Frequency Response of Hysteretic Comparators in Switching Converters

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Abstract

Hysteretic switching dc-dc converters are popular because they are (*i*) relatively simple (i.e., self-oscillating and self-compensating), (*ii*) fast (i.e., able to respond within one switching cycle: $f_{0dB} = f_{SW}$), and (*iii*) robust (i.e., reliably stable). Although the time-domain operation of a hysteretic comparator can at times be intuitive, its ac transfer response in a switching converter (i.e., gain and phase) is not because linearizing what is already an inherently nonlinear circuit is difficult. This presentation illustrates how to derive a *describing* (rather than *transfer*) *function* that conveys more ac insight and shows how the oscillator circuit ensures there is just enough phase shift across the feedback loop to sustain oscillations (i.e., reach 180^{O} of phase shift at $f_{0dB} = f_{SW}$).

Summary

- Hysteretic switching converter ≡ **Oscillator** (i.e., $f_{SW} = f_{0dB} = f_{180o}$);
- AC **Response** ≡ **Large Signal** (i.e., loop processes **f_{SW} signal**);
- **LC eliminates higher-than-f_{SW} frequencies**

  ∴ **Only comparator's f_{SW} component is relevant**.

$$\text{Gain} \equiv \frac{\Delta v_{OUT}(f_{SW})}{\Delta v_{IN}(f_{SW})} = \frac{\left(\frac{4}{\pi}\right)V_{DD}}{\Delta v_{IN(PP)}} \leq \frac{\left(\frac{4}{\pi}\right)V_{DD}}{V_{HYST}}$$

Square Wave's Fundamental Component

Input Ripple's Amplitude

- **If** comparator's $\mathbf{T_{DLY}} \ll \mathbf{T_{SW}}$ ∴ **No in-band** comparator **pole**;
- **Comparator waits for v_{IN} to reach trip point**

  ∴ **90^O if $\Delta v_{IN} = V_{HYST}$** AND **< 90^O if $\Delta v_{IN} > V_{HYST}$**;
- **–FB *adjusts* Δv_{IN} (*gain* and *phase*) until oscillations are sustained.**

Introduction: Problem Statement

"Hysteretic buck converters are always stable."

[1] K. -C. Lee et al., ISSCC 2010.
[2] C. -H. Tso et al., IEEE Power Electronics Letters, Sept. 2003.
[3] J. H. Park et al., IEEE COMPEL Workshop, 2006.

• **Hysteretic DC-DC Converters:**

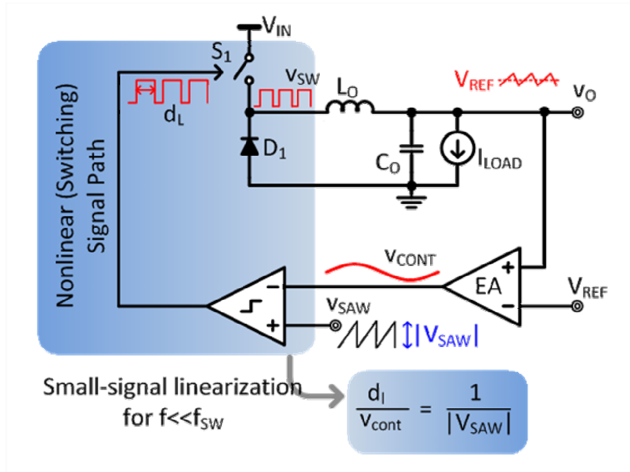
- Simple system and intuitive operation, but **always stable?**

- **Output voltage ripple often exceeds hysteretic window**, **why?**

- Output can **ring rail-to-rail when** the output impedance lacks resistive components (e.g., $\mathbf{R_{ESR}}$ **is low** in C_O) , **why?**

