ECE 3040B Microelectronic Circuits

Exam 1

June 7, 2001

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Print your name clearly and largely: ***SOLUTIONS***

Instructions:
Read all the problems carefully and thoroughly before you begin working. You are allowed to use 1 new sheet of notes (1 page front and back), your note sheet from the previous exams as well as a calculator. There are 100 total points in this exam. Observe the point value of each problem and allocate your time accordingly. SHOW ALL WORK AND CIRCLE YOUR FINAL ANSWER WITH THE PROPER UNITS INDICATED. Write legibly. If I cannot read it, it will be considered a wrong answer. Do all work on the paper provided. Turn in all scratch paper, even if it did not lead to an answer. Report any and all ethics violations to the instructor. Good luck!

Problem #	==>	**#1**	**#2**	**#3**	**#4**	**#5**	**#6**	**#7**	**#8**	**#9**	**#10**	**#11**	**#12**	**#13**	**#14**	**#15**	**#16**	**#17**	**Bonus**	**Bonus**
Number of Tests=	45																			
Point Value of problem=		2	2	2	2	2	3	3	3	3	3	5	5	5	5	5	25	25	10	5
Individual Problem Average=		86.7	100.0	93.3	91.1	75.6	97.8	84.4	95.6	82.2	91.1	92.9	96.9	83.1	88.0	65.8	31.3	41.9	7.6	30.2
Exam Average=	64.5																			
Exam Standard Deviation=	15.6																			
Exam Max=	100.0																			
Exam Min=	34.0																			

First 25% Multiple Choice and True/False (Select the most correct answer)

1.) (2-points) True or [✓ False:] Drift current results from movement of electrons and holes from areas of high concentration to areas of low concentration.

2.) (2-points) [✓ True] or False: Adding acceptors to a semiconductor results in more holes than electrons in the material.

3.) (2-points) True or [✓ False:] If Carbon (Group 4 element) is used to dope InP (In is group 3, P is group 5), a p-type semiconductor will <u>always</u> result.

4.) (2-points) [✓ True] or False: The energy band gap in a semiconductor is the energy required to free an electron that normally bonds atoms together, allowing the electron to move through the crystal.

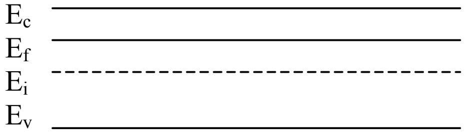
5.) (2-points) [✓ True] or False: When the fermi-energy is far above an allowed state, the state is probably occupied by an electron.

Select the **<u>best</u>** answer for 6-10:

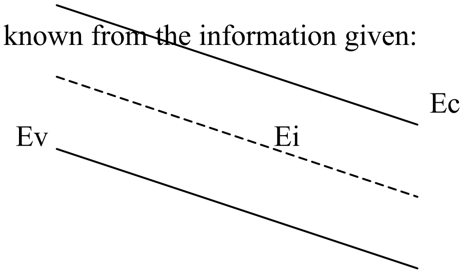
6.) (3-points) Given Si and Ge are from group 4, In and Ga are from group 3 and P is from group 5, which of the following semiconductors is a binary compound semiconductor?
   - a.) Si
   - b.) Ge
   - c.) $In_{0.47}Ga_{0.53}P$
   - [✓ d.)] InP

7.) (3-points) The electron effective mass…
   - [✓ a.)] …is needed to account for the interaction of the electron with the periodic potentials in the crystal.
   - b.) …is smaller than the mass of an electron in vacuum because the electron is moving at close to the speed of light.
   - c.) …is always larger in compound semiconductor.
   - d.) …is equal to the hole effective mass since they have equal but opposite charge.

8.) (3-points) The following energy band diagram indicates the material is:
   - a.) p-type
   - [✓ b.)] n-type
   - c.) intrinsic
   - d.) s-type



9.) (3-points) For to the following band diagram, what is known from the information given:
   - a.) The device is leaning on it's side.
   - [✓ b.)] There is a non-zero electric field in this material
   - c.) There is no current flow in this device
   - d.) There is no electric field in this material.



