ECE 3040
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Homework 5

1) Purpose: Understanding the common-emitter amplifier.
In the circuit below, assume the npn BJT is operating in forward-active mode. Let β = 130, V_A = 60 V, and V_{BE} = 0.7 V. Assume the capacitors have negligible impedance at the frequency of the ac signal.

a. What are the purposes of capacitors C_1, C_2, and C_3 in this circuit?

Capacitors C_1 and C_3 are coupling capacitors. Their purpose is to allow ac signals to enter/exit the circuit at particular nodes while leaving DC signals unchanged. Capacitor C_2 is a bypass capacitor. Its purpose is to rout ac signals around the emitter resistor, effectively allowing the emitter to act as an ac ground.

b. Determine all DC terminal voltages and currents as well as the small-signal voltage gain A_v of the amplifier circuit if R_E = 12 kΩ.

Step 1: Obtain the DC equivalent circuit by treating capacitors as open circuits and Thevenizing the base input.

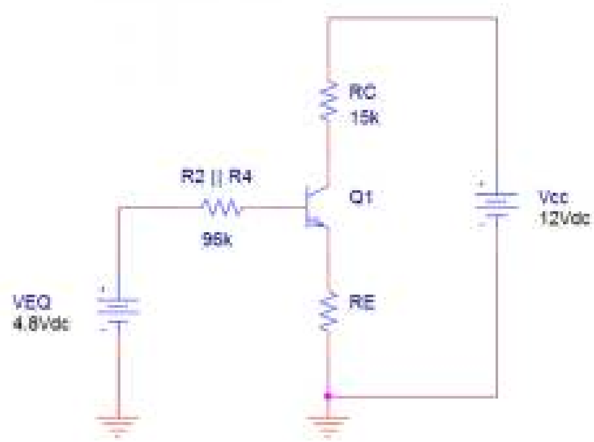

Figure 1S: The DC equivalent circuit.

Step 2: Use KVL and the assumption that the transistor is biased in forward-active mode to determine the Q-point currents and voltages.

Relevant transistor equations:

$$I_C = \beta I_B$$
$$I_E = (\beta + 1) I_B$$

Following the left side of the circuit:

$$V_{EQ} - (R_2||R_4) I_B - V_{BE} - R_E I_E = 0$$

$$V_{EQ} - (R_2||R_4)I_B - V_{BE} - R_E(\beta+1)I_B = 0$$

$$I_B = \frac{V_{EQ} - V_{BE}}{(R_2||R_4) + R_E(\beta+1)}$$

$$= \frac{4.8\text{V} - 0.7\text{V}}{96\ \text{k}\Omega + (12\ \text{k}\Omega)131}$$

$$\boxed{I_B = 2.46\ \mu\text{A}}$$

$$\boxed{I_C = \beta I_B = 130(2.46\ \mu\text{A}) = 0.320\ \text{mA}}$$

$$\boxed{I_E = (\beta+1)I_B = 131(2.46\ \mu\text{A}) = 0.322\ \text{mA}}$$

Knowing the DC terminal currents, we can find the DC terminal voltages by again using KVL.

$$V_B = V_{EQ} - (R_2||R_4)I_B = 4.8\ \text{V} - (96\ \text{k}\Omega)(2.46\ \mu\text{A})$$

$$\boxed{V_B = 4.56\ \text{V}}$$

$$V_C = V_{CC} - R_C I_C = 12\ \text{V} - (15\ \text{k}\Omega)(0.320\ \text{mA})$$

$$\boxed{V_C = 7.20\ \text{V}}$$

$$V_E = V_B - V_{BE} = 4.56\ \text{V} - 0.7\ \text{V}$$

$$\boxed{V_E = 3.86\ \text{V}}$$

<u>Step 3</u>: Obtain the ac equivalent circuit by treating all capacitors as short circuits and all DC sources as short circuits.

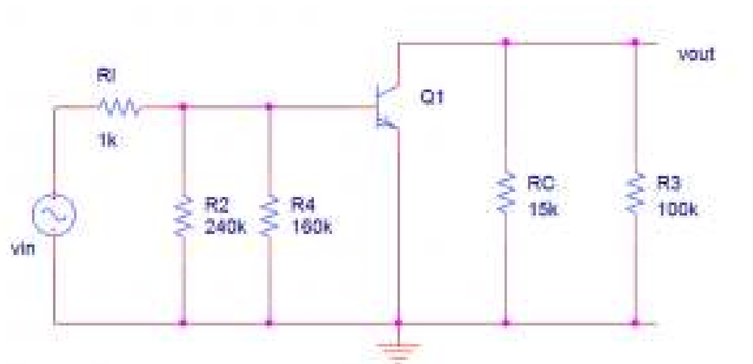

Figure 2S: The ac equivalent circuit.

<u>Step 4</u>: Obtain the Hybrid-Pi circuit by replacing the transistor with the Hybrid-Pi model. From analyzing this circuit, obtain the small-signal voltage gain.