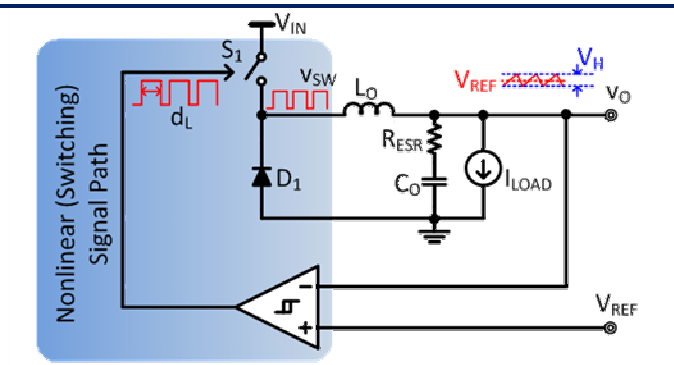
➔ How can we explain the stability and dynamics of hysteretic comparators in switching dc-dc converters?

PWM Switching Converters



➔ A PWM switching converter **can be averaged and linearized across a switching cycle**.

Hysteretic Switching Converters

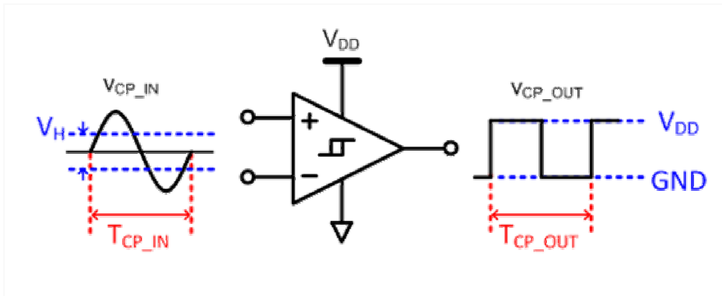


- **No error amp, Digital output, Circuit processes frequencies near f_{SW}**:

  → Hysteretic comparator is **difficult to linearize** (i.e., extract ac transfer function).

- It is well known that the circuit **sustains oscillations at f_{SW}**, **but how?**

Linearizing a Hysteretic Comparator

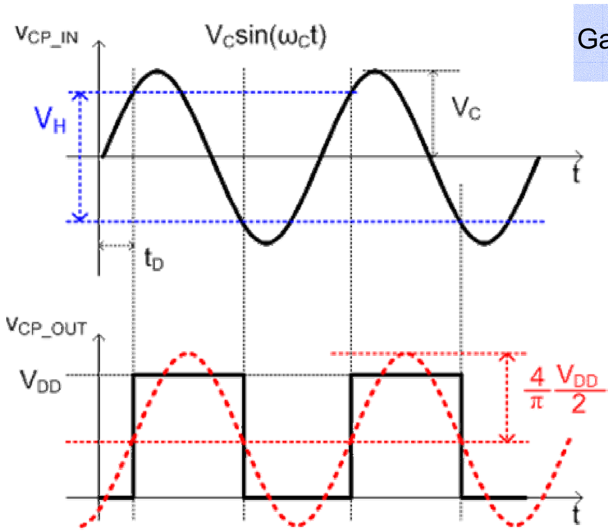


Observations:

1) Nonlinear block
2) $T_{CP\_IN} = T_{CP\_OUT} \rightarrow f_{CP\_IN(fund)} = f_{CP\_OUT(fund)}$
3) Gain and phase-shift relationships can be defined.

* Reference: Chestnut and Mayer, "Servomechanisms and Regulating System Design," New York: John Wiley & Sons, 1955.