10.) (3-points) A plane intersecting the coordinate axes at x=3a, y=3a and z=3a, where a is the lattice constant has which of the following Miller indexes:
   - a.) (100)
   - [✓ b.)] (111)
   - c.) (666)
   - d.) (333)
   - e.) Forget it, I will just quit school and go sell T-shirts at the beach.

Second 25% Short Answer ("Plug and Chug"):

11.) (5-points) A semiconductor with an intrinsic concentration, n_i=1e14 cm^{-3}, is doped with 2e14 cm^{-3} donors and 1e11 cm^{-3} acceptors. Assuming total ionization, what is the electron and hole concentrations?

$$n = \left(\frac{N_D - N_A}{2}\right) + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2}$$

$$n = \frac{2e14 - 1e11}{2} + \sqrt{\left(\frac{2e14 - 1e11}{2}\right)^2 + (1e14)^2}$$

$$\boxed{n = 2.41\ e\ 14\ cm^{-3}}$$

$$\boxed{p = \frac{n_i^2}{n} = \frac{(1e14)^2}{2.41e14} = 4.14\ e13\ cm^{-3}}$$

12.) (5-points) For the semiconductor in question 11, if the effective density of states in the conduction and valence band (Nc and Nv) is Nc=1e19 and Nv=2e19 cm-3, what is the bandgap energy of the material at room temperature?

$$n_i = \sqrt{N_c N_v}\, e^{-E_G/2kT}$$

$$1e14\ cm^{-3} = \sqrt{(1e19)(2e19)}\ e^{-E_g/[2(0.0259)]}$$

$$\boxed{E_g = 0.614\ eV}$$

13.) (5-points) For the material in questions 11 and 12, what is E_f-E_v (where E_f is the fermi energy and E_v is the top of the valence band)?

$$p = N_v\, e^{(E_v - E_f)/kT}$$

$$4.14\ e13\ cm^{-3} = 2e19\, e^{-(E_f - E_v)/0.0259}$$

$$\boxed{E_f - E_v = 0.339 eV}$$

14.) (5-points) For the material in questions 11-13, the electron mobility is 800 $cm^2/Vsec$ and the hole mobility is 200 $cm^2/Vsec$. What length of material is needed to make a resistor with resistance 1000 ohms using a cylinder with cross-sectional area 0.001 cm^2.

$R = \frac{\rho L}{A}$ and $\rho = \frac{1}{q(\mu_n n + \mu_p p)}$

$$\rho = \frac{1}{1.6e\text{-}19\,(800\,(2.41e14) + 200(4.14e13))}$$

$$\rho = 31.08 \ \Omega\text{-cm}$$

$$R = \frac{31.08\ L}{0.001} = 1000\ \Omega$$

$\boxed{L = 322\ \mu m}$ (or $0.032\ cm$)

15.) (5-points) A semiconductor has 5e19 cm^{-3} very deep acceptors (large binding energy) which are only partially ionized at room temperature and 4e17 cm^{-3} shallow donors (small binding energy). This results in 5e17 cm^{-3} holes, 4e17 cm^{-3} electrons and all the donors are ionized. How many unionized acceptors are present? (Hint: do not make this problem harder than it is)

All that is needed is the concept of Charge Neutrality!

$$p - N_A^- + N_d^+ - n = 0$$

$$5e17 - N_A^- + 4e17 - 4e17 = 0$$

$$N_A^- = 5e17\ cm^{-3}$$

$$\Downarrow$$

Unionized Acceptors, $N_A^0 = N_A - N_A^-$

$$N_A^0 = 5e19 - 5e17$$

$$\boxed{N_A^0 = 4.95e19\ cm^{-3}}$$

Third 25% Problems (3rd 25%)
16.) (25-points)
A semiconductor has the following parameters:
Mobility, μ_p=200 cm^2/VSec
Substrate relative Dielectric Constant, $\varepsilon_{r\text{-semiconductor}}=K_S=11.7$
Dielectric Constant of free space, ε_O =8.854e-14 F/cm
Substrate intrinsic concentration, n_i=1e10 cm^{-3}
Substrate Doping, $N_A(x)=1e15e^{4.6\sin(2\pi x/100um)}$ cm^{-3}

Plot and label (label the maximum and minimum values) the electric field from x=0 to x=100 um for this material in equilibrium.

