ECE 3040 Microelectronic Circuits

Exam 1

February 6, 2003

Dr. W. Alan Doolittle

Print your name clearly and largely: Solution

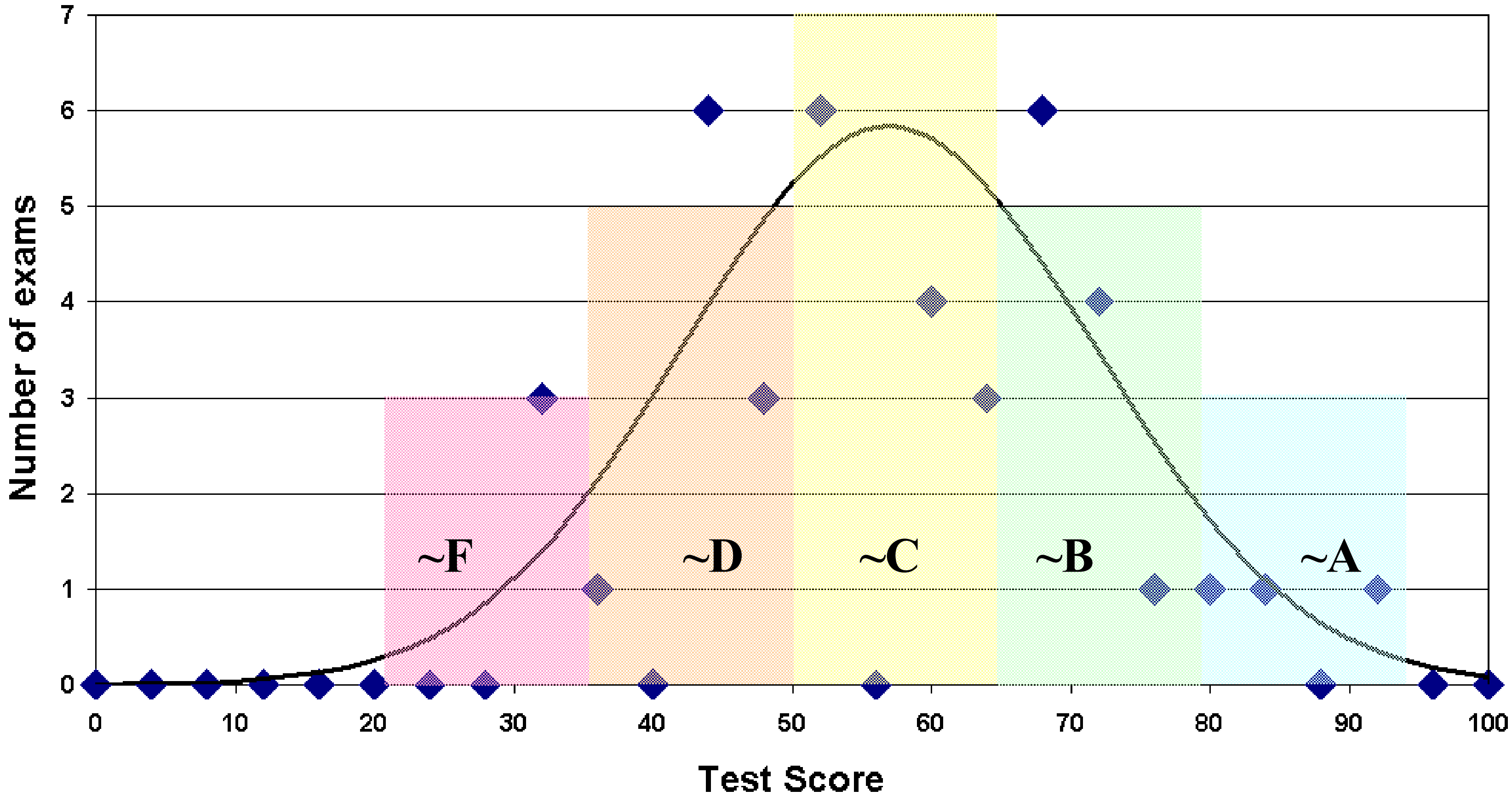
Instructions:
Read all the problems carefully and thoroughly before you begin working. You are allowed to use 1 new sheet of notes (1 page front and back) as well as a calculator. There are 100 total points. Observe the point value of each problem and allocate your time accordingly. SHOW ALL WORK AND CIRCLE YOUR FINAL ANSWER WITH THE PROPER UNITS INDICATED. Write legibly. If I cannot read it, it will be considered a wrong answer. Do all work on the paper provided. Turn in all scratch paper, even if it did not lead to an answer. Report any and all ethics violations to the instructor. Good luck!