Describing Function: Gain and Phase Shift



$$\text{Gain} = \frac{\text{Output Amplitude}}{\text{Input Amplitude}}$$

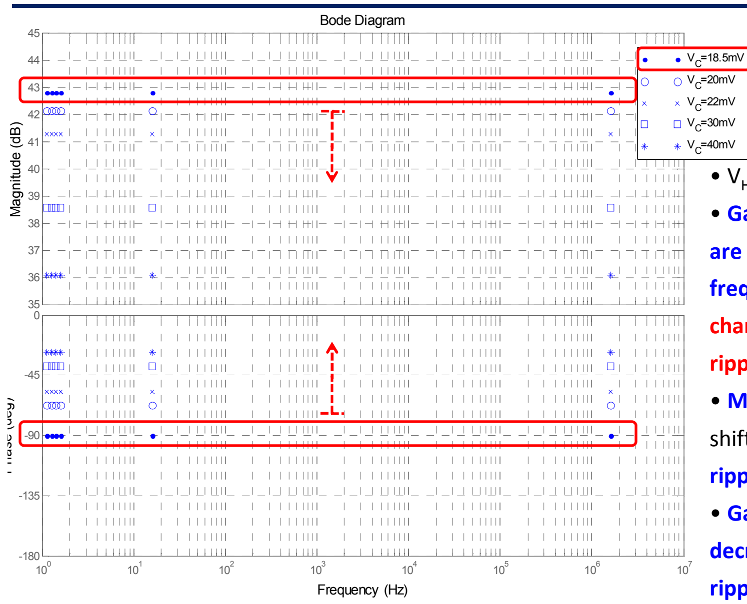
$$= \frac{\frac{4}{\pi}\frac{V_{DD}}{2}}{V_C} = \frac{2V_{DD}}{\pi V_C}$$

$$\text{Phase-Shift} = \omega_C t_D$$

$$V_C \sin(\omega_C t_D) = \frac{V_H}{2}$$

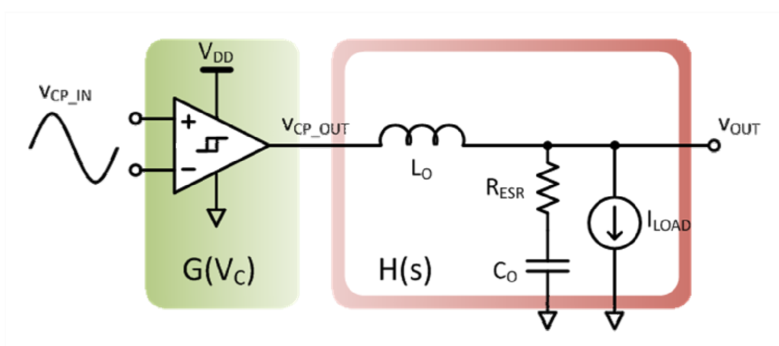
$$\omega_C t_D = \sin^{-1}\left(\frac{V_H}{2V_C}\right)$$

Frequency Response



- $V_H = 2 \times 18.5$ mV
- **Gain and phase-shift are constant over frequency**, **but they change with the input ripple** (i.e., with V_C).
- **Max Gain and phase** shift **with min. input ripple**: $V_C = V_H/2$.
- **Gain and phase shift decreases as input ripple** (V_C) **increases.**