Two ways to solve this.

Method 1: Almost exactly like our homework problem!

$$E_f - E_i = -kT \ln\left(\frac{N_A(x)}{n_i}\right)$$

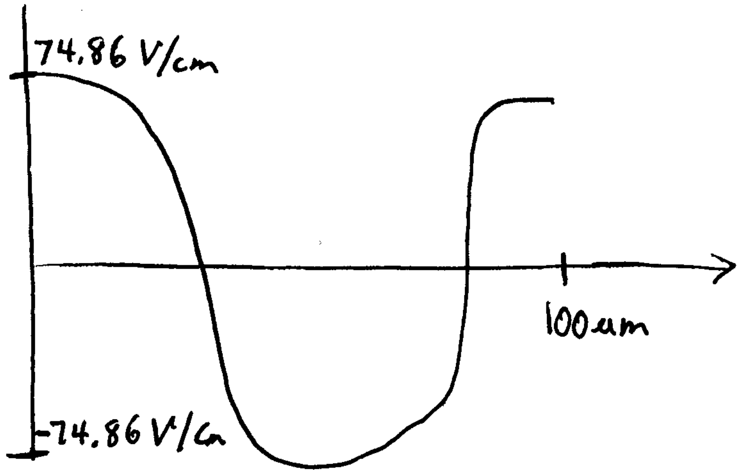
$$= -kT \ln[N_A(x)] + kT \ln(n_i)$$

$$= -kT\left[\ln(1e15) + \ln\left(e^{4.6\sin(2\pi x/100um)}\right)\right] + kT\ln(n_i)$$

$$E_i = E_f + kT\left[\ln(1e15) + 4.6\sin(2\pi x/100um)\right] - kT\ln(n_i)$$

$$\mathcal{E} = \frac{1}{q}\frac{dE_i}{dx} = \frac{kT}{q}\left(\frac{2\pi}{100um}\right) 4.6 \cos\left(\frac{2\pi x}{100um}\right)$$

$$\mathcal{E} = 75.86 \cos\left(\frac{2\pi x}{100um}\right)$$



Extra work can be done here, but clearly indicate with problem you are solving.

Method 2: In equilibrium, $J_p = J_n = 0$.

$$\Rightarrow J_p = \underbrace{q\mu_p p \mathcal{E}}_{\text{Drift}} - \underbrace{q D_p \nabla p}_{\text{Diffusion}} = 0$$

$$\mathcal{E} = \frac{q D_p \nabla p}{q \mu_p p} = \underbrace{\frac{kT}{q}}\,\frac{\nabla p}{p}$$

Einstein Relationship: $\frac{kT}{q} = \frac{D_p}{\mu_p}$

$$\mathcal{E} = \frac{kT}{q}\left[\frac{1e15\, e^{4.6\sin(2\pi x/100um)}\left[4.6\left(\frac{2\pi}{100um}\right)\cos\frac{2\pi x}{100um}\right]}{1e15\, e^{4.6\sin(2\pi x/100um)}}\right]$$

$$\boxed{\mathcal{E} = \left(\frac{kT}{q}\right) 4.6\left(\frac{2\pi}{100um}\right)\cos\left(\frac{2\pi x}{100um}\right)}$$

Same plot as before...