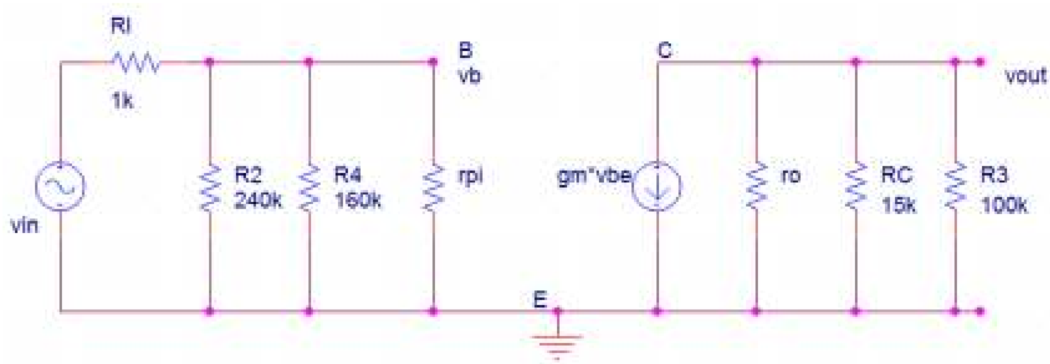

Figure 3S: The Hybrid-Pi equivalent circuit.

The small-signal parameters, as determined from the Q-point found in Step 2, are as follows.

$$g_m = \frac{I_C}{V_T} = \frac{0.320\ \text{mA}}{0.0259\ \text{V}} = 12.4\ \text{mS}$$
$$r_\pi = \frac{\beta}{g_m} = \frac{130}{12.4\ \text{mS}} = 10.5\ \text{k}\Omega$$
$$r_o = \frac{V_A + V_{CE}}{I_C} = \frac{60\ \text{V} + 7.20\ \text{V} - 3.86\ \text{V}}{0.320\ \text{mA}} = 198\ \text{k}\Omega$$

To determine $A_v = v_{out}/v_{in}$, we first relate v_{in} to v_{be} and then relate v_{be} to v_{out}.

$$v_{be} = \frac{R_2||R_4||r_\pi}{R_I + R_2||R_4||r_\pi} v_{in}$$
$$v_{out} = -g_m v_{be}(r_o||R_C||R_3) = -g_m(r_o||R_C||R_3)\frac{R_2||R_4||r_\pi}{R_I + R_2||R_4||r_\pi} v_{in}$$

$$A_v = \frac{v_{out}}{v_{in}} = -g_m(r_o||R_C||R_3)\frac{R_2||R_4||r_\pi}{R_I + R_2||R_4||r_\pi}$$
$$= -(12.4\ \text{mS})(12.92\ \text{k}\Omega)\left(\frac{9.46\ \text{k}\Omega}{1\ \text{k}\Omega + 9.46\ \text{k}\Omega}\right)$$
$$A_v = -137.3\ \text{V/V}$$

Note that the voltage gain is negative. This is because the current passing through r_o is moving upwards, while v_{out} is defined as the voltage drop across r_o in the opposite direction. Physically, this means that the output signal is 180° out of phase with the input signal.

c. Repeat part (b) with R_E = 80 kΩ.

Following the exact same procedures as part (b), the following results are obtained:

$$\boxed{I_B = 0.388\ \mu A}$$
$$\boxed{I_C = 50.4\ \mu A}$$
$$\boxed{I_E = 50.8\ \mu A}$$

$$\boxed{V_B = 4.76\ V}$$
$$\boxed{V_C = 11.24\ V}$$
$$\boxed{V_E = 4.06\ V}$$

$$g_m = 1.95\ mS$$
$$r_\pi = 66.8\ k\Omega$$
$$r_o = 1.33\ M\Omega$$

$$\boxed{A_v = -24.6\ V/V}$$

d. If the goal is to maximize the voltage gain, what general design rule for common-emitter amplifiers can you infer from the results of parts (b) and (c)?

In general, we want the emitter resistor to be as low as possible.

2) Purpose: Coupling diodes and BJTs.
In the circuit below, find the Q-point of both the Zener diode and BJT. Assume the BJT is biased in the forward-active regime, that $V_Z = 5$ V, $R_Z = 0\ \Omega$, $\beta = 100$, and $V_{BE} = 0.7$ V.

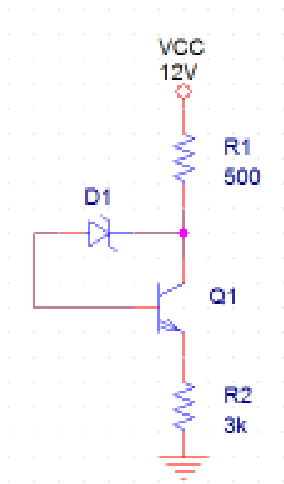

Figure 2: Zener diode with BJT.