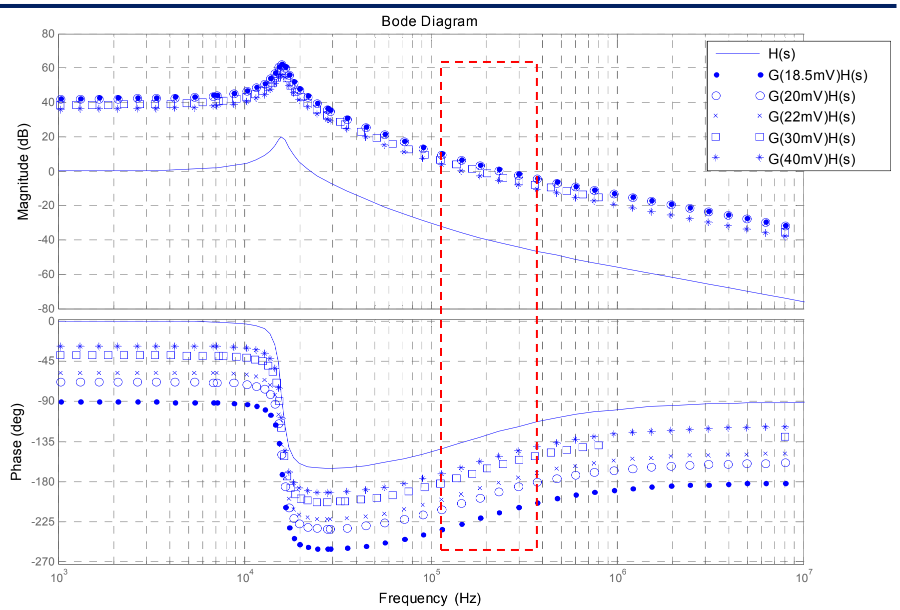
Loop-Gain Transfer Function



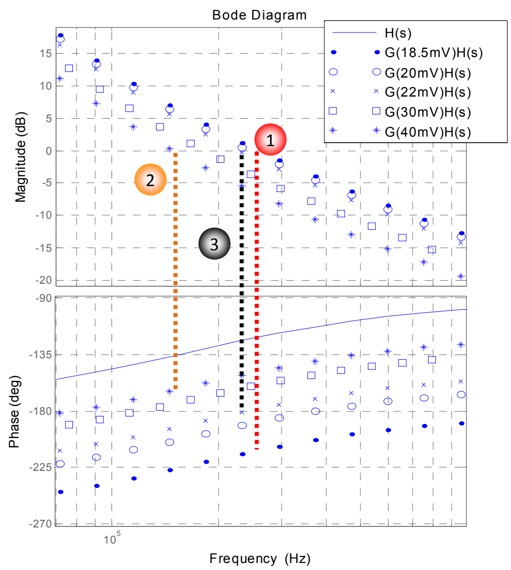
$$G(V_C) = \frac{2V_{DD}}{\pi V_C} \angle \sin^{-1}\left(\frac{V_H}{2V_C}\right) \qquad H(s) = \frac{1 + sR_{ESR}C_O}{1 + sR_{ESR}C_O + s^2 L_O C_O}$$

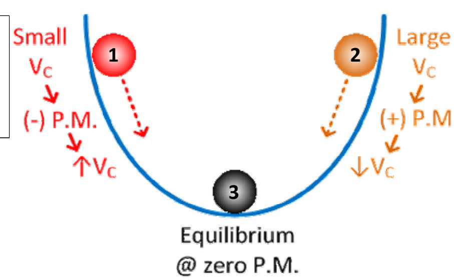
➔ Loop-Gain = $G(V_C) \times H(s)$

Frequency Response of the Loop-Gain



Frequency Response of the Loop-Gain (zoom in)



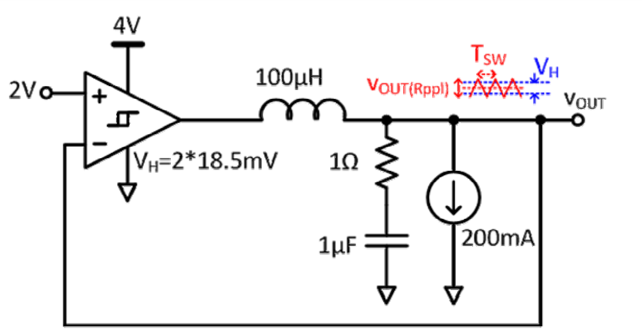


➔ The loop changes the amplitude and frequency of the input ripple to ensure $f_{0dB} = f_{180^o}$;

➔ Loop sustains oscillation;

➔ $f_{SW} = f_{0dB} = f_{180^o}$

Theory vs. Simulation



Parameters	Theoretical Estimation	Simulated Result
f_{SW}	230 KHz	262 KHz
$v_{OUT(Rppl)}$	22 mV	20 mV

* **Source of Error:** v_{OUT} is not a sinusoidal waveform.

Conclusions

Hysteretic DC-DC Switching Converters:

- **Sustained oscillation** of v_{OUT} about V_{REF} → $\mathbf{f_{0dB} = f_{180^\circ}}$;
- **Respond within 1 switching cycle** → $\mathbf{f_{0dB} = f_{SW}}$;

  ∴ Faster than PWM counterparts.

- Hysteretic comparator is **nonlinear**.

Describing Function of Hysteretic Comparators:

- Linearize by **analyzing fundamental frequency**;
- Supply **V_{DD} fixes Δv_{OUT}'s amplitude** to a constant;
- **Hysteresis delays** (phase-shifts) **response**;

  → **Gain and phase change with input ripple**'s amplitude.