Also, significant points given for a well reasoned qualitative (no #s) answer arrived at by:

$N_A(x)$ ⟹ Equilibrium Band diagram ⟹ electrostatic potential ⟹ Electric Field

Pulling all the concepts together for a useful purpose: (4th 25%)

17.) (25-points)

Steady State

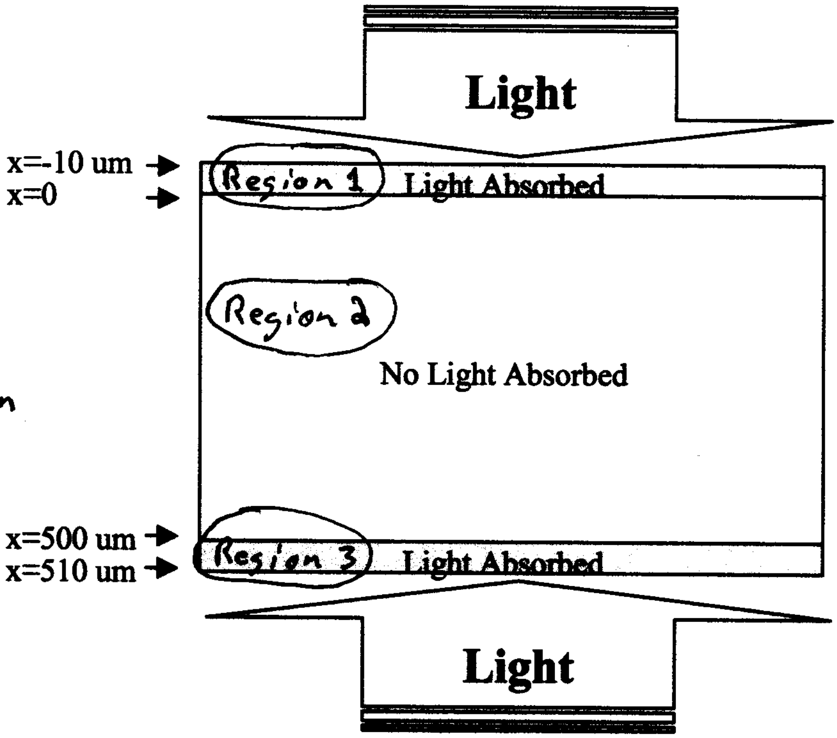
$\frac{\partial \Delta n_p}{\partial t}=0$

Regions 1+3

$\frac{d^2\Delta n_p}{dx^2}=0$

Light from two identical light sources is absorbed on BOTH sides of a silicon wafer of thickness 520 um (the wafer is similar to that passed around in class). The wafer is p-type and is uniformly doped with 10^{17} cm^{-3} acceptors. The light has been on since the dinosaurs walked the earth (a very long time!) and can be approximated as being absorbed uniformly within a 10 um region near either surface (i.e. no light penetrates past 10 um into the wafer). The diffusion coefficient for electrons is 2.5 cm^2/Sec. $= D_n$ If the excess HOLE generation rate is 10^{17} cm^{-3}/sec and the minority carrier lifetime is 1 millisecond (1e-3 seconds); $=\tau_n$

$G_L|_{holes} = G_L|_{electrons} = 10^{17}$ cm^{-3}/sec



What is the excess electron concentration for all positions in the wafer. Hint: break the problem up into three regions (two of which have identical solutions) and use the x axis pictured above. Your answer should be a numeric expression with x being the only independent variable.

(Bonus-10 points) The excess electron concentration changes with position in the wafer. Thus, an electron current density must result. What is the magnitude and direction of this current density at x=125 um from the top surface?

(Bonus-5 points) Explain why no net current would flow.

Given: $0=D_n\frac{d^2\Delta n_p}{dx^2}-\frac{\Delta n_p}{\tau_n}$ General Solution is: $\Delta n_p(x)=Ae^{-x/L_n}+Be^{+x/L_n}$

Given: $0=D_n\frac{d^2\Delta n_p}{dx^2}-\frac{\Delta n_p}{\tau_n}+G_L$ General Solution is: $\Delta n_p(x)=Ae^{-x/L_n}+Be^{+x/L_n}+G_L\tau_n$

Given: $0=D_n\frac{d^2\Delta n_p}{dx^2}$ General Solution is: $\Delta n_p(x)=A+Bx$

