# Piezoelectric CMOS Charger: Highest Output-Power Design 

Siyu Yang, Graduate Student Member, IEEE, and Gabriel A. Rincón-Mora, Fellow, IEEE<br>School of Electrical and Computer Engineering<br>Georgia Institute of Technology, Atlanta, GA USA<br>E-mail: jimsyyang@gatech.edu, Rincon-Mora@gatech.edu


#### Abstract

Wireless microsensors and internet of things that add intelligence to the surroundings need ambient energy to extend life and expand functionality. Piezoelectric transducer can turn vibrations into electrical charge, and with the recycling bridge power stage, it can keep the highest voltage across the transducer and output the most power into the battery. This paper examines how to design the optimum recycling bridge power stage. Specifically, this paper theorizes the optimum size of the switches, the optimum inductor, and how to operate the circuit so that it losses the least power. The ohmic loss should equal charge loss to yield the lowest total loss on a MOSFET switch. The loss on the switches and the inductor are co-optimized. The switched inductor charger is optimized for discontinuous conduction mode. The resulting optimized power stage is 92\% efficient. Keywords


Piezoelectric, charger, CMOS, switched-inductor, synchronous switch harvesting on inductor (SSHI), optimize power switch, lowest loss, powering internet of things.

## 1. Piezoelectric-Powered Systems

Wireless microsensors and internet of things embedded in factories, vehicles, and human bodies add intelligence to the surroundings to save money, energy, and lives [1]-[4]. To be non-intrusion to the environment, they must be tiny, and the batteries drain quickly. Fortunately, vibration is present in many applications [5]-[6], and recent research shows that piezoelectric transducers [7] can draw power from the vibration to constantly replenish the batteries to extend life for the microsensors [8]-[14].

A piezoelectric powered system, shown in Fig. 1, consists of the transducer, modeled as an alternating current source in parallel with its parasitic capacitor [8], a charger to charge the battery, and power supplies to provide power for the loads. The charger, however, has power loss and adds breakdown constraints to the system. With these constraints, it is important to optimally design the charger to increase drawn power and reduce losses in limited spaces.


Fig. 1. Piezoelectric transducer powered microsystem.
In this paper, we demonstrate an example optimization process with the following parameters: piezoelectric capacitance $C_{P Z}$ is $15 \mathrm{nF}, \mathrm{i}_{\mathrm{PZ}(\mathrm{PK})}$ is $20 \mu \mathrm{~A}$, vibration frequency $f_{V I B}$ is 100 Hz , CMOS breakdown voltage $\mathrm{V}_{\mathrm{BD}}$ is 3.0 V , and inductor volume $\mathrm{L}_{\mathrm{VOL}}$ is $3 \times 3 \times$
$0.8 \mathrm{~mm}^{3}<10 \mathrm{~mm}^{3}$. Section II compares and selects the best power stage. Sections III through V optimize the three segments of the power stage.

## 2. Highest- $\mathrm{P}_{\mathrm{PZ}}$ Charger

Piezoelectric transducers have low mechanical to electrical coupling factors [8]-[14]. As a result, the vibration is hardly affected by the electrical power it generates, and the charge, and current $i_{\text {PZ }}$ in Fig. 2, also remains the same. Since power is current multiplies voltage, the higher voltage across the transducer, the higher power it generates. Since the transducer interfaces with the CMOS charger, the voltage cannot exceed the breakdown voltage $\mathrm{V}_{\mathrm{BD}}$. Therefore, keeping $\mathrm{V}_{\mathrm{PZ}}$ near $\mathrm{V}_{\mathrm{BD}}$ across the positive half cycle, and at $V_{B D}$ across the negative, produces the highest power $P_{P Z}$.

The recycling bridge power stage, commonly referred to as Synchronous Swith Harvesting on Inductor (SSHI) [8][11], in Fig. 3 can achieve exactly that. The maximum power point (MPP) switched inductor (SL) utilizes a buck-boost converter to regulate $\mathrm{v}_{\mathrm{R}}$ near $\mathrm{V}_{\mathrm{BD}}$ and charge the battery $\mathrm{v}_{\mathrm{O}}$. Across the positive half cycle of the vibration, synchronous diodes $D_{T R}$ and $D_{B G}$ direct positive $i_{P Z}$ into rectifying capacitor $\mathrm{C}_{\mathrm{R}}$. After the positive half cycle, at 7.5 ms in Fig. 2, $\mathrm{S}_{\mathrm{X}}$ closes so that the inductor $\mathrm{L}_{\mathrm{X}}$ receives all the energy from $\mathrm{C}_{\mathrm{PZ}}$, and subsequently charges $\mathrm{C}_{\mathrm{PZ}}$ in the negative direction across $t_{x}$. Similarlily, across the negative half cycle, synchronous diodes $D_{T G}$ and $D_{B R}$ direct negative $i_{P Z}$ into rectifying capacitor $C_{R}$, after which, at $2.5 \mathrm{~ms}, \mathrm{~S}_{\mathrm{X}}$ closes so that the $L_{X}$ receives all the energy from $C_{P Z}$, and subsequently charges $C_{P Z}$ in the positive direction across $t_{X}$. The power from the transducer is


Fig. 2. Voltage and current waveforms for piezoelectric transducer.
Unfortunately, chargers lose ohmic power on the transfers, and charge loss to open and close the switches. As a result, not all the drawn power reaches the battery. The following three sections present the way to design the optimal recycling bridge power stage to maximize the power that reaches the battery.


Fig. 3. Recycling bridge power stage.

## 3. Optimize Power Switches

The power switches employed in the power stage consume ohmic loss when they conduct current. They also require charge provided from the supply to switch them on and off [15]. For a MOSFET switch, we use the minimum length device to reduce the silicon area. The on resistance $\mathrm{R}_{\text {MOs }}$ decreases with the width of the device $\mathrm{W}_{\text {MOS }}$, and the gate capacitance $\mathrm{C}_{\mathrm{G}}$ increases with it. Therefore, the wider the switches, the higher the charge loss, but the lower the ohmic loss. Optimizing the power switches comes down to finding the lowest sum of these two losses.