Sign your name on **ONE** of the two following cases:

I DID NOT observe any ethical violations during this exam:

I observed an ethical violation during this exam:

Class Totals	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12	#13	#14
Number of Tests=	40													
Point Value of problem=	2	2	2	2	2	3	3	3	3	3	10	15	25	25
Individual Problem Average=	51.5	84.8	97.0	81.8	78.8	87.9	30.3	84.8	61.6	78.8	93.9	56.2	31.3	14.5
Exam Average=	56.7													
Exam Standard Deviation=	14.4													
Exam Max=	90													
Exam Min=	29													



First 25% **Multiple Choice and True/False (Circle the letter of the most correct answer)**

1.) (2-points) True or [✓ False:] Ionized acceptors are positively charged

2.) (2-points) [✓ True] or False: The energy bandgap is the energy required to break a valence electron away and have it conduct through the material

3.) (2-points) True or [✓ False:] A metal has a much higher energy bandgap than an insulator, making it conduct better.

4.) (2-points) True or [✓ False:] Indirect semiconductors are better at absorbing and emitting light than direct bandgap materials.

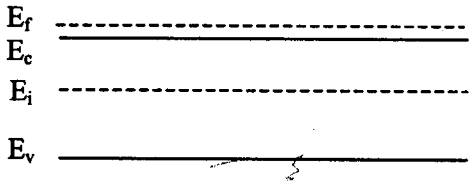
5.) (2-points) [✓ True] or False: Zincblende and diamond crystal structures are nearly the same, only the zincblende structure involves two or more elements while the diamond structure involves only one element..

Select the <u>**best**</u> answer for 6-10:

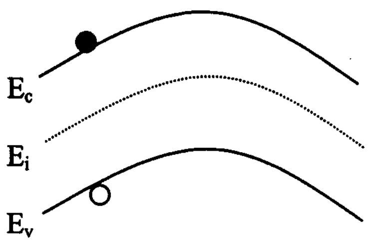
6.) (3-points) A heavily doped n-type GaAs block (n_o>>n_i) is connected to a battery. Which of the following is true:
- a.) The block is in equilibrium
- b.) The block has a flat fermi energy
- c.) Minority carriers carry most of the current
- d.) Holes carry most of the current
- [✓ e.)] Electrons carry most of the current

7.) (3-points) Assuming only partial ionization (do not make this hard, it is not): If the fermi energy, E_f, is located directly on an acceptor whose concentration is 1e19 cm^{-3}, what is the hole concentration in the material?
- a.) 1e19 cm^{-3}
- b.) 2e19 cm^{-3}
- [✓ c.)] 5e18 cm^{-3}
- d.) 0 cm^{-3}
- e.) This question is not fair!

8.) (3-points) The following energy band diagram indicates the material is:
- a.) Non-degenerate n-type
- [✓ b.)] Degenerate n-type
- c.) intrinsic
- d.) Non-degenerate p-type
- e.) Degenerate p-type



9.) (3-points) For the electron and hole shown in the following band diagram circle all that are true.
- a.) The electron will move to the right
- [✓ b.)] The hole will move to the right
- [✓ c.)] The electron will move to the left
- d.) The hole will move to the left
- e.) There is no current flow in this device
- f.) The device has to be in equilibrium



10.) (3-points) In steady state:

- a.) The number of electrons equals the number of holes.
- b.) The number of electrons is greater than holes in a n-type material
- c.) No net current can flow
- [✓ d.)] The time rate of change of the system is zero
- e.) The minority carrier concentration can not change from position to position.

Second 25% Short Answer ("Plug and Chug"):

For the following problems (11-12) use the following material parameters:

n_i=5e12 cm^{-3} N_D=8e12 cm^{-3} donors N_A=6e12 cm^{-3} acceptors.