Given: $0=D_n\frac{d^2\Delta n_p}{dx^2}+G_L$ General Solution is: $\Delta n_p(x)=Ax^2+Bx+C$

Given: $\frac{d\Delta n_p}{dt}=-\frac{\Delta n_p}{\tau_n}$ General Solution is: $\Delta n_p(t)=\Delta n_p(t=0)e^{-t/\tau_n}$

Given: $0=-\frac{\Delta n_p}{\tau_n}+G_L$ General Solution is: $\Delta n_p=G_L\tau_n$

Extra work can be done here, but clearly indicate with problem you are solving.

$$\frac{\partial \Delta n_p}{\partial t} = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L$$

For $-10\,\mu m < x < 0$ and $500 < x < 510\,\mu m$:

$$0 = -\frac{\Delta n_p}{\tau_n} + G_L$$

$$\Delta n_p = G_L \tau_n$$

$$= \frac{10^{17}\,cm^{-3}}{sec}(1e{-3}\,sec)$$

$$(*)\ \boxed{\Delta n_p = 1e14\ cm^{-3}}$$

For $0 < x < 500$:

$$0 = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n}$$

$$\Downarrow$$

$$\Delta n_p(x) = A e^{-x/L_n} + B e^{+x/L_n}$$

where $L_n = \sqrt{D_n \tau_n}$

$$= \sqrt{(2.5\,cm^2/sec)(1e{-3}\,sec)}$$

$$= 0.05\,cm = 500\,\mu m$$

Boundary Conditions:

$$\Delta n_p(x=0) = A + B = 1e14\ cm^{-3}\ (\text{from }(*))$$

$$B = 1e14 - A$$

$$\Delta n_p(x = 500\,\mu m) = A e^{-500/500} + B e^{+500/500} = 1e14$$

$$= A e^{-1} + B e^{1}$$

$$= A e^{-1} + (1e14 - A) e^{1}$$

Extra work can be done here, but clearly indicate with problem you are solving.

$$A\left[e^{-1} - e^{1}\right] = 1e14 - 1e14\,e^{1}$$

$$A = \frac{-1.72e14}{2.35} = 7.31e13\ cm^{-3}$$

$$B = 2.69e13\ cm^{-3}$$

$$\therefore\ \boxed{\Delta n_p(x) = \begin{cases} 1e14\ cm^{-3} \\ 7.31e13\, e^{-x/500\mu m} + 2.69e13\, e^{x/500\mu m} \\ 1e14\ cm^{-3} \end{cases}}$$

Bonus #1:

$$J_n = \overset{\text{0 due to no electric field}}{\cancel{q \mu_n n_p \mathcal{E}}} + q D_n \frac{dn_p}{dx} \rightarrow n_p = n_{op} + \Delta n_p$$

$$\frac{dn_p}{dx} = \overset{0}{\cancel{\frac{dn_{op}}{dx}}} + \frac{d\Delta n_p}{dx}$$

$$\Rightarrow J_n = +q D_n \frac{d\Delta n_p}{dx}$$

$$= +q D_n \left[\frac{-7.31e13\,cm^{-3}}{0.05\,cm} e^{-x/500\mu m} + \frac{2.69e13\,cm^{-3}}{0.05} e^{x/500\mu m} \right]$$

