

# ECE 3040 Microelectronic Circuits

Exam 1

February 15, 2019

Dr. W. Alan Doolittle

20 minutes

Print your name clearly and largely:

Solutions

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**Instructions:**

**DO NOT REMOVE ANY SHEETS FROM THIS EXAM!** Read all the problems carefully and thoroughly before you begin working. You are allowed to use 1 sheet of notes (1 page front and back) as well as a calculator. There are 100 total points. Observe the point value of each problem and allocate your time accordingly. **SHOW ALL WORK AND CIRCLE YOUR FINAL ANSWER WITH THE PROPER UNITS INDICATED. Write legibly. If I cannot read it, it will be considered a wrong answer.** Do all work on the paper provided. Turn in all scratch paper, even if it did not lead to an answer. **Turn in your notes sheet placed under your exam.** Report any and all ethics violations to the instructor. A periodic table is supplied on the last page. Good luck!

Sign your name on **ONE** of the two following cases:

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I DID NOT observe any ethical violations during this exam:

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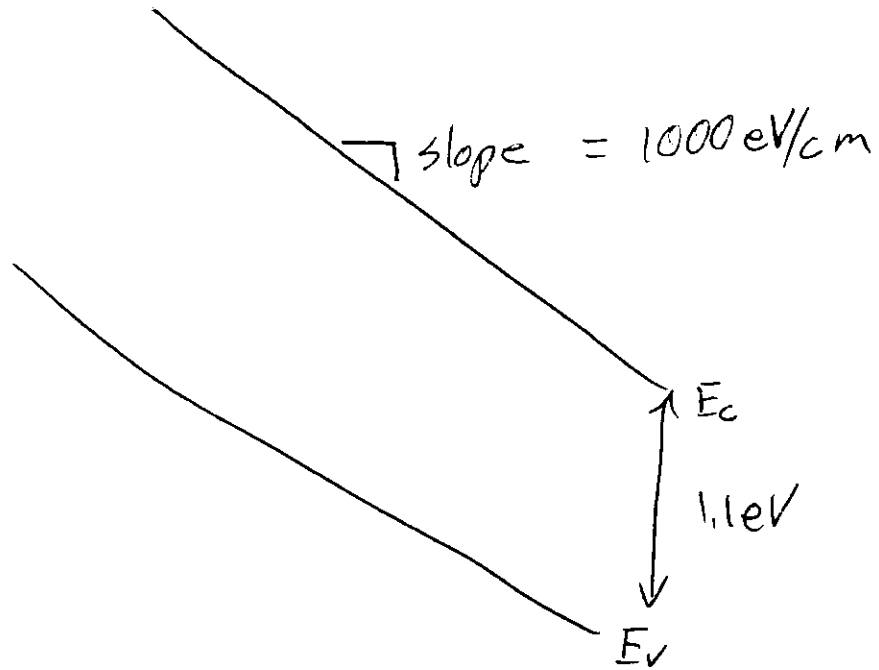
I observed an ethical violation during this exam:

**First 30% Multiple Choice and True/False**  
(Circle the letter of the most correct answer or answers )

- 1.) (3-points) True or False: The energy bandgap can be considered the energy required to rip an electron out of the material into the vacuum where it conducts electricity. X
- 2.) (3-points) True or False: The mobility is the low electric field slope of the drift velocity, in the region where the drift velocity is linearly proportional to the electric field.
- 3.) (3-points) True or False: The product of [the density of states] and [1 -Fermi-distribution function] gives the hole concentration as a function of energy in the valence band.
- 4.) (3-points) True or False: If both electrons and holes are exposed to a built in field, as in a solar cell, the charges both move to the same side of the device accumulating on the anode (p-side). X
- 5.) (3-points) True or False: For a device with a high concentration of defect states or impurity states, the minority carrier lifetime will be very high. X
- 6.) (3-points) True or False: In a degenerately doped semiconductor, more than one hole can occupy a given state.
- 7.) (3-points) True or False: Larger bond strength results in higher energy bandgaps.
- 8.) (3-points) True or False: The Fermi-Dirac integral of order  $\frac{1}{2}$ , the Fermi distribution function and the Boltzmann distribution function are all ways of describing the probability that a state is filled with an electron.
- 9.) (3-points) True or False: Auger recombination is only important at low current density or at low optical injection (low optical power). X X
- 10.) (3-points) True or False: Impact ionization can increase a small current into a large current but is generally noisy current as the multiplication of charge is random and thus, stochastic.

