ECE 3040 Homework 2

1) Purpose: Understanding what the fermi-distribution function is telling us.
The fermi-distribution function depicted in Figure 2.15 describes the probability that a state is occupied at a given energy. Plot the probability that a state is empty.
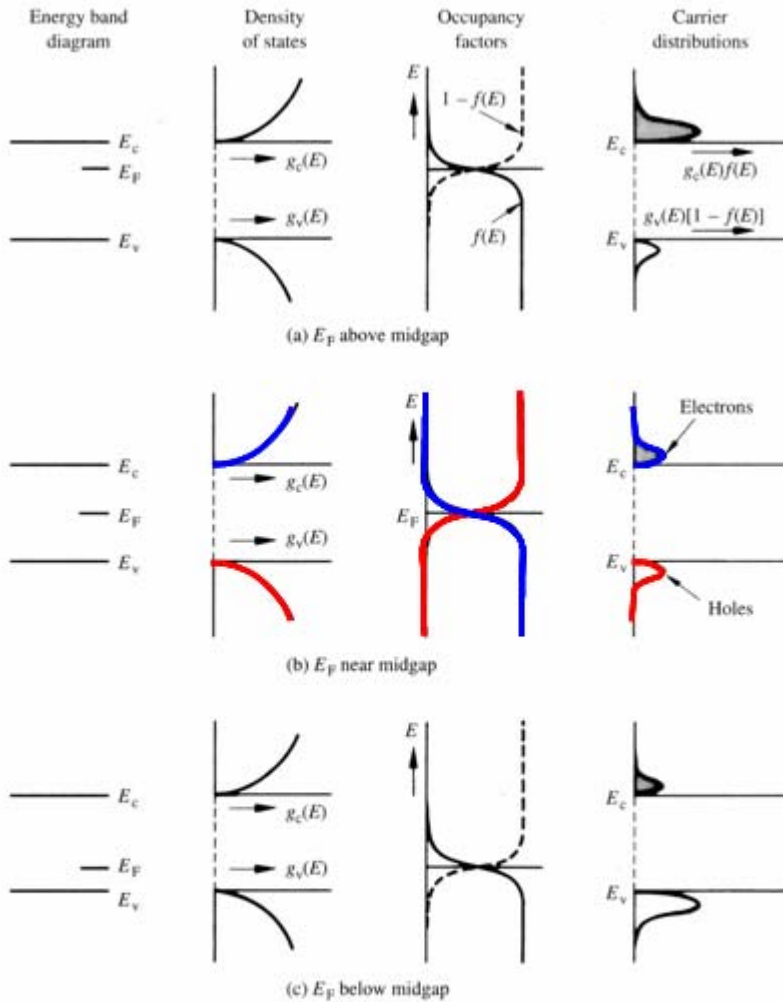This is 1-f(E) . See blue curve under occupancy factors below.

Figure 2.16 Carrier distributions (not drawn to scale) in the respective bands when the Fermi level is positioned (a) above midgap, (b) near midgap, and (c) below midgap. Also shown in each case are coordinated sketches of the energy band diagram, density of states, and the occupancy factors (the Fermi function and one minus the Fermi function).

2) Purpose: Understanding what the electron distribution is within each band - Problem Pierret 2.7

2.7

The distribution of electrons in the conduction band is given by $g_c(E)f(E)$; the distribution of holes in the valence band is given by $g_v(E)[1 - f(E)]$. Working with the electron distribution we note,

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}} \cong e^{-(E - E_F)/kT}$$

...for all $E \geq E_c$ if the semiconductor is nondegenerate

Thus

$$g_c(E)f(E) = \frac{m_n^* \sqrt{2m_n^*(E-E_c)}}{\pi^2 \hbar^3} e^{-(E-E_F)/kT}$$

$$= \kappa (E-E_c)^{1/2} e^{-(E-E_F)/kT} \qquad ...\kappa \equiv \frac{m_n^* \sqrt{2m_n^*}}{\pi^2 \hbar^3}$$

The extrema points of any function are obtained by taking the derivative of the function and setting the derivative equal to zero.

$$\frac{d}{dE}[g_c(E)f(E)] = \frac{\kappa}{2(E-E_c)^{1/2}} e^{-(E-E_F)/kT} - \frac{\kappa}{kT}(E-E_c)^{1/2} e^{-(E-E_F)/kT}$$

$$\text{set} = 0$$

Clearly

$$\frac{1}{2\sqrt{E_{peak}-E_c}} = \frac{\sqrt{E_{peak}-E_c}}{kT}$$

or

$$E_{peak} - E_c = kT/2$$

and

$$\boxed{E_{peak} = E_c + kT/2}$$ ...for electrons in the conduction band

The development leading to the peak energy of $E_{peak} = E_v - kT/2$ for holes in the valence band is completely analogous.

3) Purpose: Understanding special cases of doping - Problem Pierret 2.16

2.16

(a) As $T \rightarrow 0$, $n \rightarrow 0$ and $p \rightarrow 0$. (See the discussion in Subsection 2.5.7.)