Each power switch consumes ohmic power $\mathrm{P}_{\mathrm{R} \text { (MOS) }}$

$$
\begin{equation*}
\mathrm{P}_{\mathrm{R}(\mathrm{MOS})}=\mathrm{i}_{\text {MOS(RMS })}{ }^{2} \mathrm{R}_{\text {MOS }}\left(\frac{\mathrm{t}_{\mathrm{C}}}{\mathrm{t}_{\mathrm{VIB}}}\right)=\frac{\mathrm{K}_{\mathrm{R}(\mathrm{MOS})}}{\mathrm{W}_{\text {MOS }}} \propto \frac{1}{\mathrm{~W}_{\mathrm{MOS}}}, \tag{2}
\end{equation*}
$$

where $\mathrm{i}_{\text {MOS(RMS) }}$ is the root-mean-square (RMS) current through the device, $\mathrm{t}_{\mathrm{C}}$ is the conduction time, and $\mathrm{K}_{\mathrm{R} \text { (MOS) }}$ is the coefficient. Each switch also requires charge to switch it on and off, and consumes charge loss $\mathrm{P}_{\mathrm{C} \text { (MOS) }}$

$$
\begin{align*}
\mathrm{P}_{\mathrm{C}(\mathrm{MOS})} & =\mathrm{v}_{\mathrm{DD}} \mathrm{q}_{\mathrm{C}} \mathrm{f}_{\mathrm{SW}}=\mathrm{v}_{\mathrm{R}}\left(\mathrm{C}_{\mathrm{G}} \mathrm{v}_{\mathrm{R}}\right) \mathrm{f}_{\mathrm{SW}}=\mathrm{C}_{\mathrm{G}} \mathrm{~V}_{\mathrm{BD}}^{2} \mathrm{f}_{\mathrm{SW}},  \tag{3}\\
& =\mathrm{K}_{\mathrm{C}(\mathrm{MOS})} \mathrm{W}_{\mathrm{MOS}} \propto \mathrm{~W}_{\mathrm{MOS}}
\end{align*}
$$

where $\mathrm{q}_{\mathrm{C}}$ is the charge required to turn the switch on, $\mathrm{f}_{\mathrm{Sw}}$ is the switching frequency, $\mathrm{V}_{\mathrm{DD}}$ is the supply voltage, and $\mathrm{K}_{\mathrm{C} \text { (MOS) }}$ is the coefficient. Since $\mathrm{V}_{\mathrm{R}}$ is regulated to near $\mathrm{V}_{\mathrm{BD}}$, $\mathrm{V}_{\mathrm{DD}}$ is also near $\mathrm{V}_{\mathrm{BD}}$. To find the lowest total loss, we increase $\mathrm{W}_{\text {MOS }}$ until the change in $\mathrm{P}_{\mathrm{C}(\text { MOS })}$ cancels the change in $\mathrm{P}_{\mathrm{R} \text { (MOS) }}$.

$$
\begin{align*}
\frac{\mathrm{dP}_{\text {MOS }}}{\mathrm{dW}_{\text {MOS }}} & =\frac{\mathrm{dP}_{\mathrm{R}(\text { MOS })}}{\mathrm{dW}_{\text {MOS }}}+\frac{\mathrm{dP}_{\mathrm{C}(\text { MOS })}}{\mathrm{dW}_{\text {MOS }}}  \tag{4}\\
& =-\frac{\mathrm{K}_{\mathrm{R}(\text { MOS })}}{\mathrm{W}_{\text {MOS }}^{2}}+\mathrm{K}_{\mathrm{C} \text { (MOS) }}=0
\end{align*} .
$$

The optimum width is therefore

$$
\begin{equation*}
\mathrm{W}_{\mathrm{MOS}}{ }^{\prime}=\sqrt{\frac{\mathrm{K}_{\mathrm{R}(\mathrm{MOS})}}{\mathrm{K}_{\mathrm{C}(\mathrm{MOS})}}}, \tag{5}
\end{equation*}
$$

and the total loss is

$$
\begin{align*}
\mathrm{P}_{\mathrm{MOS}}{ }^{\prime} & =\mathrm{P}_{\mathrm{R}(\mathrm{MOS})}{ }^{\prime}+\mathrm{P}_{\mathrm{C}(\mathrm{MOS})}{ }^{\prime}=\frac{\mathrm{K}_{\mathrm{R}(\mathrm{MOS})}}{\mathrm{W}_{\mathrm{MOS}}{ }^{\prime}}+\mathrm{K}_{\mathrm{C}(\mathrm{MOS})} \mathrm{W}_{\mathrm{MOS}}{ }^{\prime} \\
& =\frac{\mathrm{K}_{\mathrm{R}(\mathrm{MOS})}}{\sqrt{\frac{\mathrm{K}_{\mathrm{R}(\mathrm{MOS})}}{\mathrm{K}_{\mathrm{C}(\mathrm{MOS})}}}+\mathrm{K}_{\mathrm{C}(\mathrm{MOS})} \sqrt{\frac{\mathrm{K}_{\mathrm{R}(\mathrm{MOS})}}{\mathrm{K}_{\mathrm{C}(\mathrm{MOS})}}}} .  \tag{6}\\
& =2 \sqrt{\mathrm{~K}_{\mathrm{R}(\mathrm{MOS})} \mathrm{K}_{\mathrm{C}(\mathrm{MOS})}}
\end{align*} .
$$



Fig. 4. Ohmic, charge, and total loss on a switch with optimum width. This occurs when $\mathrm{P}_{\mathrm{C}(\mathrm{MOS})}$ equals $\mathrm{P}_{\mathrm{R} \text { (MOS) }}$, and it is shown in Fig. 4. As a result, optimized power switches always consume equal ohmic and charge loss.
Bridge: The bridge consists of 4 power switches. Each switch on the bridge conducts half of $\mathrm{t}_{\mathrm{VIB}}$, and the current across that conduction time is $\mathrm{i}_{\mathrm{PZ}}$, with an RMS of $12.7 \mu \mathrm{~A}$. Therefore, we can use (2) and (3) to optimize the bridge, and each switch consumes 1.15 nW with and optimized width of 350 nm . Therefore, the loss of bridge $\mathrm{P}_{\text {BRG }}$ is 4.6 nW .
Recycler/Switched Inductor: The ohmic loss and charge loss on the power switches in the rest of the power stage, i.e. the two switched inductors (SL), is not only a function of the width of the device, but some other design parameters, e.g. inductance and switching frequency. However, the conclusion that the loss on the power switch is the least when charge loss equals the ohmic loss still applies. Therefore, we can use this conclusion to remove device width as a variable in the optimization, and optimize the other design parameters.