Electron mobility, μ_n = 1600 cm^2/Vsec Hole mobility , μ_p = 480 cm^2/Vsec

Temperature=27 degrees C

11.)(10-points) Assuming total ionization, what is the electron and hole concentrations and is the material p or n-type?

$$n = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2}$$

$$= \frac{8e12 - 6e12}{2} + \sqrt{\left(\frac{8e12-6e12}{2}\right)^2 + (5e12)^2}$$

$$\boxed{n = 6.1e12\ cm^{-3}}$$

$$\boxed{p = \frac{n_i^2}{n} = \frac{(5e12)^2}{6.1e12} = 4.1\,e12\ cm^{-3}}$$

$$n > p \Rightarrow \boxed{n\text{-type}}$$

12.) (15-points) A block of material is 5 mm long and has a cross-sectional area of 0.1 mm^2. If 9 Volts is placed across its length, what is the electron current density, and hole current density in the material when **a.)** it is in the dark and **b.)** when light generates an additional 1e11 cm^{-3} electrons? You may assume the mobility is fixed in all cases.

$$\mathcal{E} = \frac{9V}{0.5cm} = 18\ V/cm$$

$$J_p = \sigma_p \mathcal{E} \quad \sigma_p = (q\,\mu_p p)\ , \quad J_n = \sigma_n \mathcal{E}\ ,\ \sigma_n = (q\,\mu_n n)$$

a.) $J_p = (18\ V/cm)(1.6e-19)(480)\,4.1e12$, $J_n = 18(1.6e-19)(1600)(6.1e12)$

$$\boxed{J_p = 5.67\ mA/cm^2} \qquad \boxed{J_n = 28.1\ mA/cm^2}$$

b.) $J_p = 18(1.6e-19)(480)(4.1e12 + 1e11)$ $\qquad J_n = 18(1.6e-19)(1600)(6.1e12 + 1e11)$

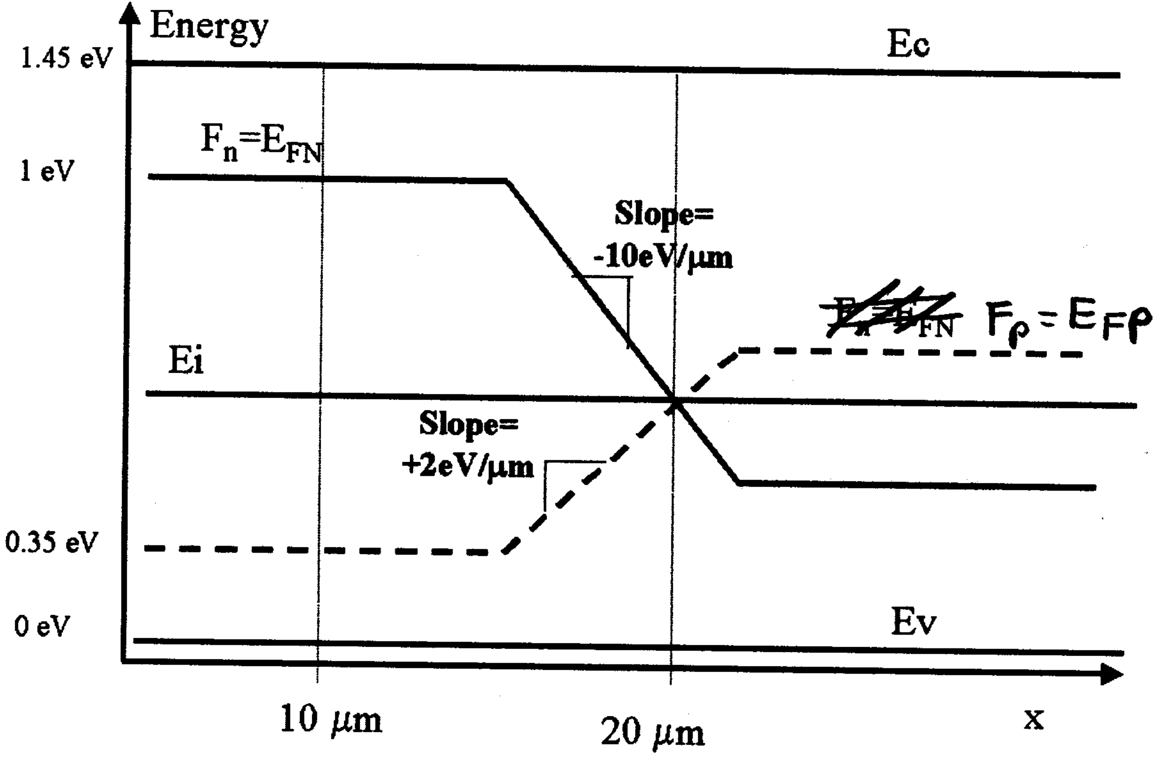
$$\boxed{J_p = 5.8\ mA/cm^2} \qquad \boxed{J_n = 28.6\ mA/cm^2}$$