$$J_n(x) = -0.000585\, e^{-x/500\mu m} + 0.000215\, e^{x/500\mu m}$$

$$\boxed{J_n(x = 125\,\mu m) = -180\ \mu A/cm^2}$$

Direction is toward the upper surface

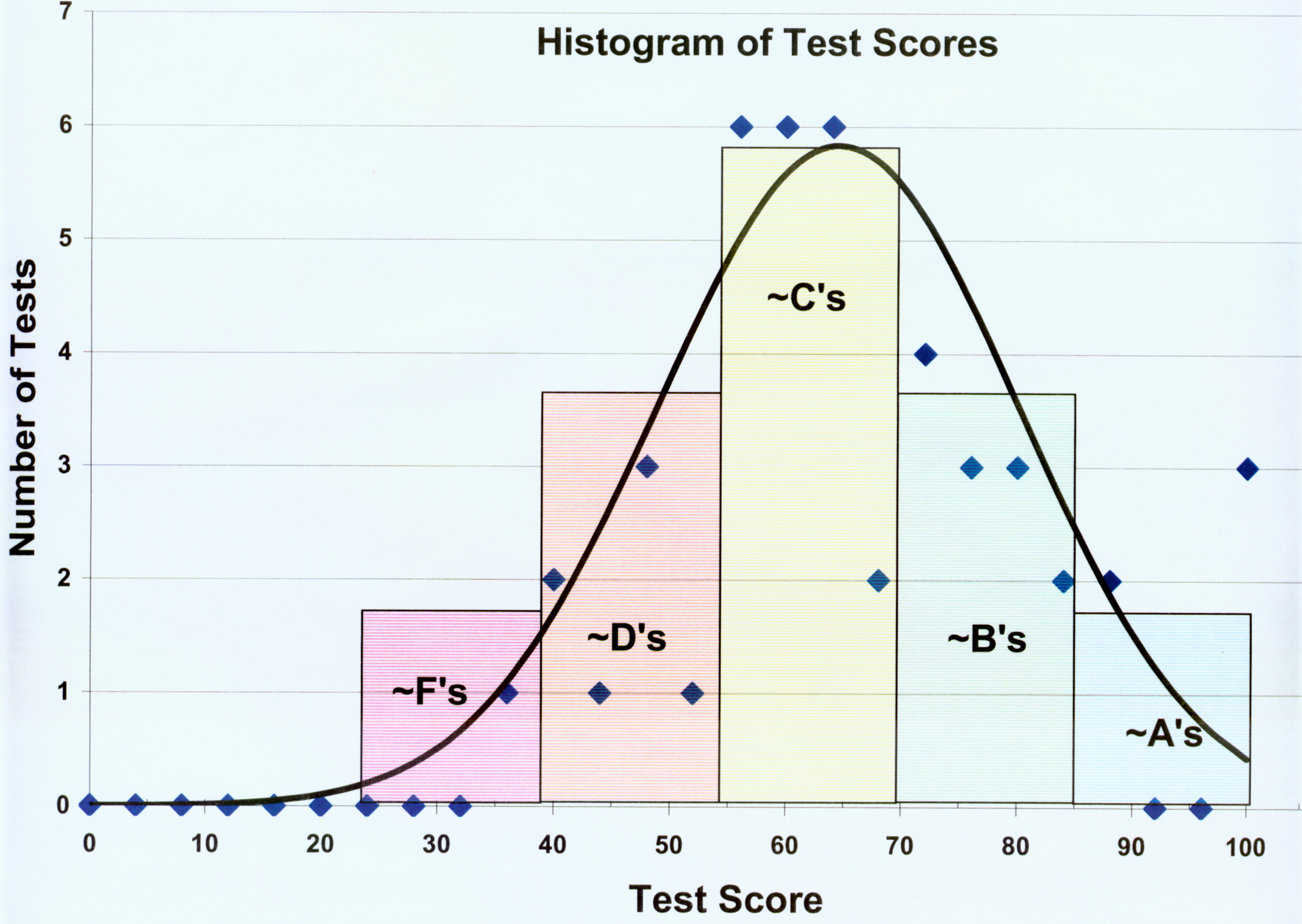
Extra work can be done here, but clearly indicate with problem you are solving.

Bonus #2: The only reason why a net current is not flowing is that the light is symetrically applied to the wafer. Thus, e^- current flows from the center toward the edges and hole current flows from the edges toward the center,

Note: Unless $\mu_p = \mu_n$, ~~the net~~

$$J_n(x=x') \neq J_p(x=x').$$

(If $\mu_p = \mu_n$ $J_n = J_p$).


Histogram of Test Scores
Number of Tests
Test Score
0
1
2
3
4
5
6
7
0
10
20
30
40
50
60
70
80
90
100
~F's
~D's
~C's
~B's
~A's