

# ECE 3040 Microelectronic Circuits

Exam 1

September 27, 2023

Dr. W. Alan Doolittle

25 minutes

Print your name clearly and largely:

Solution

## Instructions:

**DO NOT REMOVE ANY SHEETS FROM THIS EXAM!** Read all the problems carefully and thoroughly before you begin working. You are allowed to use 1 sheet of notes (1 page front and back) as well as a calculator. There are 100 total points. Observe the point value of each problem and allocate your time accordingly. **SHOW ALL WORK AND CIRCLE YOUR FINAL ANSWER WITH THE PROPER UNITS INDICATED. Write legibly. If I cannot read it, it will be considered a wrong answer.** Do all work on the paper provided. Turn in all scratch paper, even if it did not lead to an answer. **Turn in your notes sheet placed under your exam.** Report any and all ethics violations to the instructor. A periodic table is supplied on the last page. Good luck!

Sign your name on **ONE** of the two following cases:

I DID NOT observe any ethical violations during this exam:

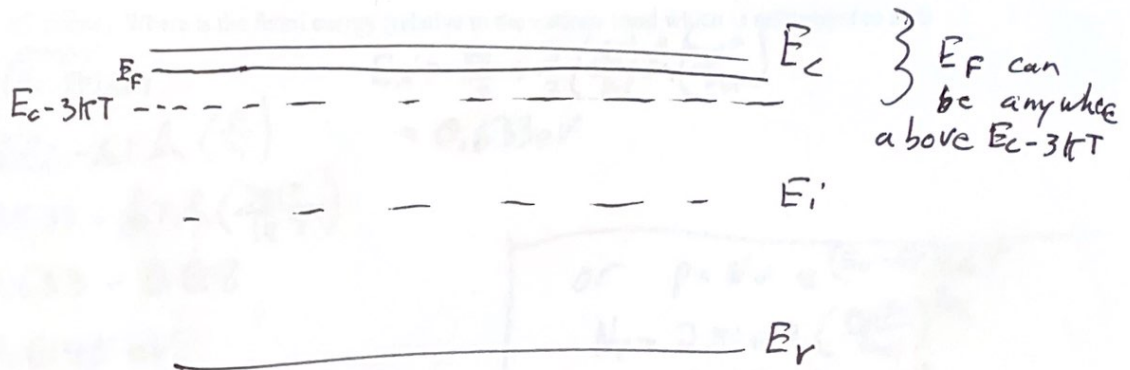
I observed an ethical violation during this exam:

**First 20% Multiple Choice and True/False**  
**(Circle the letter of the most correct answer or answers )**

- 1.) (2-points) True or **False**: The energy bandgap can be considered the energy required to rip an electron out of the material into the vacuum where it conducts electricity.
- 2.) (2-points) **True** or False: The mobility is the low electric field slope of the drift velocity, in the region where the drift velocity is linearly proportional to the electric field.
- 3.) (2-points) True or **False**: Electric fields are always applied from external sources like batteries and can never be created inside the device.
- 4.) (2-points) True or **False**: If both electrons and holes are exposed to a built in field, as in a solar cell, the charges both move to the same side of the device accumulating on the anode (p-side).
- 5.) (2-points) True or **False**: For a device with a high concentration of defect states or impurity states, the minority carrier lifetime will be very high.
- 6.) (2-points) **True** or False: In a degenerately doped semiconductor, more than one hole can occupy a given state.
- 7.) (2-points) **True** or False: Larger bond strength results in higher energy bandgaps.
- 8.) (2-points) **True** or False: The Fermi-Dirac integral of order  $\frac{1}{2}$ , the Fermi distribution function and the Boltzmann distribution function are all ways of describing the probability that a state is filled with an electron.
- 9.) (2-points) True or **False**: Auger recombination is only important at low current density or at low optical injection (low optical power).
- 10.) (2-points) **True** or False: Impact ionization can increase a small current into a large current but is generally noisy current as the multiplication of charge is random and thus, stochastic.

Short Answer ("Plug and Chug"):

- 11.) (6-points) Sketch and label the energy band diagram of any degenerately doped n-type semiconductor indicating why the material is degenerately doped and where the fermi-energy is. Points deducted for lack of neatness, clarity and missing energy labels. No numeric values are needed – just symbolic labels.