Third 25% Problems (3^{rd} 25%)

13.) (25-points total)

A semiconductor at room temperature (27 degrees C) has the following parameters:
Hole Diffusion coefficient, D_p=11.86 cm^2/Sec
Electron Diffusion coefficient, D_n=33.625 cm^2/Sec
Substrate intrinsic concentration, n_i=1e10 cm^{-3}
Also, these conversion factors may help: Amp=Coulomb/Sec, and Coulomb=Joule/Volt

The sample is in non-equilibrium with the following energy band relationships



a.) (7 points) What are the electron and hole current densities, J_n and J_p at position x=10 μm? Make sure you support your answer with equations or a discussion.

$$J_p = \mu_p p \nabla F_p \;,\; \nabla F_p = 0 \Rightarrow \boxed{J_p = 0\ \text{A/cm}^2}$$

$$J_n = \mu_n n \nabla F_n \;,\; \nabla F_n = 0 \Rightarrow \boxed{J_n = 0\ \text{A/cm}^2}$$

b.) (18 points) What are the electron and hole current densities, J_n and J_p at position x=20 μm? Make sure you support your answer with equations or a discussion.

@ 20um $F_n = F_p = E_i$

Thus, $p = n_i e^{(E_i - F_p)/kT} = n_i e^0 = n_i = 1e10\ cm^{-3}$

$n = n_i e^{(F_n - E_i)/kT} = n_i e^0 = n_i = 1e10\ cm^{-3}$

see next page

Extra work can be done here, but clearly indicate which problem you are solving.

$$\mu_p = \frac{q\, D_p}{kT} = \frac{11.86\ cm^2/sec}{0.0259\,V} = 458\ cm^2/V\text{-}sec$$

$$\mu_n = \cdots = \frac{33.625\,cm^2/sec}{0.0259} = 1298\ cm^2/V\text{-}sec$$

$$\nabla F_p = 2eV/\mu m \quad or \quad \frac{2eV\ (1.6e\text{-}19\ \frac{J}{eV})}{1e\text{-}4\,cm}$$

$$= 3.2\,e\text{-}15\ J/cm$$

$$\therefore\ J_p = (\mu_p)\,p\,(\nabla F_p)$$

$$= \left(\frac{458\,cm^2}{V\text{-}sec}\right)\left(1e10\ cm^{-3}\right)\left(3.2e\text{-}15\,\frac{J}{cm}\right)$$

$$= 0.0146\ \left(\frac{J}{V}\right)\left(\frac{1}{sec}\right)\left(\frac{1}{cm^2}\right)$$

$$J_p = 0.0146\ \frac{coul}{sec}\,\frac{1}{cm^2}$$

$$\boxed{J_p = 0.0146\ A/cm^2}$$

Similarly,

$$J_n = (\mu_n)\,n\,(\nabla F_n)$$

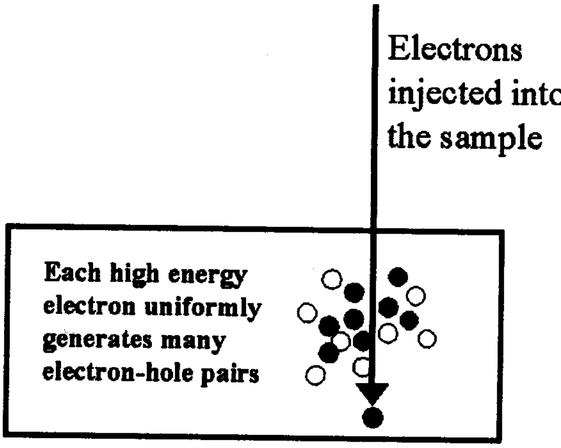
$$= (1298)(1e10)\left(-\frac{10eV\ (1.6e\text{-}19\ J/eV)}{1e\text{-}4\,cm}\right)$$

$$\boxed{J_n = -0.208\ A/cm^2}$$

Pulling all the concepts together for a useful purpose: (4th 25%)

14.) (25-points)

A thin SiC semiconductor material of thickness 2 um with energy bandgap 3.2 eV is placed in a vacuum chamber and is bombarded with electrons at a rate equivalent to 1 nanoampere of current per square cm (1 nA/cm^2). The extra electrons are absorbed in the material. Due to the high energy of the electrons used for this bombardment, valence electrons are "knocked" into the conduction band in a process analogous to light generation. Each high-energy bombarding electron has 10,000 eV of energy and thus, can generate multiple electron-hole pairs before all of its energy is dissipated. The wafer is p-type and is uniformly doped with 10^{18} cm^{-3} acceptors. The electron beam was turned on January 1, 2003 and is completely absorbed throughout the material uniformly. At 12 noon on February 6th 2003, a Clemson engineer knocked out power and the electron beam instantly shut off. Determine the excess electron concentration in the SiC for all positions AFTER the beam is turned off. Assume a minority carrier lifetime of 0.1 μS and a mobility of 30 cm^2/V-sec.



