile transmitters," *IEEE Trans. on Circ. and Syst.*, vol. 56, no. 1, pp. 268–279, Jan. 2009.

[5] L. Kahn, "Single-sideband transmission by envelope elimination and restoration," *Proc. IRE*, vol. 40 no. 7, pp. 803–806, Jul. 1952.

[6] B. Sahu and G. Rincón-Mora, "A low voltage, dynamic, noninverting, synchronous buck-boost converter for portable applications," *IEEE Trans. Power Electron.*, vol. 19, no. 2, pp. 443–452, Mar. 2004.

[7] V. Yousefzadeh, N. Wang, Z. Popović, and D. Maksimović, "A digitally controlled DC/DC converter for an RF power amplifier," *IEEE Trans. Power Electron.*, vol. 19, no. 2, pp. 443–452, Mar. 2004.

[8] B. Sahu and G. Rincón-Mora, "A high efficiency WCDMA RF power amplifier with adaptive, dual-mode buck-boost supply and bias-current control," *IEEE Microw. and Wireless Comp. Lettters*, vol. 17, no. 3, pp. 238–240, Mar. 2007.

[9] F. Wang et al., "A monolithic high-efficiency 2.4 GHz 20-dBm SiGe BiCMOS envelope-tracking OFDM power amplifier," *IEEE J. Solid-State Circuits*, vol. 42, no. 6, pp. 1271–1281, Feb. 2007.

[10] F. Wang et al., "Design of wide-bandwidth envelope tracking power amplifiers for OFDM applications," *IEEE Trans. Microw. Theory Tech.*, vol. 53, no. 5, pp. 1244–1255, Apr. 2005.

[11] N. Schlumpf, M. Declercq, and C. Dehollain, "A fast modulator for dynamic supply linear RF power amplifier," *IEEE J. Solid-State Circuits*, vol. 39, no. 7, pp. 1015–1025, July 2004.

[12] N. Wang et al., "Linearity of *X*-band class-E power amplifiers in EER operation," *IEEE Trans. Microw. Theory Tech.*, vol. 53, no. 3, pp. 1096–1102, Mar. 2005.

[13] J. Pedro, J. A. Garcia, and P. Cabral, "Nonlinear distortion analysis of polar transmitters," *IEEE Trans. Microw. Theory Tech.*, vol. 55, no. 12, pp. 2757–2765, Dec. 2007.

[14] J. Kitchen, C. Chu, S. Kiaei, and B. Bakkaloglu, "Combined linear and Δ-modulated switch-mode PA supply modulator for polar transmitters," *IEEE J. Solid-State Circuits*, vol. 44, no. 2, pp. 404–413, Feb. 2009.

[15] B. Sahu, "Integrated, dynamically adaptive supplies for linear RF power amplifiers in portable applications," Ph.D. dissertation, Dept. Elect. and Comp. Eng., Georgia Inst. Tech., Atlanta, GA, 2004.

[16] J. B. Groe and L. E. Larson, *CDMA Mobile Radio Design*, Artech House, Inc., Boston, MA, 2000.