## 4. Optimize Recycler

After the bridge steers current into rectifying capacitor $\mathrm{C}_{\mathrm{R}}$ across a half cycle, the recycler, consists of switch $\mathrm{S}_{\mathrm{X}}$ and inductor $L_{X}$ with equivalent series resistance $R_{L X}$. The inductor receives all the energy from $\mathrm{C}_{\mathrm{PZ}}$ across a quarter of the LC oscillation cycle, as the current and voltage waveforms in Fig. 5 indicate, and back to the $\mathrm{C}_{\mathrm{PZ}}$ in the opposite direction across the next quarter cycle. The transition takes a half cycle

$$
\begin{equation*}
\mathrm{t}_{\mathrm{X}}=0.5 \mathrm{t}_{\mathrm{LC}}=0.5 \sqrt{2 \pi \mathrm{~L}_{\mathrm{X}} \mathrm{C}_{\mathrm{PZ}}} \tag{7}
\end{equation*}
$$



Fig. 5. Current and voltage waveforms for the recycler.
The energy transferred $\mathrm{E}_{\mathrm{X}}$ is

$$
\begin{equation*}
\mathrm{E}_{\mathrm{X}}=\mathrm{E}_{\mathrm{C}}=0.5 \mathrm{C}_{\mathrm{PZ}} \mathrm{~V}_{\mathrm{BD}}{ }^{2} \approx \mathrm{E}_{\mathrm{LX}}=0.5 \mathrm{~L}_{\mathrm{X}} \mathrm{i}_{\mathrm{LX}(\mathrm{PK})}{ }^{2} . \tag{8}
\end{equation*}
$$



Fig. 6. Inductance and resistance for a $3 \times 3 \times 0.8 \mathrm{~mm}^{3}$ inductor.


Fig. 7. Power loss on inductor $L_{X}$, switch $S_{X}$, and total for the recycler.
The period of the LC oscillation is so short compared with a vibration period that the transition looks instantaneous in Fig. 2, as highlighted by the gray region labeled at $t_{\mathrm{X}}$. The ohmic loss on $\mathrm{R}_{\mathrm{LX}}$ is

$$
\begin{align*}
\mathrm{P}_{\mathrm{R}(\mathrm{LX})} & =\mathrm{i}_{\mathrm{LX}(\mathrm{RMS})}{ }^{2} \mathrm{R}_{\mathrm{LX}}\left(\frac{\mathrm{t}_{\mathrm{C}}}{\mathrm{t}_{\mathrm{VIB}}}\right) \\
& =\frac{1}{2}\left(\frac{\mathrm{C}_{\mathrm{PZ}} \mathrm{~V}_{\mathrm{BD}}^{2}}{\mathrm{~L}_{\mathrm{X}}}\right)\left(\mathrm{L}_{\mathrm{X}} \mathrm{k}_{\mathrm{L}}\right)\left(\frac{0.5 \sqrt{2 \pi \mathrm{~L}_{\mathrm{X}} \mathrm{C}_{\mathrm{PZ}}}}{\mathrm{t}_{\mathrm{VIB}}}\right),  \tag{9}\\
& =\mathrm{K}_{\mathrm{R}(\mathrm{LX})} \sqrt{\mathrm{L}_{\mathrm{X}}} \propto \sqrt{\mathrm{~L}_{\mathrm{X}}}
\end{align*}
$$

where $k_{L}$ is the ratio of $R_{L X}$ to $L_{X}$, and $K_{R(L X)}$ is the coefficient. Since the inductor is limited to a $3 \times 3 \times 0.8 \mathrm{~mm}^{3}$ box, inductance increases with the number of turns, which leads to higher series resistance. To better demonstrate the optimization, DC resistance values are used for derivations. As shown in Fig. 6, where the solid line is a linear approximation of the values of inductance and resistance, the coefficient $k_{L}$ is $55 \Omega / \mathrm{mH}$. Since $L_{X}$ is the only variable in the final expression, we can simplify and see that loss of $L_{X}$ is proportional to square root of $L_{X}$.

Similarly, the ohmic loss on $S_{X}$ is

$$
\begin{align*}
\mathrm{P}_{\mathrm{R}(\mathrm{SX})} & =\mathrm{i}_{\mathrm{LX}(\mathrm{RMS})}{ }^{2} \mathrm{R}_{\mathrm{SX}}\left(\frac{\mathrm{t}_{\mathrm{C}}}{\mathrm{t}_{\mathrm{VIB}}}\right) \\
& =\frac{1}{2}\left(\frac{\mathrm{C}_{\mathrm{PZ}} \mathrm{~V}_{\mathrm{BD}}{ }^{2}}{\mathrm{~L}_{\mathrm{X}}}\right) \mathrm{R}_{\mathrm{SX}}\left(\frac{0.5 \sqrt{2 \pi \mathrm{~L}_{\mathrm{X}} \mathrm{C}_{\mathrm{PZ}}}}{\mathrm{t}_{\mathrm{VIB}}}\right),  \tag{10}\\
& =\frac{\mathrm{K}_{\mathrm{R}(\mathrm{SX})}}{\mathrm{W}_{\mathrm{SX}} \sqrt{\mathrm{~L}_{\mathrm{X}}}} \propto \frac{1}{\mathrm{~W}_{\mathrm{SX}} \sqrt{\mathrm{~L}_{\mathrm{X}}}}
\end{align*}
$$

where $\mathrm{K}_{\mathrm{R}(\mathrm{SX})}$ is the constant coefficient. Interestingly, the ohmic loss on $R_{L X}$ increases with higher $L_{X}$, but the ohmic loss on $\mathrm{S}_{\mathrm{X}}$ decreases with higher $\mathrm{L}_{\mathrm{X}}$. That is because although higher $L_{X}$ means lower peak inductor current, the transfer time increases with $L_{X}$, and $R_{L X}$ also increases with $\mathrm{L}_{\mathrm{X}}$, but $\mathrm{R}_{\mathrm{SX}}$ is completely decoupled with $\mathrm{L}_{\mathrm{X}}$.

From (3), the switch $\mathrm{S}_{\mathrm{X}}$ also consumes charge loss that is proportional with $\mathrm{W}_{\mathrm{SX}}$, From Section III, the total loss on $\mathrm{S}_{\mathrm{X}}$ is at its minimum when $\mathrm{P}_{\mathrm{R}(\mathrm{SX})}=\mathrm{P}_{\mathrm{C}(\mathrm{SX})}$, and from (6) the total loss $\mathrm{P}_{\mathrm{SX}}$ is the least when

$$
\begin{align*}
\mathrm{P}_{\mathrm{SX}}^{\prime} & =2 \sqrt{\mathrm{~K}_{\mathrm{R}(\mathrm{MOS})} \mathrm{K}_{\mathrm{C}(\mathrm{MOS})}}=2 \sqrt{\left(\frac{\mathrm{~K}_{\mathrm{R}(\mathrm{SX})}}{\sqrt{\mathrm{L}_{\mathrm{X}}}}\right) \mathrm{K}_{\mathrm{C}(\mathrm{MOS})}} .  \tag{11}\\
& =\frac{\mathrm{K}_{\mathrm{SX}}}{\mathrm{~L}_{\mathrm{X}}^{0.25}} \propto \frac{1}{\mathrm{~L}_{\mathrm{X}}^{0.25}}
\end{align*}
$$