(b) Since $N \gg n_i$, one would have

$$n = N_D \quad \text{and} \quad p = n_i^2/N_D \quad \text{...if a donor}$$

$$p = N_A \quad \text{and} \quad n = n_i^2/N_A \quad \text{...if an acceptor}$$

We are told $n = N$ and $p = n_i^2/N$. Clearly the impurity is a **donor**.

(c) Here we are given the minority carrier concentration, $n = 10^5/\text{cm}^3$. As long as the Si is nondegenerate, one can always write

$$np = n_i^2$$

Thus

$$p = n_i^2/n = \frac{(10^{10})^2}{10^5} = \mathbf{10^{15}/cm^3}$$

Note: From previous problems we recognize that the above carrier concentrations do indeed correspond to a nondegenerate semiconductor.

(d) Given $E_F - E_i = 0.259\text{eV}$ and $T = 300\text{K}$,

$$n = n_i \, e^{(E_F - E_i)/kT} = (10^{10}) \, e^{0.259/0.0259} = \mathbf{2.20 \times 10^{14}/cm^3}$$

$$p = n_i \, e^{(E_i - E_F)/kT} = (10^{10}) \, e^{-0.259/0.0259} = \mathbf{4.54 \times 10^5/cm^3}$$

(e) Employing the np product relationship,

$$np = n^2/2 = n_i^2$$

$$n = \sqrt{2}\,n_i = \mathbf{1.414 \times 10^{13}/cm^3}$$

Next employing the charge neutrality relationship,

$$p - n + N_D - N_A = n/2 - n + N_D = 0$$

$$N_D = n/2 = n_i/\sqrt{2} = \mathbf{0.707 \times 10^{13}/cm^3}$$

4) Purpose: Understanding the relationships between various energies - Problem Pierret 2.22

<u>2.22</u>

(a) Because $m_n^* \ll m_p^*$ in GaAs, the density of states in the conduction band is considerably smaller than the density of states in the valence band at a given energy displacement from the respective band edges. To be precise, using the effective mass values from Table 2.1,

$$\frac{g_c(E_c+\Delta E)}{g_v(E_v-\Delta E)} = \left(\frac{m_n^*}{m_p^*}\right)^{3/2} = \left(\frac{0.066}{0.52}\right)^{3/2} = 0.045$$

The required sketch is shown below.
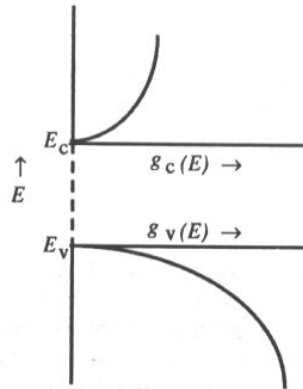
(b) E_i will lie **ABOVE** midgap. Because the density of states is smaller in the conduction band, the Fermi level must be displaced from the middle of the band gap toward the conduction band edge to achieve an equal number of filled states in the two bands.

(c) Employing $kT = 0.0259$ eV and the effective mass values from Table 2.1, we find

$$E_i = \frac{E_c + E_v}{2} + \frac{3}{4}kT \ln(m_p^*/m_n^*) \qquad \text{...Eq.(2.36)}$$

$$= \frac{E_c + E_v}{2} + \frac{3}{4}(0.0259)\ln(0.52/0.066)$$

or

$$E_i = \frac{E_c + E_v}{2} + 0.040 \text{ eV}$$

E_i lies **approximately 0.04 eV above midgap** in GaAs at room temperature. (Note that the displacement from midgap is considerably larger for GaAs compared to Si. However, the room temperature displacement of 0.04 eV is still only 0.04/1.42 = 2.8% of the total band gap.)

(d) Using the results of Problems 2.13 and 2.20, we know

$$N_C = 4.26 \times 10^{17}/cm^3$$

...GaAs

$$N_V = 9.41 \times 10^{18}/cm^3 \qquad \text{at 300K}$$

and

$$N_{DImax} = N_C \, e^{-3} = (4.26 \times 10^{17}) \, e^{-3} = 2.12 \times 10^{16}/cm^3$$

$$N_{AImax} = N_V \, e^{-3} = (9.41 \times 10^{18}) \, e^{-3} = 4.68 \times 10^{17}/cm^3$$

Please note that *n-type GaAs becomes degenerate at relatively low donor dopings.* This fact is very important in the modeling of certain GaAs devices; constructive use is made of this fact in other GaAs devices.