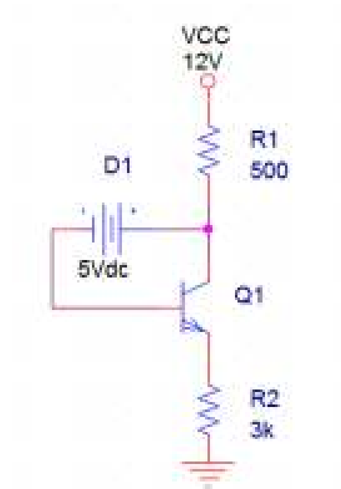

Figure 2S: Zener diode in breakdown regime, represented as a DC voltage source.

From inspection of the circuit, and if we are assuming the BJT is biased in the forward-active regime, it is safe to assume the Zener diode is operating in the breakdown regime. Therefore, we can replace it with a 5 V DC source, as seen in Figure 2S.

Starting KVL from the power supply,

$$\mathbf{12\ V} - (I_C + I_B)R_C - V_Z - V_{BE} - I_E R_E = 0$$
$$\mathbf{12\ V} - (\beta+1)I_B R_C - V_Z - V_{BE} - (\beta+1)I_B R_E = 0$$
$$I_B = \frac{12\ \text{V} - V_Z - V_{BE}}{(\beta+1)(R_C + R_E)} = \frac{6.3\ \text{V}}{(101)(3500\ \Omega)}$$
$$\boxed{I_B = 17.8\ \mu\text{A}}$$
$$\boxed{I_C = \beta I_B = 1.78\ \text{mA}}$$
$$\boxed{I_E = (\beta+1)I_B = 1.80\ \text{mA}}$$

$$V_C = 12\ \text{V} - I_E R_C = 12\ \text{V} - (1.80\ \text{mA})(500\ \Omega)$$
$$\boxed{V_C = 11.1\ \text{V}}$$
$$V_B = V_C - V_Z = 11.1\ \text{V} - 5\ \text{V}$$
$$\boxed{V_B = 6.1\ \text{V}}$$
$$V_E = V_B - V_{BE} = 6.1\ \text{V} - 0.7\ \text{V}$$
$$\boxed{V_E = 5.4\ \text{V}}$$

Obviously, from inspection, the current through the Zener diode is the current that goes directly into the base of the BJT. Therefore,

$$\boxed{I_Z = I_B = 17.8\ \mu A}$$

And, of course, given from the problem statement, $\boxed{V_Z = 5\ V}$

3. Purpose: BJT application in circuit designing

Assume forward active mode bias and identical BJTs Q_1 and Q_2 in the following "current mirror" circuit.

Given, R_2= 10kΩ, R_3= 1kΩ, R_7= 100Ω, R_8= 100Ω, β=416.4, and I_S = 6.73 fA.

(a) Find the current flowing in R_3 and compare it to the current flowing in R_2.
Note: it may be helpful to use Ebers Moll model only for determining collector currents in the two transistors, but otherwise use Beta/CVD model.

(b) What happens to the currents if R_3 is replaced with a 5kΩ resistor?

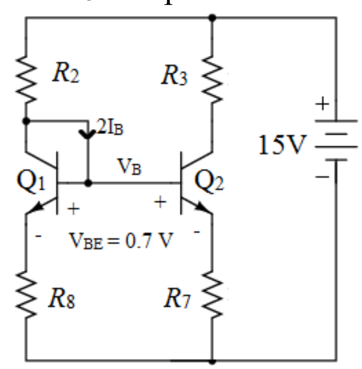

Figure 3. Current mirror circuit.

(a)

V_B is same for both Q_1 and Q_2 transistors, so is V_E. Also, V_{C1}=V_{B1}.
Since, $V_{E1} = V_{E2}$ and $R_8 = R_7$, we can calculate, I_{E1}=I_{E2}.
We know,

$$I_E = \left(\frac{\beta+1}{\beta}\right) I_C$$

$$I_B = \frac{I_C}{\beta}$$

Now,

$$15\text{ V} = I_{E1}R_8 + 0.7 + R_2(I_{C1} + 2I_B)$$

$$\Rightarrow I_{C1} = \frac{15 - 0.7}{R_2\left(\frac{2}{\beta}\right) + R_2 + \left(\frac{\beta+1}{\beta}\right)R_8} = 1.409\text{ mA}$$

So,

$$I_{E1} = \left(\frac{\beta+1}{\beta}\right) I_{C1} = 1.412\text{ mA}$$

Since, $V_{BE1} = V_{BE2}$ and $I_C = I_S e^{V_{BE}/V_T}$,

$$\boxed{I_{C1} = I_{C2} = 1.409\text{ mA}}$$

(b)
Nothing changes if R_3=5kΩ, since the current through collector (I_{C2}) is set by Q_1. The circuit acts as a dc current source as long as Q_1 and Q_2 remain forward active. For this to happen, I_C must not be too high. I_C are set by R_2 (primarily) and R_8+R_7. Also, for $I_{C1} = I_{C2}$, the choice of $R_8 = R_7$ was important.

Note, multiple current sources can be made from this configuration. For this configuration, common power supply is not required, only common ground is necessary.