where $K_{S X}$ is the coefficient. From (9) and (11), we can see that increasing $L_{X}$ would result in higher loss on the inductor, but lower loss on the switch. Therefore, we increase $\mathrm{L}_{\mathrm{X}}$ until the gain in $\mathrm{P}_{\mathrm{R}(\mathrm{LX})}$ cancels the reduction in $\mathrm{P}_{\mathrm{SX}}$

$$
\begin{align*}
\frac{\mathrm{dP}_{\mathrm{X}}}{\mathrm{dL}_{\mathrm{X}}} & =\frac{\mathrm{dP}_{\mathrm{R}(\mathrm{LX})}}{\mathrm{dL}_{\mathrm{x}}}+\frac{\mathrm{dP}_{\mathrm{SX}}}{\mathrm{dL}_{\mathrm{X}}}  \tag{12}\\
& =\frac{0.5 \mathrm{~K}_{\mathrm{R}(\mathrm{LX})}}{\sqrt{\mathrm{L}_{\mathrm{X}}}}-\frac{\mathrm{K}_{\mathrm{SX}}}{4 \mathrm{~L}_{\mathrm{X}}^{0.25}}=0
\end{align*}
$$

As shown in Fig. 7, the total loss on the recycler is 944 nW with an optimum $\mathrm{L}_{\mathrm{X}}{ }^{\prime}=7 \mu \mathrm{H}$, and $\mathrm{W}_{\mathrm{SX}}{ }^{\prime}=9.8 \mathrm{~mm}$.

## 5. Optimize MPP Switched Inductor

The last part of the power stage is the MPP switchedinductor (SL) charger that regulates the rectifying voltage at just under $\mathrm{V}_{\mathrm{BD}}$ and charges the battery $\mathrm{v}_{\mathrm{B}}$. It is needed because $\mathrm{P}_{\mathrm{PZ}}$ is proportional to the rectifying voltage, so it cannot directly connect to the battery, whose voltage varies from 0 to $\mathrm{V}_{\mathrm{BD}}$.

A buck-boost converter is used to lower loss. Switches $\mathrm{S}_{\mathrm{R}}$ and $\mathrm{S}_{\mathrm{GO}}$ close to establish energizing voltage $\mathrm{V}_{\mathrm{E}}$, which is $\mathrm{V}_{\mathrm{R}}$ near $\mathrm{V}_{\mathrm{BD}}$, to energize the inductor $\mathrm{L}_{\mathrm{O}}$ over energizing time $t_{E}$ :

$$
\begin{equation*}
\mathrm{t}_{\mathrm{E}}=\left(\frac{\Delta \mathrm{i}_{\mathrm{LO}}}{\mathrm{v}_{\mathrm{E}}}\right) \mathrm{L}_{\mathrm{O}} \tag{13}
\end{equation*}
$$

where $\Delta \mathrm{i}_{\text {LO }}$ is the peak to peak inductor current, shown in Fig. 8 and Fig. 10. Similarly, switches $\mathrm{S}_{\mathrm{GR}}$ and $\mathrm{S}_{\mathrm{O}}$ close to drain $L_{O}$ with draining voltage $v_{D}$ equals $v_{O}$ across draining time $\mathrm{t}_{\mathrm{D}}$ :

$$
\begin{equation*}
\mathrm{t}_{\mathrm{D}}=\left(\frac{\Delta \mathrm{i}_{\mathrm{LO}}}{\mathrm{v}_{\mathrm{D}}}\right) \mathrm{L}_{\mathrm{O}} . \tag{14}
\end{equation*}
$$

A buck-boost can operate in either continuous conduction mode (CCM) and discontinuous conduction mode (DCM), and they need to be analyzed individually.

## a. CCM

In continuous conduction mode [16], the inductor conducts current the entire switching cycle $t_{0}$, as shown in Fig. 8. Therefore, the power transferred by the inductor is its maximum energy minus its minimum energy

$$
\begin{align*}
\mathrm{P}_{\mathrm{O}} & \approx\left(\mathrm{E}_{\mathrm{L}(\mathrm{MAX})}-\mathrm{E}_{\mathrm{L}(\mathrm{MIN})}\right) \mathrm{f}_{\mathrm{O}} \\
& =0.5 \mathrm{~L}_{\mathrm{O}}\left(\mathrm{i}_{\mathrm{LO}(\mathrm{MAX})}{ }^{2}-\mathrm{i}_{\mathrm{LO}(\mathrm{MIN})}{ }^{2}\right) \mathrm{f}_{\mathrm{O}},  \tag{15}\\
& =\mathrm{L}_{\mathrm{O}} \mathrm{i}_{\mathrm{LO}(\mathrm{AVG})} \Delta \mathrm{i}_{\mathrm{LO}} \mathrm{f}_{\mathrm{O}} \approx \mathrm{P}_{\mathrm{PZ}}-\mathrm{P}_{\mathrm{X}}-\mathrm{P}_{\mathrm{B}}
\end{align*}
$$

where $i_{\text {LO(AVG) }}$ is the average inductor current, and $f_{O}=1 / t_{0}$ is the switching frequency. The energy transferred is also the drawn power from the piezoelectric transducer minus the losses on the recycler and the bridge.

Again, the transfers are not lossless. The inductors resistance $R_{\text {LO }}$ burns ohmic power

$$
\begin{align*}
\mathrm{P}_{\mathrm{R}(\mathrm{LO})} & =\mathrm{i}_{\mathrm{LO}(\mathrm{RMS})}{ }^{2} \mathrm{R}_{\mathrm{LO}}=\left(\mathrm{i}_{\mathrm{LO}(\mathrm{DC})}{ }^{2}+\mathrm{i}_{\mathrm{LO}(\mathrm{AC}, \mathrm{RMS})}{ }^{2}\right) \mathrm{R}_{\mathrm{LO}} \\
& =\left[\mathrm{i}_{\mathrm{LO}(\mathrm{AVG})}{ }^{2}+\left(\frac{0.5 \Delta \mathrm{i}_{\mathrm{LO}}}{\sqrt{3}}\right)^{2}\right] \mathrm{L}_{\mathrm{X}} \mathrm{k}_{\mathrm{L}} \tag{16}
\end{align*}
$$