5) Purpose: Understanding of simple electron-hole concentration relationships.
Find the electron and hole concentrations as well as the fermi level and intrinsic energy positions for a silicon sample with the following conditions: (You may need data from tables 2.1 and 2.3 and assume **Total ionization** and a 27 C temperature)

a) 10^{16} cm^{-3} Al
Al is an acceptor, Si has n_i=1e10cm^{-3} , since N_a>>N_d and N_a>>n_i : p=Na=1e16 cm^{-3} and $n=n_i^2/p$=1e4 cm^{-3}

b) 9x10^{17} cm^{-3} As and 8.95x10^{17} cm^{-3} Al
Al is an acceptor, As is a donor, Si has n_i=1e10cm^{-3} , since N_a~N_d but (N_d - N_a)>>n_i : n=(N_d - N_a)=5e15 cm^{-3} and p=n_i^2/n=2e4 cm^{-3}
Note: here I used the reduction discussed in class of:

$$n = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2} \approx N_D - N_A \quad for \quad N_D - N_A \gg n_i$$

c) What is the majority carrier concentration for parts a, and b?
Majority carrier is holes for a) and electrons for b)

d) What is the minority carrier concentration for parts a, and b?
Minority carrier is electrons for a) and holes for b)

e) Is the material n-type or p-type in parts a, and b?
p-type for a) and n-type for b)

6) Purpose: Understanding of the partial ionization "real world" situation.
Find the electron and hole concentrations as well as the fermi level position for a silicon sample with the following conditions: (You may need data from tables 2.1 and 2.3, assume *Partial ionization* with the valence band edge, Ev, set equal to 0, and a 27 C temperature). For each case below, make a table including Ef guesses, each of the 4 terms in the partial ionization equation (free p, fixed ionized acceptors, free n and fixed ionized donors), and the left and right hand side values. Using a spreadsheet or math package would also be useful (and allowed) but be sure to show the work/codes.

a) 9×10^{17} cm^{-3} B and 8.95×10^{17} cm^{-3} Sb

b) 9×10^{17} cm^{-3} In and 8.95×10^{17} cm^{-3} Sb

c) Speculate why is B more commonly used as an acceptor than In?

Notes on problem 6: Given the partial ionization equation from lecture 6:

$$p - \frac{N_A}{1 + g_A \, e^{(E_A - E_f)/kT}} = n - \frac{N_D}{1 + g_D \, e^{(E_f - E_D)/kT}}$$

$$N_v e^{(E_v - E_f)/kT} - \frac{N_A}{1 + g_A \, e^{(E_A - E_f)/kT}} = N_c e^{(E_f - E_c)/kT} - \frac{N_D}{1 + g_D \, e^{(E_f - E_D)/kT}}$$

You will almost never have the residuals (the individual 4 terms in the partial ionization equation) equate exactly. Instead, one needs to assume a fermi energy and find the value of the left and right hand sides. If they do not equal (which they will almost never exactly equal), assume a new fermi energy. If the second guess you made is closer than the first guess, the two sides will approach each other. This indicates your guess is headed in the correct direction. Make a 3rd guess. In each round, if the guess results in left/right side terms that diverge from one another your guess is headed in the wrong direction. If the sign of the left/right sides switches, you passed over the correct value. It helps greatly to include a table of values to see the trends in your guesses.

Solution of each section a) and b) from an excel spreadsheet is:

Non-degenerate forms of n and p

For B and Sb

Ef=		Free Holes=	Ionized Acceptors=	free electrons=	Ionized Donors=	Left Side=	Right Side =	Left - Right=	Comment
a	0.56	6.472E+09	9.000E+17	1.138E+10	8.950E+17	-9.000E+17	-8.950E+17	-5.000E+15	
b	0.28	3.430E+14	8.996E+17	2.148E+05	8.950E+17	-8.993E+17	-8.950E+17	-4.267E+15	residual reduced no sign change
c	0.14	7.895E+16	8.183E+17	9.329E+02	8.950E+17	-7.394E+17	-8.950E+17	1.556E+17	residual increased and sign flip
d	0.21	5.204E+15	8.941E+17	1.415E+04	8.950E+17	-8.889E+17	-8.950E+17	6.086E+15	residual increased and sign flip from b
e	0.245	1.336E+15	8.985E+17	5.513E+04	8.950E+17	-8.971E+17	-8.950E+17	-2.147E+15	residual reduced no sign change from b
f	0.2275	2.637E+15	8.970E+17	2.794E+04	8.950E+17	-8.944E+17	-8.950E+17	6.267E+14	residual decreased and sign flip from b
g	0.230544763	2.342E+15	8.973E+17	3.144E+04	8.950E+17	-8.950E+17	-8.950E+17	0.000E+00	Exact answer (2 decimal places would be fine)

For In and Sb use a similar proceedure

Ef=	Free Holes=	Ionized Acceptors=	free electrons=	Ionized Donors=	Left Side=	Right Side =	Left - Right=	
0.329469511	5.019E+13	8.951E+17	1.468E+06	8.950E+17	-8.950E+17	-8.950E+17	0.000E+00	Note the fermi energy is dramatically different and the hole concentration varies by ~2 decades

Example of degenerate forms of n and p

Ef=	Free Holes=	Ionized Acceptors=	free electrons=	Ionized Donors=	Left Side=	Right Side =	Left-Right=
0.329469511	5.015E+13	8.951E+17	1.467E+06	8.950E+17	-8.950E+17	-8.950E+17	-3.327E+10

Note I have included for In and Sb a solution using the full Fermi-Dirac integral of order ½ just for comparison. Notice the very close agreement.

c) In has a deeper accepter and thus is not as efficient an acceptor.