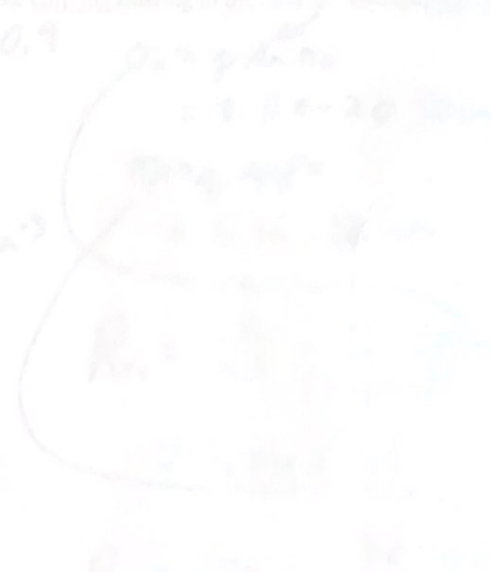


$n = \left[ \frac{3200}{\pi} \right] \pi = 2.12 \times 10^{21} \text{ cm}^{-3}$   
 $p_0 = N_A$   
 $n_0 = \frac{n^2}{p_0} = \frac{(2.12 \times 10^{21})^2}{10^{21}} = 4.5 \times 10^{21} \text{ cm}^{-3}$

$I = \frac{V}{R} = \frac{0.7 \text{ V}}{100 \Omega} = 7 \times 10^{-3} \text{ A}$

$I_p = \frac{V}{R} = \frac{0.7 \text{ V}}{100 \Omega} = 7 \times 10^{-3} \text{ A}$

$n = 2.12 \times 10^{21} \text{ cm}^{-3}$   
 $p_0 = 10^{21} \text{ cm}^{-3}$   
 $n_0 = 4.5 \times 10^{21} \text{ cm}^{-3}$   
 $E_g - q = 0.09 \text{ eV}$



For the following problems (12-13) use the following material parameters and assuming total ionization:

For InP:

$$n_i = 1.3e7 \text{ cm}^{-3} \quad N_D = 3e13 \text{ cm}^{-3} \text{ donors} \quad N_A = 3e17 \text{ cm}^{-3} \text{ acceptors}$$

$$m_p^* = 0.6m_0 \quad m_n^* = 0.08m_0$$

$$E_G = 1.344 \text{ eV} \quad \text{Electron mobility, } \mu_n = 900 \text{ cm}^2/\text{Vsec} \quad \text{Hole mobility, } \mu_p = 120 \text{ cm}^2/\text{Vsec}$$

$$\text{Temperature} = 27 \text{ degrees C}$$

12.) (7-points) Where is the fermi energy (relative to the valence band which is referenced to zero energy)?

$$p = n_i e^{(E_i - E_f)/kT}$$

$$E_f = E_i - kT \ln\left(\frac{p}{n_i}\right)$$

$$= 0.633 - kT \ln\left(\frac{3e17}{1e7}\right)$$

$$= 0.633 - 0.618$$

$$E_f = 0.0145 \text{ eV}$$

$$E_i = \frac{E_g}{2} + \frac{3}{2} \left(\frac{kT}{2}\right) \ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$= 0.633 \text{ eV}$$

$$\text{or } p = N_v e^{(E_v - E_f)/kT}$$

$$N_v = 2.51e19 \left(\frac{m_p^*}{m_0}\right)^{3/2}$$

$$= 1.167e19 \text{ cm}^{-3}$$

$$3e17 = 1.167e19 e^{(E_v - E_f)/kT}$$

$$E_f - E_v = 0.09482 \text{ eV}$$

13.) (10-points) A 52 nm (1 nm = 1e-9 m) diameter x 300 nm long cylindrical semiconductor resistor is made from the semiconductor from problem 12 is biased on two opposing sides (longest dimension) with 0.9 volts. Determine both the electron and hole currents flowing in the device.  $900 \text{ cm}^2/\text{V-sec}$

$$A = \left[\frac{52e-9}{2}\right]^2 \pi = 2.12e-11 \text{ cm}^2 \quad V = 0.9$$

$$p_0 = N_A \quad (N_A \gg N_D + N_A \gg n_i)$$

$$= 3e17 \text{ cm}^{-3}$$

$$n_0 = \frac{n_i^2}{p_0} = \frac{(1.3e7)^2}{3e17} = 5.63e-4 \text{ cm}^{-3}$$

$$I_n = \frac{V}{R_n} = \frac{0.9 \text{ V}}{1.74e25 \Omega} = \boxed{5.2e-26 \text{ A}}$$

