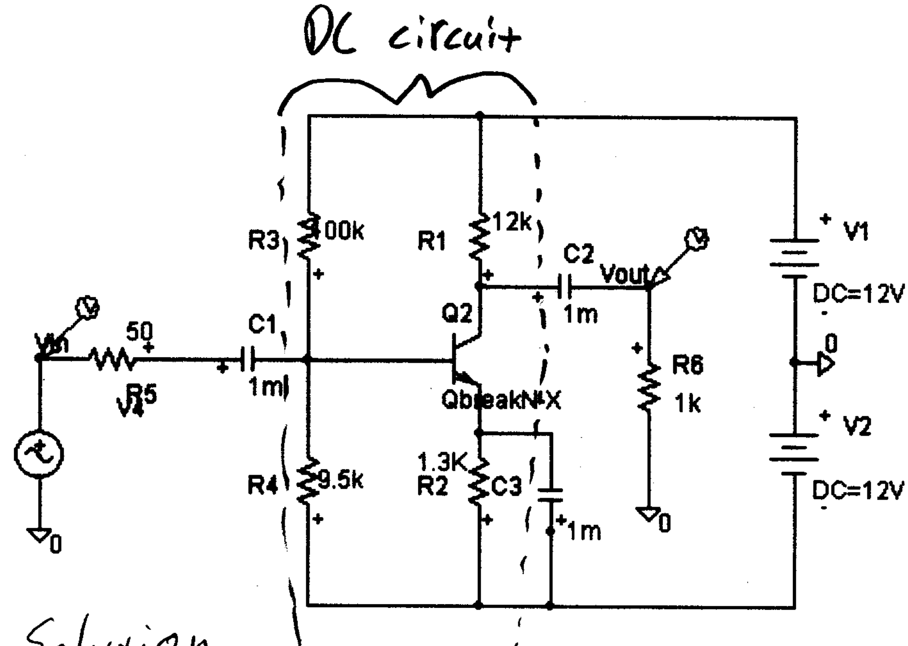
ECE 3040 Homework #6

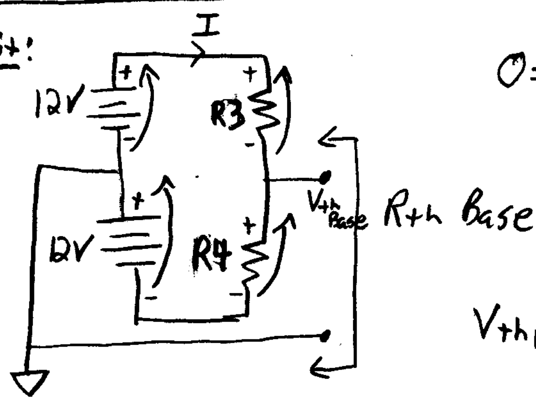
For each of the following three circuits, verify the mode of operation of the following circuit is forward active. Determine the Q-point and all relevant small signal parameters. Determine the small signal AC voltage gain A_v=Vout/Vin. What is the "worst-case" maximum voltage swing of the output based on the Collector bias voltage. (NOTE: the actual output voltage swing of this circuit may me smaller due to large input signals driving the transistor out of active mode and into saturation.) You may want to simulate this with PSPICE to get a feel for the circuit operation. Assume IS=1.83e-15 A (for PSPICE) to get a turn on voltage for 1 mA current as 0.7 V and β=100.

1.)



D.C. Solution

Base Circuit:



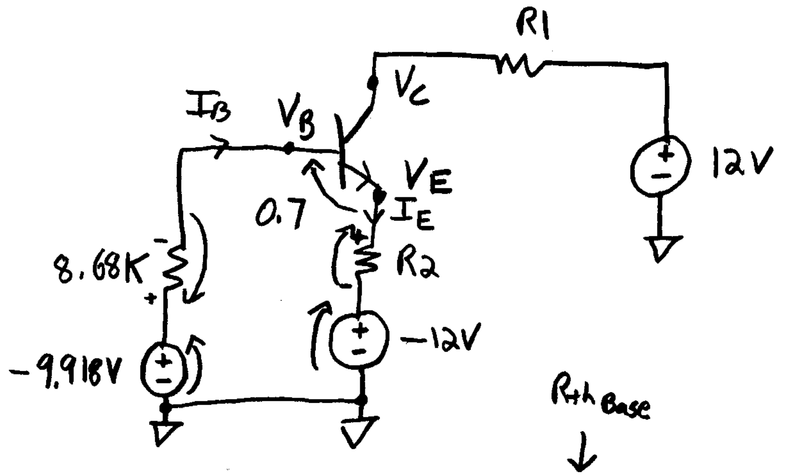
$$0 = 12V - IR_4 - IR_3 + 12V$$

$$I = \frac{24V}{R_4 + R_3} = \frac{24V}{109.5K}$$

$$I = 0.219\ mA$$

$$V_{th\,Base} = 12 - IR_3$$
$$= 12 - (0.219e{-3})\,100K$$
$$= -9.918\ V$$

$$R_{th\,Base} = R3 \,||\, R4 = 8.68K$$



$R_{th\,Base}$ (8.68K), 1.3K (R_2)

$$0 = +(-9.918) - 8.68K\, I_B - 0.7 - I_E R_2 - (-12V)$$

$$1.382 = (8.68k) I_B + (\beta+1) I_B (1.3k)$$

(β = 100)

$$I_B = 9.873\ \mu A$$

$$I_C = \beta I_B = 0.987\ mA$$

$$I_E = (\beta+1) I_B = 0.997\ mA$$

$$V_C = 12V - I_C R_1 = 12V - (0.987e{-3})\, 12k$$
$$= 0.156\ V$$

$$V_E = -12V + I_E R_2 = -12 + (0.997e{-3})\, 1.3K$$
$$= -10.7\ V$$

$$V_B = -9.918 - I_B (8.68k)$$
$$= -10\ V$$

$$V_{BE} = V_B - V_E = 0.7 \Rightarrow \text{Forward biased}$$

$$V_{BC} = V_B - V_E = -10.156\ V \Rightarrow \text{Reverse biased}$$

Active mode Verified

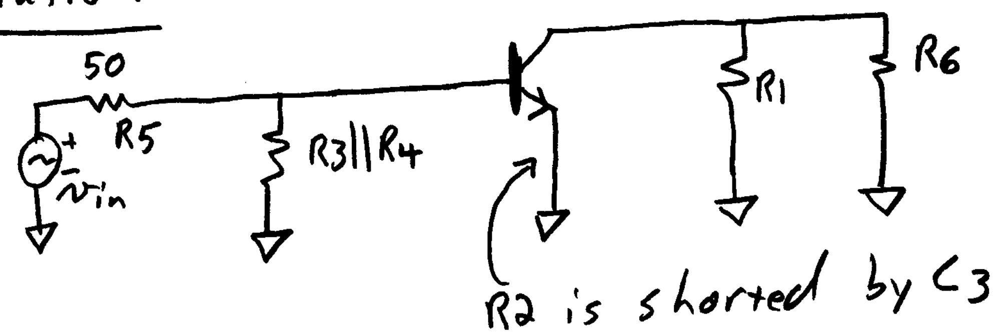
$$g_m = \frac{I_C}{V_T} = \frac{(0.987e{-3})}{0.0259} = 0.0381\ (A/V)$$

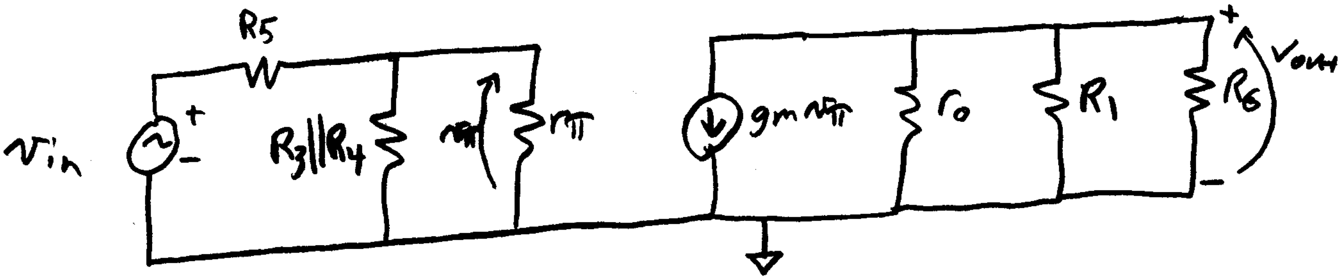
$$r_\pi = \frac{\beta}{g_m} = \frac{100}{0.0381} = 2.62\ K\Omega$$

$$r_o = \frac{V_A + V_{CE}}{I_C} = \frac{75 + 10.856}{(0.987e{-3})} = 87K\Omega$$