The energizing switches $\mathrm{S}_{\mathrm{EI}}$ and $\mathrm{S}_{\mathrm{EG}}$ burn ohmic power

$$
\begin{align*}
\mathrm{P}_{\mathrm{R}(\mathrm{EI} / \mathrm{G})} & =\mathrm{i}_{\mathrm{LO}(\mathrm{RMS}, \mathrm{E})}{ }^{2} \mathrm{R}_{\mathrm{EI} / \mathrm{G}}\left(\frac{\mathrm{t}_{\mathrm{E}}}{\mathrm{t}_{\mathrm{SW}}}\right) \\
& \propto \frac{\mathrm{i}_{\mathrm{LO}(\mathrm{AVG})}{ }^{2}+\left(\frac{0.5 \Delta \mathrm{i}_{\mathrm{LO}}}{\sqrt{3}}\right)^{2},}{\mathrm{~W}_{\mathrm{EI} / \mathrm{G}}} \tag{17}
\end{align*}
$$

where $i_{\text {LO(RMS,E) }}, \mathrm{R}_{\mathrm{EI} / \mathrm{G}}$, and $\mathrm{W}_{\mathrm{EI} / \mathrm{G}}$ are the RMS current, resistance, and width of $\mathrm{S}_{\mathrm{EI}}$ and $\mathrm{S}_{\mathrm{EG}}$. Similarly, draining switches $\mathrm{S}_{\mathrm{DG}}$ and $\mathrm{S}_{\mathrm{DO}}$ burn

$$
\begin{align*}
\mathrm{P}_{\mathrm{R}(\mathrm{DG} / \mathrm{O})} & =\mathrm{i}_{\mathrm{LO}(\mathrm{RMS}, \mathrm{D})}{ }^{2} \mathrm{R}_{\mathrm{DG} / \mathrm{O}}\left(\frac{\mathrm{t}_{\mathrm{D}}}{\mathrm{t}_{\mathrm{SW}}}\right) \\
& \propto \frac{\mathrm{i}_{\mathrm{LO}(\mathrm{AVG})}{ }^{2}+\left(\frac{0.5 \Delta \mathrm{i}_{\mathrm{LO}}}{\sqrt{3}}\right)^{2},}{\mathrm{~W}_{\mathrm{DG} / \mathrm{O}}} \tag{18}
\end{align*}
$$

where $i_{\mathrm{LO}(\mathrm{RMS}, \mathrm{D})}, \mathrm{R}_{\mathrm{DG} / \mathrm{O}}$, and $\mathrm{W}_{\mathrm{DG} / \mathrm{O}}$ are the RMS current, resistance, and width of $\mathrm{S}_{\mathrm{DG}}$ and $\mathrm{S}_{\mathrm{DO}}$. All four switches require charge loss

$$
\begin{equation*}
\mathrm{P}_{\mathrm{C}(\mathrm{SW})}=\mathrm{q}_{\mathrm{C}} \mathrm{v}_{\mathrm{DD}} \mathrm{f}_{\mathrm{O}} \propto \mathrm{~W}_{\mathrm{E} / \mathrm{D}} \mathrm{f}_{\mathrm{O}} . \tag{19}
\end{equation*}
$$

From (6), the loss on all the switches $\mathrm{P}_{\mathrm{SW}}$ is

$$
\begin{equation*}
\mathrm{P}_{\mathrm{SW}}=\sum \mathrm{P}_{\mathrm{SE} / \mathrm{D}} \propto \sqrt{\left[\mathrm{i}_{\mathrm{LO}(\mathrm{AVE})}{ }^{2}+\left(\frac{0.5 \Delta \mathrm{i}_{\mathrm{LO}}}{\sqrt{3}}\right)^{2}\right] \mathrm{f}_{\mathrm{O}}} \tag{20}
\end{equation*}
$$

The total loss is the sum of (16) and (20), and with the help of (11), it can be reduced to a function of 2 variables, i.e. $L_{O}$ and $f_{O}$. Since $L_{O}$ is limited to $1 \mu \mathrm{H}$ to $180 \mu \mathrm{H}$, by setting $L_{O}$ and sweeping $f_{O}$, it is shown in Fig. 9 that the optimal CCM charger is with the following settings: $\mathrm{L}_{\mathrm{O}}{ }^{\prime}$ is $180 \mu \mathrm{H}, \mathrm{f}_{\mathrm{O}}{ }^{\prime}$ is $111 \mathrm{MHz}, \mathrm{R}_{\mathrm{LO}}{ }^{\prime}$ is $10 \Omega, \mathrm{i}_{\mathrm{LO}(\mathrm{AVG})}$ ' is $38.2 \mu \mathrm{~A}$, $\Delta \mathrm{i}_{\mathrm{LO}}{ }^{\prime}$ is $50 \mu \mathrm{~A}$. With the operation set, switches can be optimized from Section III, with $\mathrm{W}_{\mathrm{EI}}$ at $1.2 \mu \mathrm{~m}, \mathrm{~W}_{\mathrm{EG}}$ at 860 $\mathrm{nm}, \mathrm{W}_{\mathrm{DG}}{ }^{\prime}$ at $1.2 \mu \mathrm{~m}, \mathrm{~W}_{\mathrm{Do}}{ }^{\prime}$ at $1.5 \mu \mathrm{~m}$, and total loss is 7.5 $\mu \mathrm{W}$. Note that the width for 2 energizing switches are different, and similarly the width for 2 energizing switches are different, because $\mathrm{S}_{\mathrm{EI}}$ and $\mathrm{S}_{\mathrm{DO}}$ are PMOS, and $\mathrm{S}_{\mathrm{EG}}$ and $\mathrm{S}_{\mathrm{DG}}$ are NMOS.


Fig. 8. Inductor current waveform in continuous conduction mode.


Fig. 9. Total loss on optimized SL in CCM for different inductances.

## b. DCM

In discontinuous conduction mode [17], on the other hand, the inductor does not conduct current across the entire switching cycle. Instead, the inductor is energized from 0 to $\mathrm{i}_{\mathrm{LO}(\mathrm{PK})}$ across $\mathrm{t}_{\mathrm{E}}$, and drained across $\mathrm{t}_{\mathrm{D}}$. After that, it stays at 0 current, until the next switching cycle begins, as Fig. 10 shows. The input power is the power from the piezoelectric transducer minus the losses on the recycler and the bridge, and the energy transferred by the inductor is approximately the peak energy it holds:


Fig. 10. Inductor current waveform in discontinuous conduction mode.