Short Answer (“Plug and Chug”):

- 11.)(4-points) Sketch and label the energy band diagram of Silicon (1.1 eV bandgap) subjected to an electric field of 1000V/cm showing the conduction and valance bands and labeling the diagram with some indication of how one could derive the material is exposed to this particular electric field. Points deducted for lack of neatness, clarity and missing energy labels and numeric values.



For the following problems (11-12) use the following material parameters and assuming total ionization:

For InP:

$$n_i = 1.3 \times 10^7 \text{ cm}^{-3} \quad N_D = 1.8 \times 10^{13} \text{ cm}^{-3} \text{ donors} \quad N_A = 1.6 \times 10^{17} \text{ cm}^{-3} \text{ acceptors}$$

$$m_p^* = 0.6 m_0 \quad m_n^* = 0.08 m_0$$

$$E_G = 1.344 \text{ eV} \quad \text{Electron mobility, } \mu_n = 900 \text{ cm}^2/\text{Vsec} \quad \text{Hole mobility, } \mu_p = 120 \text{ cm}^2/\text{V-sec}$$

$$\text{Temperature} = 27 \text{ degrees C}$$

12.) (5-points) Where is the fermi energy (relative to the valence band which is referenced to zero energy)?

$$p_0 = \frac{N_A - N_D}{2} + \sqrt{\frac{(N_A - N_D)^2}{4} + n_i^2} = 1.6 \times 10^{17} \text{ cm}^{-3} \quad n_0 = \frac{n_i^2}{p_0} = 1.06 \times 10^{-3} \quad E_i = \frac{E_g}{2} + \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right) = 0.711 \text{ eV}$$

Several Approaches

1)  $p_0 = n_i e^{(E_i - E_f)/kT}$   
 $E_f = 0.109 \text{ eV}$

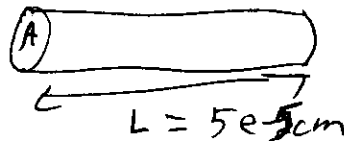
2)  $n_0 = n_i e^{(E_f - E_i)/kT}$   
 $E_f = 0.109 \text{ eV}$

3)  $N_C = 2.51 \times 10^{19} \left(\frac{m_n^*}{m_0}\right)^{3/2} = 5.68 \times 10^{17} \text{ cm}^{-3}$   
 $n_0 = \frac{N_C e^{(E_f - E_i)/kT}}{1} = 1.06 \times 10^{-3}$   
 $E_f = 0.109 \text{ eV}$

4)  $N_V = 2.51 \times 10^{19} \left(\frac{m_p^*}{m_0}\right)^{3/2} = 1.16 \times 10^{19} \text{ cm}^{-3}$   
 $p_0 = N_V e^{(E_f - E_i)/kT} = 1.6 \times 10^{17} \text{ cm}^{-3}$   
 $E_f = 0.109 \text{ eV}$

13.) (8-points) A 25 nm (1 nm = 1e-9 m) diameter x 500 nm long cylindrical semiconductor resistor is made from the semiconductor from problem 12 is biased on two opposing sides (longest dimension) with 0.9 volts. Determine both the electron and hole currents flowing in the device.

$$A = \left(\frac{25 \times 10^{-7} \text{ cm}}{2}\right)^2 \pi = 4.909 \times 10^{-12} \text{ cm}^2$$



$$R_p = \rho_p \frac{L}{A} = 0.326 \Omega\text{-cm}$$

$$\rho_n = \rho_n \frac{L}{A} = 6.57 \times 10^{18} \Omega\text{-cm}$$

wow!