A.C. Solution

Note: With practice this step can be skipped →





$$\frac{v_{out}}{v_{in}} = \frac{v_\pi}{v_{in}} \frac{v_{out}}{v_\pi}$$

$$\frac{v_\pi}{v_{in}} = \frac{R_3 \| R_4 \| r_\pi}{R_5 + R_3 \| R_4 \| r_\pi}$$

$$v_{out} = (-g_m v_\pi)\, r_o \| R_1 \| R_6$$

$$\frac{v_{out}}{v_\pi} = -g_m \; r_o \| R_1 \| R_6$$

$$\frac{v_{out}}{v_{in}} = -\frac{R_3 \| R_4 \| r_\pi}{R_5 + R_3 \| R_4 \| r_\pi} \left(g_m \left(r_o \| R_1 \| R_6 \right) \right)$$

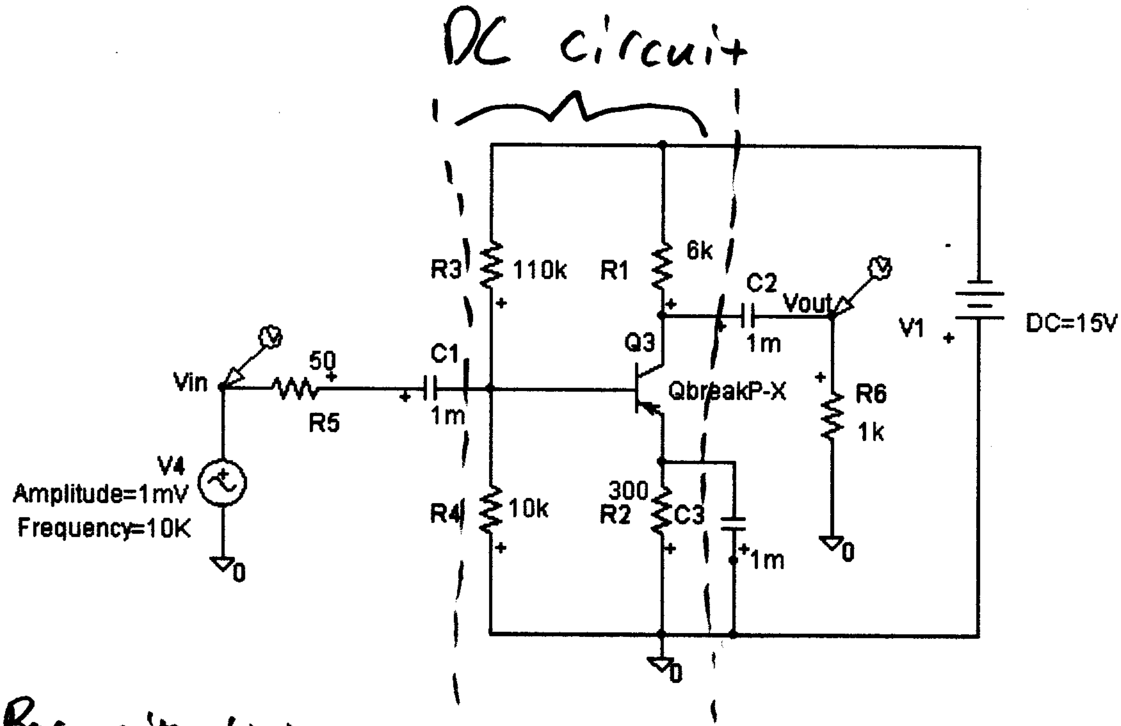
Plugging in #s,

$$\boxed{\frac{v_{out}}{v_{in}} = -34 \; V/V}$$

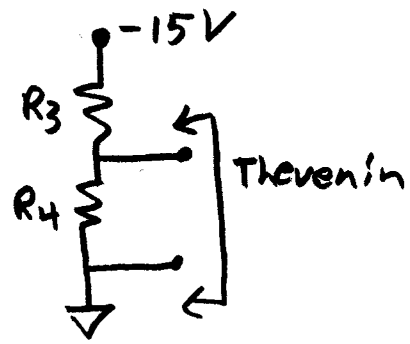
⇒ **Note:** gain is limited by moderately small value of Load resistance, R6

Since $V_C = 0.156$ V, the maximum swing would be
$12V - 0.156 = 11.844$ V.