$$
\begin{equation*}
\mathrm{P}_{\mathrm{O}} \approx 0.5 \mathrm{~L}_{\mathrm{O}} \mathrm{i}_{\mathrm{LO}(\mathrm{PK})}^{2} \mathrm{f}_{\mathrm{O}}-\mathrm{P}_{\mathrm{SL}} \approx \mathrm{P}_{\mathrm{PZ}}-\mathrm{P}_{\mathrm{X}}-\mathrm{P}_{\mathrm{B}}-\mathrm{P}_{\mathrm{SL}} \tag{21}
\end{equation*}
$$

where PSL is the loss on the SL. Since drawn power $\mathrm{P}_{\mathrm{PZ}}$, loss on the recycler $\mathrm{P}_{\mathrm{X}}$, and loss on the bridge $\mathrm{P}_{\mathrm{BRG}}$ are all known variables for the SL , the term $\mathrm{L}_{\mathrm{O}} \mathrm{i}_{\mathrm{LO}(\mathrm{PK})}{ }^{2} \mathrm{f}_{\mathrm{O}}$ is a constant

$$
\begin{equation*}
\mathrm{L}_{\mathrm{O}} \mathrm{i}_{\mathrm{LO}(\mathrm{PK})}{ }^{2} \mathrm{f}_{\mathrm{O}} \approx 2\left(\mathrm{P}_{\mathrm{PZ}}-\mathrm{P}_{\mathrm{X}}-\mathrm{P}_{\mathrm{B}}\right) . \tag{22}
\end{equation*}
$$

The switched inductor in DCM still consumes ohmic loss on the inductor's resistance:

$$
\begin{align*}
\mathrm{P}_{\mathrm{R}(\mathrm{LO})} & =i_{\mathrm{LO}(\mathrm{RMS})}{ }^{2} \mathrm{R}_{\mathrm{LO}}=\left(\frac{i_{\mathrm{LO}(\mathrm{PK})}}{\sqrt{3}}\right)^{2} \mathrm{R}_{\mathrm{LO}}\left(\frac{\mathrm{t}_{\mathrm{C}}}{\mathrm{t}_{\mathrm{O}}}\right) \\
& =\left(\frac{i_{\mathrm{LO}(\mathrm{PK})}}{\sqrt{3}}\right)^{2}\left(\mathrm{~L}_{\mathrm{o}} \mathrm{k}_{\mathrm{L}}\right)\left[\left(\frac{\mathrm{L}_{\mathrm{O}}}{v_{\mathrm{E}}}+\frac{\mathrm{L}_{\mathrm{O}}}{v_{\mathrm{D}}}\right) i_{\mathrm{LO}(\mathrm{PK})}\right] \mathrm{f}_{\mathrm{O}} .  \tag{23}\\
& \propto \mathrm{i}_{\mathrm{LO}(\mathrm{PK})}{ }^{3} \mathrm{~L}_{\mathrm{O}}{ }^{2} \mathrm{f}_{\mathrm{O}}=2\left(\mathrm{P}_{\mathrm{PZ}}-\mathrm{P}_{\mathrm{X}}-\mathrm{P}_{\mathrm{B}}\right) \mathrm{i}_{\mathrm{LO(PK})} \mathrm{L}_{\mathrm{O}} \\
& \propto \mathrm{i}_{\mathrm{LO}(\mathrm{PK})} \mathrm{L}_{\mathrm{O}}
\end{align*}
$$

The last step is because from (21), $\mathrm{L}_{\mathrm{O}} \mathrm{i}_{\mathrm{LO}(\mathrm{PK})}{ }^{2} \mathrm{f}_{\mathrm{O}}$ is constant. It also burns ohmic loss on the energizing switches $\mathrm{S}_{\mathrm{EI}}$ and $\mathrm{S}_{\mathrm{EG}}$

$$
\begin{align*}
\mathrm{P}_{\mathrm{R}(\mathrm{EI} / \mathrm{G})} & =\mathrm{i}_{\mathrm{LO}(\mathrm{RMS}, \mathrm{E})}{ }^{2} \mathrm{R}_{\mathrm{EI} / \mathrm{G}}=\left(\frac{\mathrm{i}_{\mathrm{LO}(\mathrm{PK})}}{\sqrt{3}}\right)^{2}\left(\frac{\mathrm{t}_{\mathrm{E}}}{\mathrm{t}_{\mathrm{O}}}\right) \mathrm{R}_{\mathrm{EI} / \mathrm{G}} \\
& =\left(\frac{\mathrm{i}_{\mathrm{LO}(\mathrm{PK})}}{\sqrt{3}}\right)^{2}\left(\frac{\mathrm{i}_{\mathrm{LO}(\mathrm{PK})} \mathrm{L}_{\mathrm{O}}}{\mathrm{v}_{\mathrm{E}}}\right) \mathrm{f}_{\mathrm{O}} \mathrm{R}_{\mathrm{EI} / \mathrm{G}}  \tag{24}\\
& \propto \frac{\mathrm{~L}_{\mathrm{O}} \mathrm{i}_{\mathrm{LO}(\mathrm{Pk})}^{3} \mathrm{f}_{\mathrm{O}}}{\mathrm{~W}_{\mathrm{EI} / \mathrm{G}}}=2\left(\mathrm{P}_{\mathrm{PZ}}-\mathrm{P}_{\mathrm{X}}-\mathrm{P}_{\mathrm{B}}\right) \frac{\mathrm{i}_{\mathrm{LO}(\mathrm{Pk})}}{\mathrm{W}_{\mathrm{EI} / \mathrm{G}}} \\
& \propto \frac{\mathrm{i}_{\mathrm{LO}(\mathrm{Pk})}}{\mathrm{W}_{\mathrm{EI} / \mathrm{G}}} \propto \frac{\mathrm{~K}_{\mathrm{R}(\mathrm{MOS})}}{\mathrm{W}_{\mathrm{EI} / \mathrm{G}}}
\end{align*}
$$

Therefore, ohmic loss coefficient $\mathrm{K}_{\mathrm{R}(\mathrm{MOS})}$ is proportional to $\mathrm{i}_{\mathrm{LO}(\mathrm{PK})}$. Similarly, the draining switches $\mathrm{S}_{\mathrm{DG}}$ and $\mathrm{S}_{\mathrm{DO}}$ burn ohmic power