$$R_p = \frac{\rho_p L}{A} = 3.32 \text{ M}\Omega$$

$$R_n = \frac{\rho_n L}{A} = 6.7 \times 10^{25} \Omega$$

$$\frac{V}{R_p} = I_p = 2.7 \times 10^{-7} \text{ A} = 0.27 \mu\text{A}$$

$$I_n = \frac{V}{R_n} = 1.34 \times 10^{-26} \text{ A}$$

**Section 3 (more short answer)**

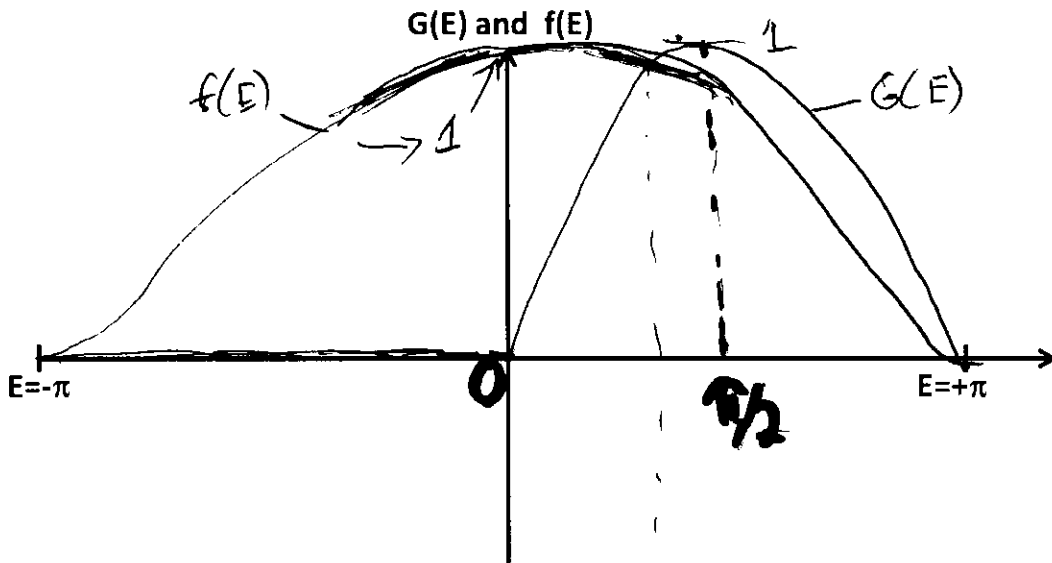
14.) (13-points total) The brilliant and humble Professor Doolittle has found a semiconductor that obeys new "Doolittian Physics". It is found the density of states of this material follows the function:

$$G(E) = \begin{cases} \sin(E) & \text{for } 0 < E < \pi \\ 0 & \text{elsewhere} \end{cases}$$

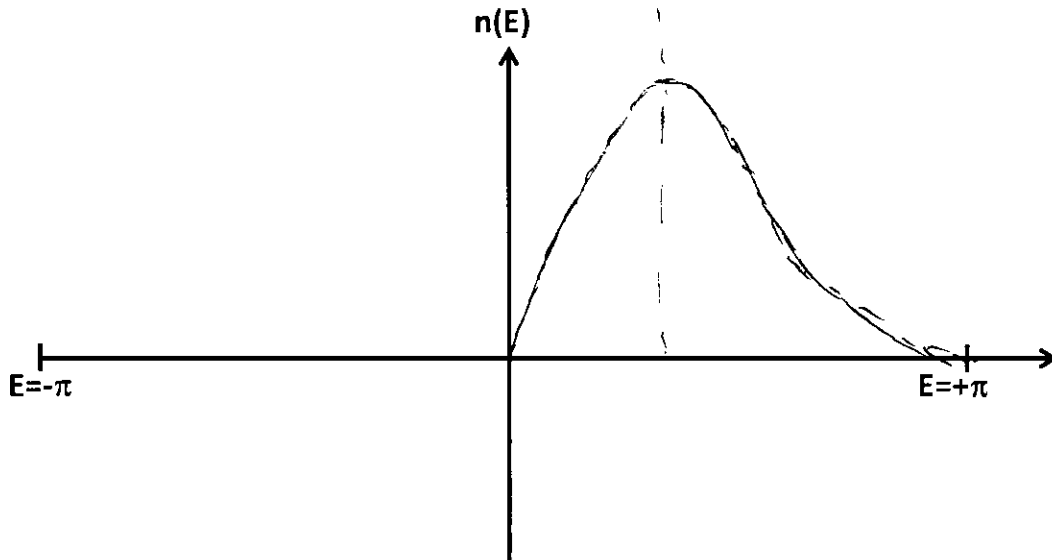
and the fermi distribution function for this new physics is:

$$f(E) = \frac{1}{2} [1 + \cos(E)] \text{ for } -\pi < E < \pi$$

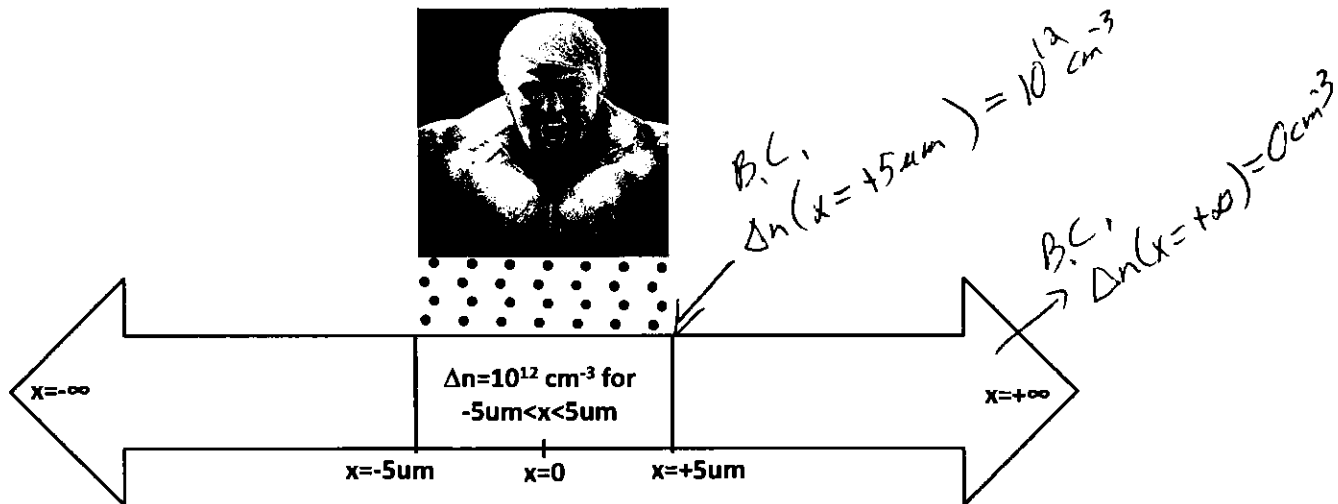
On the same Graph, sketch and label peak magnitudes and peak energies for  $G(E)$  and  $f(E)$ .



Sketch (do not label magnitudes) of the electron concentration versus energy function  $n(E)$ .



Pulling all the concepts together for a useful purpose:



14.) (40-points) : A semi-infinite slab of GaN semiconductor is to be used as part of new nuclear battery known as a Betavoltaic device. The GaN extends from negative infinity to positive infinity as pictured and is to be considered in 1 dimension ( $x$ ). The Incredible Hulk walks into the room in 1994 and radiates extremely high energy nuclear particles that generate electron hole pairs in a process identical to that as if they originated from light ONLY in the region from  $-5$  to  $+5 \mu\text{m}$ . All other regions are considered in the dark. The resultant excess electron concentration from  $+5$  to  $-5 \mu\text{m}$  is uniform and equal to  $10^{12} \text{ cm}^{-3}$ . The semiconductor is doped p-type with an acceptor concentration of  $6 \times 10^{15} \text{ cm}^{-3}$ , has an intrinsic concentration of  $1 \times 10^{-14} \text{ cm}^{-3}$  and has a minority carrier lifetime of 5 nanoseconds. If the semiconductor is held at room temperature (27 degrees C), determine the minority carrier diffusion current density at all positions in the semiconductor ( $+\infty \geq x \geq -\infty \mu\text{m}$ ). Assume a minority carrier mobility of  $1930.5 \text{ cm}^2/\text{Vsec}$ .

Given:  $0 = D_n \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n}$       General Solution is:  $\Delta n_p(x) = Ae^{-x/l_n} + Be^{+x/l_n}$

Given:  $0 = D_n \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n} + G_L$       General Solution is:  $\Delta n_p(x) = Ae^{-x/l_n} + Be^{+x/l_n} + G_L \tau_n$

Given:  $0 = D_n \frac{d^2 \Delta n_p}{dx^2}$       General Solution is:  $\Delta n_p(x) = A + Bx$

Given:  $0 = D_n \frac{d^2 \Delta n_p}{dx^2} + G_L$       General Solution is:  $\Delta n_p(x) = Ax^2 + Bx + C$