NOTE: This is a real world example from a system known as an Electron Beam Induced Current (EBIC) system.

Given: $0 = D_n \frac{d^2\Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n}$ General Solution is: $\Delta n_p(x) = Ae^{-x/L_n} + Be^{+x/L_n}$

Given: $0 = D_n \frac{d^2\Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n} + G_L$ General Solution is: $\Delta n_p(x) = Ae^{-x/L_n} + Be^{+x/L_n} + G_L\tau_n$

Given: $0 = D_n \frac{d^2\Delta n_p}{dx^2}$ General Solution is: $\Delta n_p(x) = A + Bx$

Given: $0 = D_n \frac{d^2\Delta n_p}{dx^2} + G_L$ General Solution is: $\Delta n_p(x) = Ax^2 + Bx + C$

Given: $0 = D_n \frac{d^2\Delta n_p}{dx^2} + G_{LO} f(x)$ General Solution is: $\Delta n_p(x) = \left[\frac{G_{LO}}{D_N}\iint f(x)dx\right] + Bx + C$

Given: $\frac{d\Delta n_p}{dt} = -\frac{\Delta n_p}{\tau_n}$ General Solution is: $\Delta n_p(t) = \Delta n_p(t=0)e^{-t/\tau_n}$

Given: $0 = -\frac{\Delta n_p}{\tau_n} + G_L$ General Solution is: $\Delta n_p = G_L\tau_n$

Pulling all the concepts together for a useful purpose: (4th 25%)

14.) (25-points)

A thin SiC semiconductor material of thickness 2 um with energy bandgap 3.2 eV is placed in a vacuum chamber and is bombarded with electrons at a rate equivalent to 1 nanoampere of current per square cm (1 nA/cm^2). The extra electrons are absorbed in the material. Due to the high energy of the electrons used for this bombardment, valence electrons are "knocked" into the conduction band in a process analogous to light generation. Each high-energy bombarding electron has 10,000 eV of energy and thus, can generate multiple electron-hole pairs before all of its energy is dissipated. The wafer is p-type and is uniformly doped with 10^{18} cm^{-3} acceptors. The electron beam was turned on January 1, 2003 and is completely absorbed throughout the material uniformly. At 12 noon on February 6th 2003, a Clemson engineer knocked out power and the electron beam instantly shut off. Determine the excess electron concentration in the SiC for all positions AFTER the beam is turned off. Assume a minority carrier lifetime of 0.1 µS and a mobility of 30 cm^2/V-sec.

$\frac{\partial \Delta n_p}{\partial t} = 0$ for Case A:

$\frac{\partial^2 \Delta n_p}{\partial x^2} = 0$

$\to \tau_n$

$\frac{\partial \Delta n_p}{\partial t} \neq 0$ for case B:

NOTE: This is a real world example from a system known as an Electron Beam Induced Current (EBIC) system.

Given: $0 = D_n \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n}$ General Solution is: $\Delta n_p(x) = Ae^{-x/L_n} + Be^{+x/L_n}$

Given: $0 = D_n \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n} + G_L$ General Solution is: $\Delta n_p(x) = Ae^{-x/L_n} + Be^{+x/L_n} + G_L \tau_n$

Given: $0 = D_n \frac{d^2 \Delta n_p}{dx^2}$ General Solution is: $\Delta n_p(x) = A + Bx$

Given: $0 = D_n \frac{d^2 \Delta n_p}{dx^2} + G_L$ General Solution is: $\Delta n_p(x) = Ax^2 + Bx + C$

Given: $0 = D_n \frac{d^2 \Delta n_p}{dx^2} + G_{LO} f(x)$ General Solution is: $\Delta n_p(x) = \left[\frac{G_{LO}}{D_N} \iint f(x) dx \right] + Bx + C$