$$
\begin{align*}
\mathrm{P}_{\mathrm{R}(\mathrm{DG} / \mathrm{O})} & =\mathrm{i}_{\mathrm{LO}(\mathrm{RMS}, \mathrm{D})}{ }^{2} \mathrm{R}_{\mathrm{DG} / \mathrm{O}}=\left(\frac{\mathrm{i}_{\mathrm{LO}(\mathrm{PK})}}{\sqrt{3}}\right)^{2}\left(\frac{\mathrm{t}_{\mathrm{D}}}{\mathrm{t}_{\mathrm{O}}}\right) \mathrm{R}_{\mathrm{DG} / \mathrm{O}} \\
& =\left(\frac{\mathrm{i}_{\mathrm{LO}(\mathrm{PK})}}{\sqrt{3}}\right)^{2}\left(\frac{\mathrm{i}_{\mathrm{LO}(\mathrm{PK})} \mathrm{L}_{\mathrm{O}}}{\mathrm{v}_{\mathrm{D}}}\right) \mathrm{f}_{\mathrm{O}} \mathrm{R}_{\mathrm{DG} / \mathrm{O}}  \tag{25}\\
& \propto \frac{\mathrm{~L}_{\mathrm{O}} \mathrm{i}_{\mathrm{LO}(\mathrm{Pk})}{ }^{3} \mathrm{f}_{\mathrm{O}}}{\mathrm{~W}_{\mathrm{DG} / \mathrm{O}}} \propto 2\left(\mathrm{P}_{\mathrm{PZ}}-\mathrm{P}_{\mathrm{X}}-\mathrm{P}_{\mathrm{B}}\right) \frac{\mathrm{i}_{\mathrm{LO}(\mathrm{Pk})}}{\mathrm{W}_{\mathrm{DG} / \mathrm{O}}} \\
& \propto \frac{\mathrm{i}_{\mathrm{LO}(\mathrm{Pk})}}{\mathrm{W}_{\mathrm{DG} / \mathrm{O}}} \propto \frac{\mathrm{~K}_{\mathrm{R}(\mathrm{MOS})}}{\mathrm{W}_{\mathrm{DG} / \mathrm{O}}}
\end{align*}
$$

Again, the second to last step of (24) and (25) is because from (22), $\mathrm{L}_{\mathrm{O}} \mathrm{i}_{\mathrm{LO}(\mathrm{PK})}{ }^{2} \mathrm{f}_{\mathrm{O}}$ is a constant. The charge loss is the same as the CCM case in (19), and $\mathrm{K}_{\mathrm{C}(\mathrm{MOS})}$ is proportional to $\mathrm{f}_{\mathrm{O}}$. Therefore, the balanced ohmic loss and charge loss result in the optimum switch with total loss $\mathrm{P}_{\mathrm{Sw}}{ }^{\prime}$

$$
\begin{align*}
\mathrm{P}_{\mathrm{SW}} & =\sum \mathrm{P}_{\mathrm{S}(\mathrm{E} / \mathrm{D})}=\sum 2 \sqrt{\mathrm{~K}_{\mathrm{R}(\mathrm{MOS})} \mathrm{K}_{\mathrm{C}(\mathrm{MOS})}} \\
& \propto \sqrt{\mathrm{i}_{\mathrm{LO}(\mathrm{PK})} \mathrm{f}_{\mathrm{O}}}=\frac{\sqrt{\mathrm{L}_{\mathrm{O}} \mathrm{i}_{\mathrm{LO}(\mathrm{PK})}{ }^{2} \mathrm{f}_{\mathrm{O}}}}{\sqrt{\mathrm{i}_{\mathrm{LO}(\mathrm{PK})} \mathrm{L}_{\mathrm{O}}}} \propto \frac{1}{\sqrt{\mathrm{i}_{\mathrm{LO}(\mathrm{PK})} \mathrm{L}_{\mathrm{O}}}} \tag{26}
\end{align*}
$$

The total loss on the SL is the sum of (23) and (26), and there is an optimum $\mathrm{i}_{\mathrm{LO}(\mathrm{PK})} \mathrm{L}_{\mathrm{O}}$ that yields the lowest total loss on the SL. In other words, no matter what inductor we choose, we can always modify to switches and controller accordingly such that it is least lossy, shown in the thick solid black trace in Fig. 11. The loss stays flat because for each inductance value, the $\mathrm{i}_{\mathrm{LO}(\mathrm{PK})}$ is inversely proportional to $\mathrm{L}_{\mathrm{O}}$ so that their product stays constant. As a result, from (22), $\mathrm{f}_{\mathrm{O}}$ is also proportional to $\mathrm{L}_{\mathrm{O}}$. It consumes $\mathrm{P}_{\mathrm{SL}}{ }^{\prime}=2.11 \mu \mathrm{~W}$ in the following optimal settings: $\mathrm{L}_{\mathrm{O}}{ }^{\prime}$ is $130 \mu \mathrm{H}, \mathrm{R}_{\mathrm{LO}}{ }^{\prime}$ is $7.2 \Omega, \mathrm{f}_{\mathrm{O}}{ }^{\prime}$ is 40 kHz , and $\mathrm{i}_{\mathrm{LO}(\mathrm{PK})}{ }^{\prime}$ is 3.9 mA . With the waveforms set, the optimum switch size can be obtained from Section III: $\mathrm{W}_{\mathrm{EI}, \mathrm{G}}{ }^{\prime}$ is $459 \mu \mathrm{~m}$ and $\mathrm{W}_{\mathrm{DG}, \mathrm{O}^{\prime}}$ is $640 \mu \mathrm{~m}$. The thin traces in Fig. 11 also show the optimized losses with different inductor volume constraint. Therefore, DCM is less lossy than CCM, while operating at a much lower frequency, so we would operate it at DCM.


Fig. 11. Total loss on optimized SL in CCM for different inductor volumes.

## 6. Overall Performance

From the previous three sections, the optimum piezoelectric charger when the vibration provides $20 \mu \mathrm{~A}$ on the transducer is built. The charger draws $38.2 \mu \mathrm{~W}$ from $\mathrm{P}_{\mathrm{PZ}}$, and loses 944 nW on the recycler, 4.6 nW on the bridge, and $2.11 \mu \mathrm{~W}$ on the SL, resulting in an overall efficiency of $92 \%$. Simulation of the circuits consumes 990 nW on the recycler, 8.6 nW on the bridge, and $2.35 \mu \mathrm{~W}$ on the SL, resulting in an overall efficiency of $91 \%$. Repeating the same process for $\mathrm{i}_{\mathrm{PZ}(\mathrm{PK})}$
from $1 \mu \mathrm{~A}$ to $50 \mu \mathrm{~A}$, the results are presented in Fig. 12. $\mathrm{P}_{\mathrm{PZ}}$, $\mathrm{P}_{\mathrm{SL}}$, and $\mathrm{P}_{\mathrm{D}}$ all scale linearly with vibration strength, but $\mathrm{P}_{\mathrm{X}}$ does not. That is because the recycler operation is the same across all conditions, and the loss also remains the same.


Fig. 12. Drawn power and optimized losses across vibration strength.
Increasing volume of the inductor can reduce the loss on the recycler $\mathrm{P}_{\mathrm{X}}$ and switched inductor $\mathrm{P}_{\text {SL, }}$, as shown in Fig. 13. The efficiency stays above $90 \%$ for all inductor sizes and reaches $95 \%$ with a $5 \times 5 \times 3 \mathrm{~mm}^{3}$ inductor. Choosing the largest inductor the application allows yields the highest efficiency.

Table 1 lists and compares the technology used, components, power level, and performance for the design in this paper and several state-of-the-arts. Despite constraint to the smallest inductor, the aforementioned design has the highest efficiency, and the design guideline should be followed for all future recycling bridge designs.