Given:  $0 = D_n \frac{d^2 \Delta n_p}{dx^2} + G_{LO} f(x)$       General Solution is:  $\Delta n_p(x) = \left[ -\frac{G_{LO}}{D_n} \iint f(x) dx^2 \right] + Bx + C$

Given:  $\frac{d\Delta n_p}{dt} = -\frac{\Delta n_p}{\tau_n}$       General Solution is:  $\Delta n_p(t) = \Delta n_p(t=0)e^{-t/\tau_n}$

Given:  $0 = -\frac{\Delta n_p}{\tau_n} + G_L$       General Solution is:  $\Delta n_p = G_L \tau_n$

$$D_n = \mu_n \frac{kT}{q}$$

$$= 1930.49 (0.0259) = 50 \text{ cm}^2/\text{sec}$$

$$L_n = \sqrt{D_n \tau} = \sqrt{50 \times 5 \times 10^{-9}} = 5 \mu\text{m}$$

Extra work can be done here, but clearly indicate which problem you are solving.

$x \geq 5 \mu\text{m}$ :

$$\Delta n(x) = A e^{-x/L_n} + B e^{+x/L_n}$$

B.C.  $x = +\infty$

$$\Delta n(x \rightarrow \infty) = A(0) + B \infty$$

$$\implies B = 0$$

B.C.  $x = 5 \mu\text{m}$

$$\Delta n(x = 5 \mu\text{m}) = A e^{-5/5 \mu\text{m}}$$

$$10^{12} = A e^{-1}$$

$$= A 0.3678$$

$$A = 2.718 e^{12} \text{ cm}^{-3}$$

$x \geq 5 \mu\text{m}$ :  $\Delta n(x) = 2.718 e^{12} \text{ cm}^{-3} e^{-x/5 \mu\text{m}} \text{ cm}^{-3}$

by symmetry  $x \leq -5$   $\Delta n(x) = 2.718 e^{12} \text{ cm}^{-3} e^{+x/5 \mu\text{m}} \text{ cm}^{-3}$

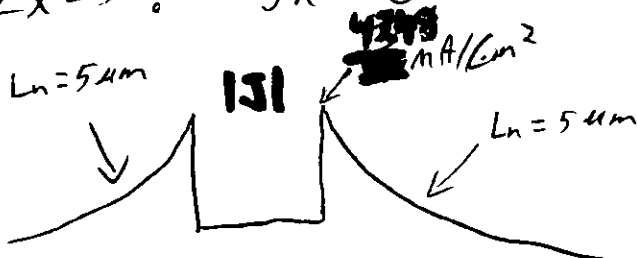
$-5 < x < 5$ :  $\Delta n(x) = 10^{12} \text{ cm}^{-3}$

$$J_n = q D_n \nabla n = (1.6 \times 10^{-19}) (50 \text{ cm}^2/\text{sec}) \left( \frac{2.718 e^{12}}{5 \times 10^{-4} \text{ cm}} e^{-x/5 \mu\text{m}} \right)$$

$x > 5 \mu\text{m}$ :  $J_n = \left[ \begin{matrix} -43.48 \\ \text{mA/cm}^2 \end{matrix} \right] \left[ e^{-x/5 \mu\text{m}} \right]$

$x < -5 \mu\text{m}$ :  $J_n = \left[ \begin{matrix} +43.48 \\ \text{mA/cm}^2 \end{matrix} \right] \left[ e^{+x/5 \mu\text{m}} \right]$

$-5 < x < 5$ :  $J_n = 0$



Extra work can be done here, but clearly indicate which problem you are solving.



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# PERIODIC TABLE OF THE ELEMENTS