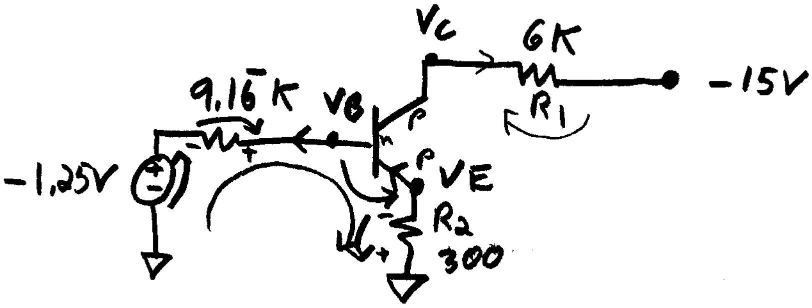
2.)



Base circuit:



$$V_{th_{Base}} = -15 \frac{R_4}{R_3+R_4}$$
$$= -1.25\,V$$
$$R_{th_{Base}} = R_3 \| R_4$$
$$= 9.1\bar{6}\,K$$

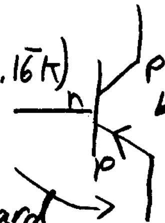


$$0 = +(-1.25) + I_B(9.1\bar{6}k) + 0.7 + I_E\,300$$
$$0.55 = I_B\left(9.1\bar{6}K + (\beta+1)300\right)$$
(β = 100)

$$I_B = 13.9\,\mu A$$
$$I_C = 1.39\,mA = \beta I_B$$
$$I_E = 1.41\,mA = (\beta+1) I_B = \left(\frac{\beta+1}{\beta}\right) I_C$$
$$g_m = \frac{I_C}{V_T} = 0.054\,S \qquad r_\pi = \frac{\beta}{g_m} = 1863\,\Omega$$

$V_C = -15 + I_C 6K$
$= -6.66\ V$

$V_B = -1.25 + I_B(9.16K)$
$= -1.12\ V$



Reverse Bias

⟹ Forward Biased

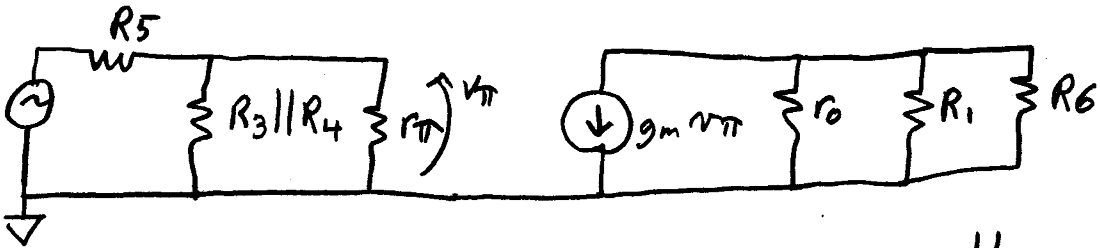
$V_E = -I_E(300)$
$= -0.423\ V$

⟹ Forward Active

$V_{EC} = -0.423 - (-6.66)$
$= 6.237\ V$

$$r_o = \frac{V_A + V_{EC}}{I_C} = \frac{75 + 6.237}{1.39e{-3}} = 58.4\,K$$

A.C. Solution:



The a.c. solution is the same as in problem #1,

⋮

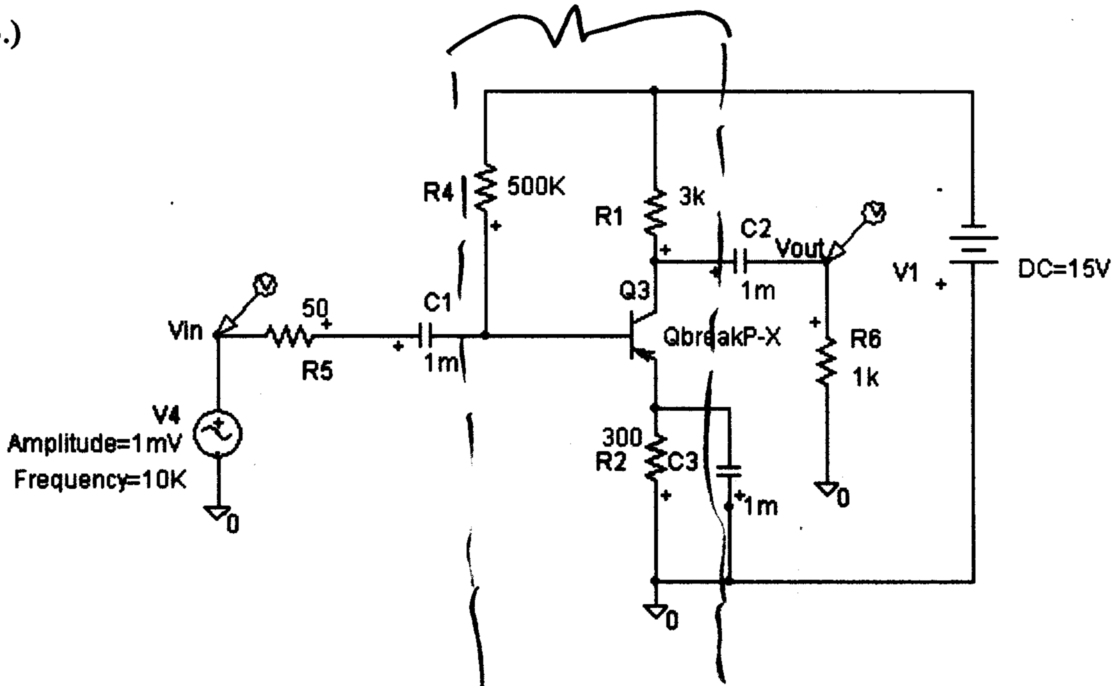
$$\frac{v_{out}}{v_{in}} = -\frac{R_3||R_4||r_\pi}{R_5 + R_3||R_4||r_\pi}\left(g_m\left(r_o||R_1||R_6\right)\right)$$

$$\boxed{\frac{v_{out}}{v_{in}} = -44.2\ V/V}$$

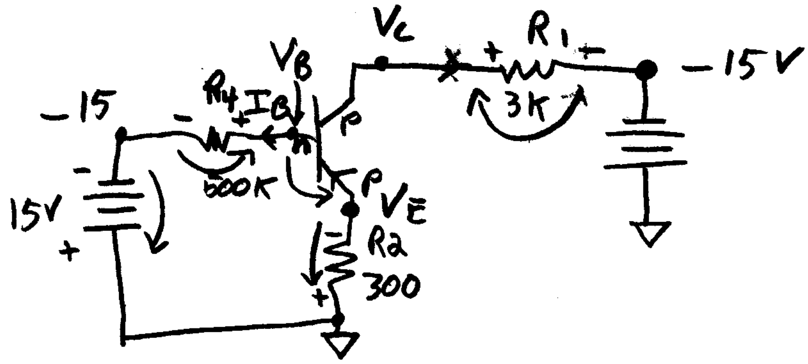
Since $V_C = -6.66\,V$ the maximum output voltage swing would be **6.66 V** (Note: $|-6.66V|$ is less than $|-15-(-6.66)| = 8.34V$)

DC Circuit

3.)