$$I_p = \frac{V}{R_p} = \frac{0.9 \text{ V}}{2.45e5 \Omega} = \boxed{3.67 \mu\text{A}}$$

$$\sigma_n = q \mu_n n_0$$

$$= 8.11e-20 \text{ } \Omega^{-1}\text{cm}$$

$$\sigma_p = q \mu_p p_0$$

$$= 5.76 \text{ } \Omega^{-1}\text{cm}$$

$$R_n = \frac{L}{\sigma_n A} = \frac{3e-5 \text{ cm}}{(2.12e-11) \text{ cm}^2}$$

$$= 1.74e25 \Omega$$

$$R_p = \frac{L}{\sigma_p A} = \frac{3e-5 \text{ cm}}{(2.12e-11) (5.76)}$$

$$= 2.45e5 \Omega$$

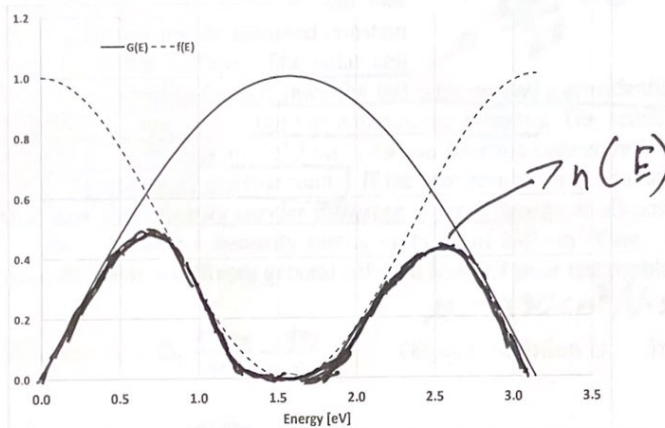
**Section 3 (more short answer)**

14.) (13 -points total) The brilliant and humble Professor Doolittle has found a semiconductor that obeys new "Doolittian Physics". It is found the density of states of this material follows the function:

$$G(E) = \begin{cases} \sin(E) & \text{for } 0 < E < \pi \\ 0 & \text{elsewhere} \end{cases}$$

and the fermi distribution function for this new physics is:

$$f(E) = \frac{1}{2} [1 + \cos(2E)] \text{ for } 0 < E < \pi$$



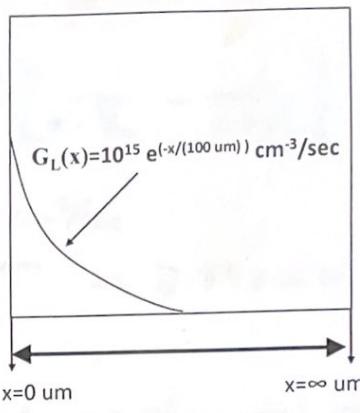
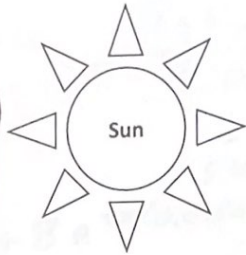
Sketch (do not label magnitudes) of the electron concentration versus energy function  $n(E)$  for  $0 < E < \pi$ .

**Pulling all the concepts together for a useful purpose:**

15.) (44-points)

B.C. @  
 $x=0$   
 $G_L(x=0)\tau_n = \Delta n(x=0)$

A semi-infinite length section of semiconductor is to be used as a solar cell. The sun's light is absorbed such that the surface ( $x=0$ ) excess minority carrier concentration,  $\Delta n(x=0)$ , always equals the surface generation rate times the minority carrier lifetime. (The generation rate throughout the semiconductor is dependent on position as:



$G_L(x) = G_{LO} e^{-\alpha x} = 10^{15} e^{-(x/(100 \text{ um}))}$  cm<sup>-3</sup>/sec for all  $x$  and cannot be assumed constant nor only on the surface. The solar cell has been exposed to the sun since the last time we had a presidential nominee that was competent for

$\frac{\partial \Delta n}{\partial x} = 0$

the office (a long time - but we will assume infinity). The semiconductor is doped p-type with an acceptor concentration of  $1e17 \text{ cm}^{-3}$ , has an intrinsic concentration of  $1e10 \text{ cm}^{-3}$  and has a minority carrier lifetime of 40 microseconds. If the semiconductor is held at room temperature (27 degrees C), determine the minority carrier diffusion current density at all positions in the semiconductor ( $0 \leq x < \infty \text{ um}$ ). Assume a minority carrier mobility of  $230 \text{ cm}^2/\text{Vsec}$ . I note that this problem "may" or "may not" have a different general solution from all prior test problems but is given.