Table of Selected Radioactive Isotopes

GROUP IA		Table of Selected Radioactive Isotopes										VIII																																																																																										
1 1.00794 H Hydrogen	2 4.002602 He Helium	3 6.941 Li Lithium	4 9.012182 Be Beryllium	5 10.811 B Boron	6 12.011 C Carbon	7 14.003074 N Nitrogen	8 15.9994 O Oxygen	9 18.998403 F Fluorine	10 20.179 Ne Neon	11 22.989769 Na Sodium	12 24.304 Mg Magnesium	13 26.981538 Al Aluminum	14 28.0855 Si Silicon	15 30.973762 P Phosphorus	16 32.06 S Sulfur	17 35.45 Cl Chlorine	18 39.948 Ar Argon	19 39.0983 K Potassium	20 40.078 Ca Calcium	21 44.955912 Sc Scandium	22 47.867 Ti Titanium	23 50.9415 V Vanadium	24 51.9961 Cr Chromium	25 54.938044 Mn Manganese	26 55.934936 Fe Iron	27 58.933195 Co Cobalt	28 58.933195 Ni Nickel	29 63.546 Cu Copper	30 65.38 Zn Zinc	31 69.723 Ga Gallium	32 72.64 Ge Germanium	33 74.9216 As Arsenic	34 78.96 Se Selenium	35 79.904 Br Bromine	36 83.904 Kr Krypton	37 85.4678 Rb Rubidium	38 87.62 Sr Strontium	39 88.905848 Y Yttrium	40 90.907303 Zr Zirconium	41 91.224 Nb Niobium	42 92.90638 Mo Molybdenum	43 95.94 Tc Technetium	44 101.07 Ru Ruthenium	45 101.07 Rh Rhodium	46 106.42 Pd Palladium	47 107.8682 Ag Silver	48 112.411 Cd Cadmium	49 114.818 In Indium	50 118.710 Sn Tin	51 121.757 Sb Antimony	52 127.4595 Te Tellurium	53 128.905 I Iodine	54 131.29 Xe Xenon	55 132.90545 Cs Cesium	56 137.327 Ba Barium	57 138.90473 La Lanthanum	58 140.90764 Ce Cerium	59 140.90764 Pr Praseodymium	60 144.242 Nd Neodymium	61 145.90951 Pm Promethium	62 150.91961 Sm Samarium	63 151.964 Eu Europium	64 157.25 Gd Gadolinium	65 158.92534 Tb Terbium	66 162.50 Dy Dysprosium	67 164.93032 Ho Holmium	68 167.259 Er Erbium	69 168.93402 Tm Thulium	70 173.045 Yb Ytterbium	71 174.967 Lu Lutetium	72 175.04 Hf Hafnium	73 178.49 Ta Tantalum	74 180.94788 W Tungsten	75 183.84 Re Rhenium	76 186.207 Os Osmium	77 188.906 Ir Iridium	78 194.222 Pt Platinum	79 196.966569 Au Gold	80 200.59 Hg Mercury	81 200.59 Tl Thallium	82 208.980389 Pb Lead	83 208.980389 Bi Bismuth	84 208.980389 Po Polonium	85 208.980389 At Astatine	86 222.017578 Rn Radon	87 223.01851 Fr Francium	88 226.02541 Ra Radium	89 227.02771 Ac Actinium	90 232.03772 Th Thorium	91 231.036888 Pa Protactinium	92 238.02891 U Uranium	93 238.02891 Np Neptunium	94 238.02891 Pu Plutonium	95 238.02891 Am Americium	96 238.02891 Cm Curium	97 238.02891 Bk Berkelium	98 238.02891 Cf Californium	99 238.02891 Es Einsteinium	100 238.02891 Fm Fermium	101 238.02891 Md Mendelevium	102 238.02891 No Nobelium	103 238.02891 Lr Lawrencium

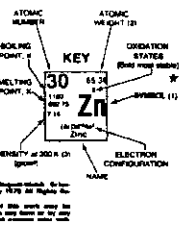


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58 140.90764 Ce Cerium	59 140.90764 Pr Praseodymium	60 144.242 Nd Neodymium	61 145.90951 Pm Promethium	62 150.91961 Sm Samarium	63 151.964 Eu Europium	64 157.25 Gd Gadolinium	65 158.92534 Tb Terbium	66 162.50 Dy Dysprosium	67 164.93032 Ho Holmium	68 167.259 Er Erbium	69 168.93402 Tm Thulium	70 173.045 Yb Ytterbium	71 174.967 Lu Lutetium
90 232.03772 Th Thorium	91 231.036888 Pa Protactinium	92 238.02891 U Uranium	93 238.02891 Np Neptunium	94 238.02891 Pu Plutonium	95 238.02891 Am Americium	96 238.02891 Cm Curium	97 238.02891 Bk Berkelium	98 238.02891 Cf Californium	99 238.02891 Es Einsteinium	100 238.02891 Fm Fermium	101 238.02891 Md Mendelevium	102 238.02891 No Nobelium	103 238.02891 Lr Lawrencium

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