D.C. solution: Much easier,



$$0 = (+15V) + I_B R_4 + 0.7 + I_E R_2$$

$$14.3 = I_B \left(R_4 + (\beta+1) R_2\right)$$

$$= I_B \left(500K + (101)300\right)$$

$$I_B = 26.9\ \mu A$$

$$I_C = 2.69\ mA \quad (= \beta I_B)$$

$$I_E = 2.72\ mA \quad = \left(\frac{\beta+1}{\beta}\right) I_C$$

$V_C = -15V + I_C R_1 = -6.93V$

$V_E = -I_E R_2 \simeq -0.816\ V$

$V_B = -15 + I_B R_4 = -1.55V$

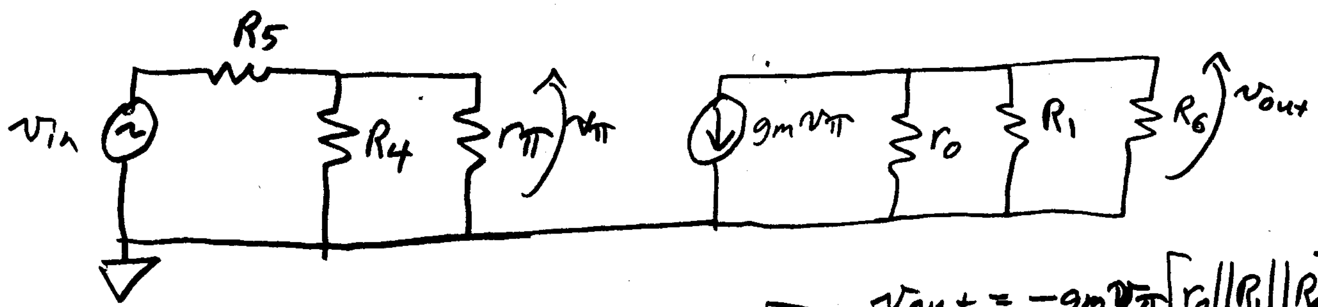
$\Rightarrow$ Forward active confirmed

$$g_m = \frac{I_c}{V_T} = \frac{2.69e-3}{0.0259} = 0.104 \ S$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{\quad} = 963 \ \Omega$$

$$r_o = \frac{V_A + V_{EC}}{I_c} = \frac{75 + 6.114}{(2.69e-3)} = 30.2 \ K\Omega$$

A.C. Solution:



$$v_{out} = -g_m v_\pi \left[r_o || R_1 || R_6 \right]$$

$$\frac{v_{out}}{v_{in}} = \frac{v_\pi}{v_{in}} \frac{v_{out}}{v_\pi}$$

$$= \left(\frac{R_4 || r_\pi}{R_5 + R_4 || r_\pi} \right) \left(-g_m \left(r_o || R_1 || R_6 \right) \right)$$

$$\boxed{\frac{v_{out}}{v_{in}} = -72.3 \quad V/V}$$

Since $V_c = -6.93V$ the maximum output voltage swing would be $6.93V$

This part of the Solution was not required to receive credit.

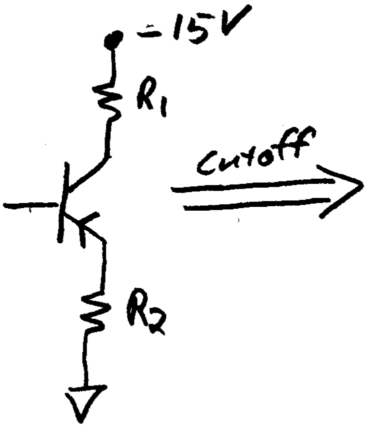
Notes about the maximum output voltage swing:

The "worst case" would be the smaller of $|+V_{CC} - V_C|$ or $|V_{EE} - V_C|$.

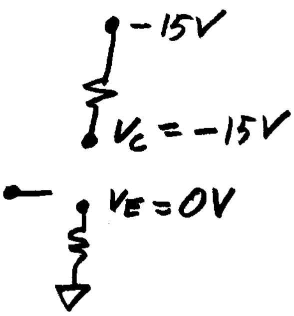
The actual maximum voltage swing is found by ~~solving~~ the circuit for the onset of saturation and again for the onset of cutoff and comparing the collector voltage calculated to that at DC.

Consider problem #3:

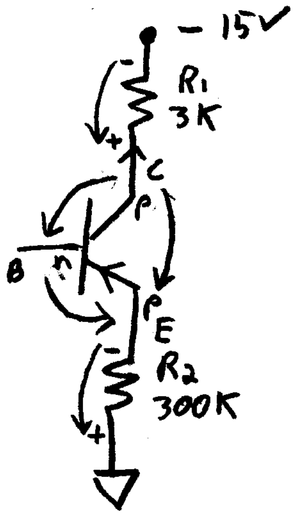
Cutoff:



Implies almost no current, so we can take the transistor out of the circuit (i.e. open circuit)



for onset of saturation:



$V_{EB} \sim 0.7V$

$V_{BC} \sim 0V$

$\Downarrow$

$V_{EC} = 0.7V$

$0 = -15V + I_c R_1 + V_{EC} + I_E R_2$

$14.3V = I_c \left(R_1 + \left(\frac{\beta+1}{\beta} \right) R_2 \right)$

↑

This is only valid because we are at the boundary between saturation and forward active. If we were well into saturation (B-C forward biased) we could not use this.

$$I_c = \frac{14.3V}{3k + \frac{101}{100} 300}$$

$I_c = 4.33 mA$

$V_c = -15V + I_c R_1$

$V_c = -2.01 V$

So the actual voltage swing would be assymetric around $V_c = -6.93V$

Largesignal

$v_C(t)$