$\tau_n = 40 \mu s$

$\mu_n = 230 \text{ cm}^2/\text{V-sec} \rightarrow D_n = \frac{kT}{q} \mu_n = 5,957 \text{ cm}^2/\text{sec}$

a) Given:  $0 = D_n \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n}$

General Solution is:  $\Delta n_p(x) = A e^{-x/L_n} + B e^{+x/L_n}$

$L_n = \sqrt{D_n \tau_n} = 0.0154 \text{ cm or } \sim 154 \mu m$

b) Given:  $0 = D_n \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n} + G_L \dots$

... General Solution is:  $\Delta n_p(x) = A e^{-x/L_n} + B e^{+x/L_n} + G_L \tau_n \sim 154 \mu m$

c) Given:  $0 = D_n \frac{d^2 \Delta n_p}{dx^2}$

General Solution is:  $\Delta n_p(x) = A + Bx$

d) Given:  $0 = D_n \frac{d^2 \Delta n_p}{dx^2} + G_L$

General Solution is:  $\Delta n_p(x) = Ax^2 + Bx + C$

e) Given:  $0 = D_n \frac{d^2 \Delta n_p}{dx^2} + G_{LO} f(x) \dots$

... General Solution is:  $\Delta n_p(x) = \left[ -\frac{G_{LO}}{D_n} \iint f(x) dx^2 \right] + Bx + C$

f) Given:  $0 = D_n \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n} + G_{LO} e^{-\alpha x} \dots$   
 ... General Solution is:  $\Delta n_p(x) = A e^{-x/L_n} + B e^{+x/L_n} - \frac{L_n G_{LO} e^{-\alpha x}}{2D_n} \left[ \frac{1}{\left\{ \alpha - \left( \frac{1}{L_n} \right) \right\}} - \frac{1}{\left\{ \alpha + \left( \frac{1}{L_n} \right) \right\}} \right]$

g) Given:  $\frac{d \Delta n_p}{dt} = -\frac{\Delta n_p}{\tau_n}$

General Solution is:  $\Delta n_p(t) = \Delta n_p(t=0) e^{-t/\tau_n}$

h) Given:  $0 = -\frac{\Delta n_p}{\tau_n} + G_L$

General Solution is:  $\Delta n_p = G_L \tau_n$

Extra work can be done here, but clearly indicate which problem you are solving.

General Solution:

$$\Delta n(x) = A e^{-x/L_n} + B e^{+x/L_n} - \frac{L_n G_{LO}}{2 D_n} \left[ \frac{1}{\alpha - 1/L_n} - \frac{1}{\alpha + 1/L_n} \right] e^{-\alpha x}$$

$\uparrow$   $0.0154 \text{ cm}$       $\nearrow$   $10^{15} \text{ cm}^{-3}$       $\uparrow$   $100 \text{ cm}^{-1}$   
 $\downarrow$   $5.957 \text{ cm}^2/\text{sec}$

$$= A e^{-x/0.0154 \text{ cm}} + B e^{+x/0.0154 \text{ cm}} - 2.89 \times 10^9 e^{-100x}$$

B.C.:

$\Delta n(x \rightarrow \infty) \Rightarrow B = 0$  to avoid unphysical results

$\Delta n(x=0) \equiv G_L(x=0) \tau_n = 4 \times 10^9 = A - 2.89 \times 10^9$

$A = 6.689 \times 10^9 \text{ cm}^{-3}$

$\therefore \Delta n(x) = 6.689 \times 10^9 e^{-x/0.0154 \text{ cm}} - 2.89 \times 10^9 e^{-100x}$

$J_n = q D_n \nabla n_p(x)$

but  $\nabla n_0 = 0$  so ...  $J_n = q D_n \nabla(\Delta n)$

~~$J_n = q D_n \nabla n$~~

$J_n = (1.6 \times 10^{19}) (5.957) \left[ \frac{-6.689 \times 10^9}{0.0154} e^{-x/L_n} + (100) 2.89 \times 10^9 e^{-100x} \right]$

$J_n = [-4.26 \mu\text{A}/\text{cm}^2] e^{-x/0.0154 \text{ cm}} + [2.76 \mu\text{A}/\text{cm}^2] e^{-100x}$