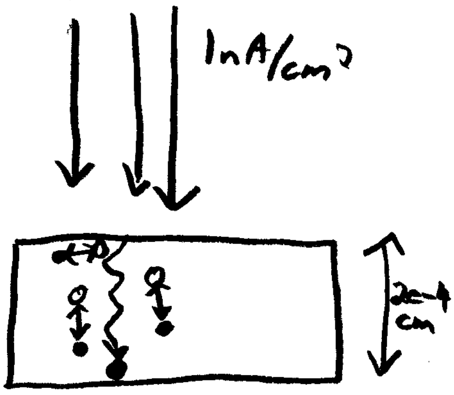
Given: $\frac{d\Delta n_p}{dt} = -\frac{\Delta n_p}{\tau_n}$ General Solution is: $\Delta n_p(t) = \Delta n_p(t=0) e^{-t/\tau_n}$

Case A: Given: $0 = -\frac{\Delta n_p}{\tau_n} + G_L$ General Solution is: $\Delta n_p = G_L \tau_n$

Before Beam Shutsoff:

$$\cancelto{0}{\frac{\partial \Delta n_p}{\partial t}} = D_n \cancelto{0}{\frac{\partial^2 \Delta n_p}{\partial x^2}} - \frac{\Delta n_p}{\tau_n} + G_L$$

$$\Delta n_p = G_L \tau_n$$



$G_L = ?$

= electrons bombarding + Generated electrons

see next page

Extra work can be done here, but clearly indicate which problem you are solving.

1 [illegible]A/cm² is absorbed into 2e-4cm thick sample.

$$\frac{\#\,\text{electrons}}{\text{cm}^2/\text{sec}} = \frac{1\ \text{nA/cm}^2}{1.6\text{e-}19\ \text{coul./electron}} = \frac{1\text{e-}9\ \frac{\text{coul}}{\text{sec}}/\text{cm}^2}{1.6\text{e-}19\ \text{coul/elec}}$$

$$= 6.25\text{e}9\ \frac{\text{electrons}}{\text{cm}^2}/\text{sec}$$

$$\frac{\#\,\text{electrons}}{\text{cm}^3}/\text{sec} = \frac{6.25\text{e}9}{2\text{e-}4\text{cm}} = \underline{\underline{3.125\text{e}13\ \text{cm}^{-3}/\text{sec}}}$$

Each electron has 10,000 eV but only needs 3.2 eV to generate a hole-electron pair. Thus, each electron generates an additional $10{,}000/3.2 = 3125$ electron-hole pairs.

$$\therefore G_L = 3.125\text{e}13\ \frac{\text{cm}^{-3}}{\text{sec}} + 3.125\text{e}13\,(3125)\,\frac{\text{cm}^{-3}}{\text{sec}}$$

$$G_L = 9.77\text{e}16\ \text{cm}^{-3}/\text{sec}$$

$$\Delta n_p = G_L \tau_n = (9.77\text{e}16\ \text{cm}^{-3}/\text{sec})(0.1\ \mu\text{s})$$

$$\underline{\Delta n_p = 9.77\text{e}9\ \text{cm}^{-3}}$$

Now shut off the Beam!

Extra work can be done here, but clearly indicate which problem you are solving.

Case B:
Beam off:

$$\frac{\partial \Delta n_p}{\partial t}^{\neq 0} = D_n \cancelto{0}{\frac{\partial^2 \Delta n_p}{\partial x^2}} - \frac{\Delta n_p}{\tau_n} + \cancelto{0}{G_L}$$

General Solution

$$\Delta n_p(t) = \underset{\substack{\uparrow \\ 9.77e9\ cm^{-3}\ \text{from before}}}{\Delta n_p(t=0)}\, e^{-t/\tau_n}$$

$$\boxed{\Delta n_p(t) = 9.77e9 \left(e^{-t/0.1\mu s}\right) cm^{-3}}$$

In a real EBIC System, the beam is not uniformly absorbed making the solution 3-demensional including the diffusion term. Additionally, each electron creats $\frac{1}{3}$ of its energy (i.e $\frac{10000}{3(3.2)}$ in this example) with the remaining energy wasted as heat. See EBIC image on the next page to see a map of "defects" lowering the minority carrier lifetime.

The following is an actual EBIC image of a SiC material containing 2 types of defects that will reduce the minority carrier diffusion length, lowering the current collected in the measurements. The small dots are dislocations (missing rows of atoms) while the large dark image is an inclusion (basically a trashed region of the crystal). The source of the current is from excess electron-hole pairs generated by a high energy electron beam (25 KV in this case) formed in a scanning electron microscope.