Fig. 13. Total loss on optimized SL in CCM for different inductor volumes.

Table 1. Performance Comparison with the State of the Art.

|  | JSSC <br> $\mathbf{1 0}[8]$ | TPE 19' <br> [11] | ISSCL <br> $\mathbf{2 0} \mathbf{}^{\prime}[12]$ | This <br> Work |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{L}_{\mathrm{MIN}}[\mathrm{nm}]$ | 500 | 350 | 130 | 300 |
| $\mathrm{~V}_{\mathrm{BD}}[\mathrm{V}]$ | 5.0 | 5.0 | 3.3 | 3.0 |
| $\Delta \mathrm{v}_{\mathrm{PZ}(\mathrm{OC})}[\mathrm{V}]$ | 4.8 | 5.0 | 3.2 | 4.3 |
| $\mathrm{f}_{\mathrm{VIB}}[\mathrm{Hz}]$ | 225 | 225 | 143 | 100 |
| $\mathrm{~L}_{\mathrm{X}}[\mu \mathrm{H}]$ | 22 | 3300 | 3300 | $7 ; 130$ |
| $\mathrm{R}_{\mathrm{L}}[\Omega]$ | $\mathrm{N} / \mathrm{A}$ | 1.8 | $\mathrm{~N} / \mathrm{A}$ | $0.9 ; 6$ |
| $\mathrm{P}_{\mathrm{PZ}}[\mu \mathrm{W}]$ | 36 | 53 | 31 | 38.2 |
| $\eta_{\mathrm{C}}$ | $88 \%$ | $95 \%$ | $90 \%$ | $92 \%$ |

## 7. Conclusions

The optimized recycling bridge power stage with $92 \%$ efficiency has been theorized and designed. To optimize the power switches, width of the device should be selected such that the ohmic loss equal the charge loss. On the recycler, we increase inductance until the gain in the loss on the inductor cancels the reduction on the switch. On the switched inductor, DCM performs better than CCM. In the end, the optimized power stage can be implemented to power internet of things and microsensors to extend life and expand functionality.

## ACKNOWLEDGMENT

The authors thank Dr. Lazaro, Dr. Blanco, Dr. Morroni, and Texas Instruments for their sponsorship and support.

## References

[1] D. Puccinelli and M. Haenggi, "Wireless sensor networks: applications and challenges of ubiquitous sensing," IEEE Circuits and Syst. Mag., vol. 3, no. 3, pp. 19-29, 2005.
[2] S.P. Beeby, M.J. Tudor, and N.M. White, "Energy harvesting vibration sources for micro-systems applications," Measurement Science and Technology, vol. 17, no. 12, pp. 175-195, Oct. 2006.
[3] P.D. Mitcheson, E.M. Yeatman, G.K. Yao, A.S. Homes, and T.C. Green, "Energy harvesting from human and machine motion for wireless electronic devices," Proc. IEEE, vol. 96, no. 9, pp. 1457-1486.
[4] F. Goodarzy, E. Skafidas, and S. Bambini, "Feasibility of energy-autonomous wireless microsensors for biomedical applications: powering and communication," IEEE Rev. Biomed. Eng., vol. 8, pp. 17-29. Aug. 2015.
[5] S. Suduvalayam and P. Kulkarni, "Energy harvesting sensor nodes: Survey and implications," IEEE Commun. Surveys Tut., vol. 13, no. 3, pp. 443-543 461, Jul.-Sep. 2011.
[6] S. Roundy, P.K. Wright, and J. Rabaey, "A study of low level vibrations as a power source for wireless sensor nodes," Comput. Commun., vol. 26, no. 11, pp. 1131-1144, Jul. 2003.
[7] G.A. Rincón-Mora and S. Yang, "Tiny piezoelectric harvesters: Principles, constraints, and power conversion," IEEE Trans. on Circuits Syst. I, vol. 63, no. 5, pp. 639-649, May 2016.
[8] Y.K. Ramadass and A.P. Chandrakasan, "An efficient piezoelectric energy harvesting interface circuit using a bias-flip rectifier and shared inductor," IEEE J. of Solid-State Circuits, vol. 45, no. 1, pp. 189-204, Jan. 2010.
[9] D. Kwon and G.A. Rincón-Mora, "A single-inductor $0.35-\mu \mathrm{m}$ CMOS energy-investing piezoelectric harvester," IEEE J. of Solid-State Circuits, vol. 49, no. 10. pp. 2277-2291, Oct. 2014.
[10] D.A. Sanchez, J Leicht, F. Hagedorn, E. Jodka, E. Fazel, and Y. Manoli, "A parallel-SSHI rectifier for piezoelectric energy harvesting of periodic and shock
excitations," IEEE J. of Solid-State Circuits, vol. 51, no. 12. pp. 2867-2879, Dec. 2016.
[11] S. Li, A. Roy, and B.H. Calhoun, "A piezoelectric energy-harvesting system with parallel-SSHI rectifier and integrated maximum-power-point tracking," IEEE Solid-State Circuit Letters., vol. 2, no. 12, pp. 301304, Dec. 2019.
[12] P.D. Mitcheson, T.C. Green, and E.M Yeatman, "Power processing circuits for electromagnetic, electrostatic and piezoelectric inertial energy scavengers," Microsyst. Technol., vol. 13, no. 11, pp. 1629-1635, Jul. 2007.
[13] S. Yang and G.A. Rincón-Mora, "Energy-harvesting piezoelectric-powered CMOS series switched-inductor bridge", IEEE Trans. on Power Electronics, vol. 34, no. 7, pp. 6489-6497, July 2019.
[14] S. Du, Y. Jia, and C. Zhao, "A fully integratred splitelectrode SSHC rectifier for piezoelectric energy harvesting", IEEE J. of Solid-State Circuits, vol. 54, no. 6, pp. 1733-1743, June. 2019.
[15] R.D. Prabha and G.A. Rincón-Mora, "Maximizing Power-Transfer Efficiency in Low-Power DC-DC Converters," IET Electronic Letters, vol. 51, no. 23, pp. 1918-1920, Nov. 2015.
[16] P.J. Liu and C.W. Chang, "CCM noninverting buckboost converter with fast duty-cycle calculation control for line transient improvement," IEEE Trans. on Power Electronics, vol. 33, no. 6, pp. 5097-5107, June 2018.
[17] W. Fu, S.T. Tan, M. Radhakrishnan, R. Byrd, and A.A. Fayed, "A DCM-only buck regulator with hysteretic-assisted adaptive minimum-on-time control for low-power microcontrollers," IEEE Trans. on Power Electronics, vol. 31, no. 1, pp. 418-429, Jan. 2016.

