Lecture 3

Wave Properties

Reading:

Notes

Properties of Waves

Classical Mechanics describes the dynamical* state variables of a particle as x, y, z, p, etc...

Quantum Mechanics takes a different approach. QM describes the state of any particle by an abstract "Wave Function", $\Psi(x, y ,z ,t)$, we will describe in more detail later.

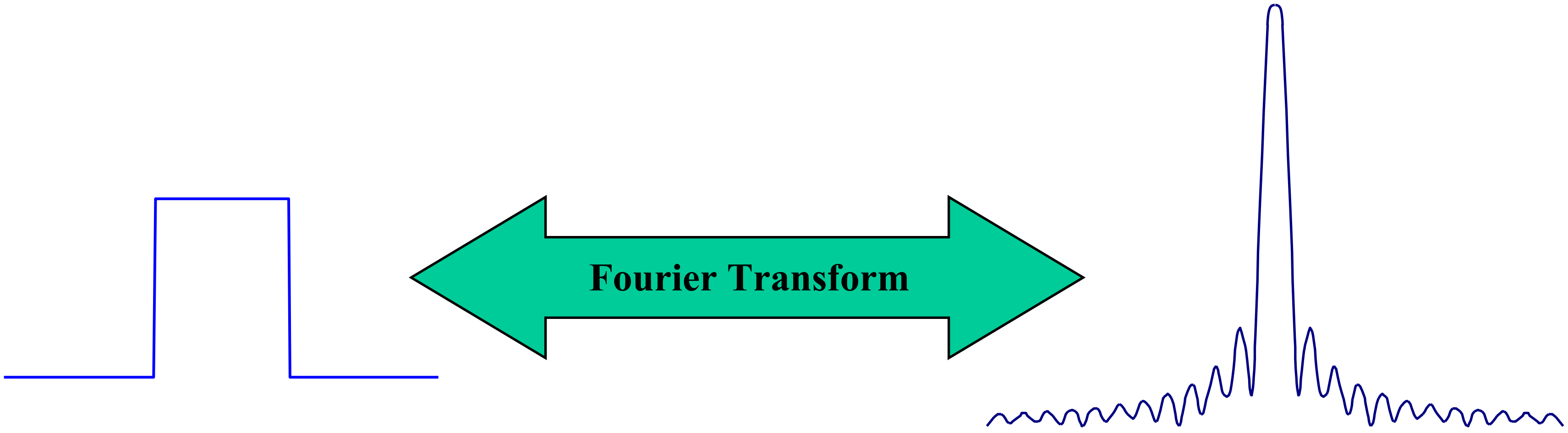
Thus, we will review some properties of waves.

***There are also classical static variable such as mass, electronic charge, etc... that do not change during physical processes.**

Properties of Waves

As waves are important in Quantum Mechanics, it is worth re-examining some properties of waves.

1) **Generality of Waves and Superposition: Any complex wave or shape of any kind can be decomposed into a set of orthogonal plane waves using a Fourier Series. (Same as in signal processing).**



Properties of Waves

2) Phase Velocity: Given a wave of the form,

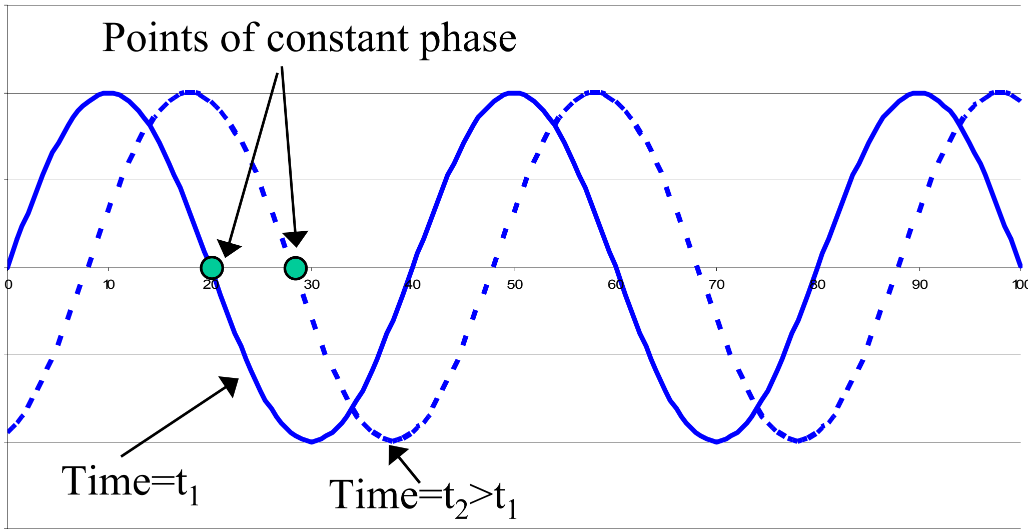
$$\Psi(x,t) = Ae^{i(kx-\omega t)}$$

Points of constant phase can be found by the following relationship:

$$kx - \varpi\, t = \text{constant}$$

Thus, we can differentiate this equation to find the phase velocity (speed of propagation) of the wave

$$v_{phase} = \frac{dx}{dt} = \frac{\varpi}{k}$$



Properties of Waves

3) **Group Velocity: Any combination of waves can result in a complex wave through superposition. This complex wave moves in space.**

$$\Psi(x,t) = A\int_{-\infty}^{\infty} g(k) e^{i(kx-\omega t)} dk$$

Consider a simple sum of two cosine functions:

$$\Psi(x,t) = A\cos(k_1 x - \varpi_1 t) + B\cos(k_2 x - \varpi_2 t)$$

Defining ω as the average of ω_1 and ω_2

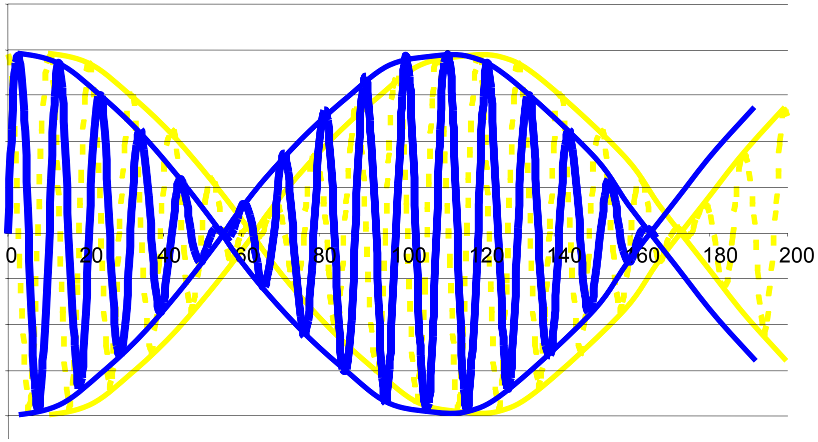
and k as the average of k_1 and k_2,

$\omega_1 = \omega - \Delta\omega$ and $\omega_2 = \omega + \Delta\omega$

$k_1 = k - \Delta k$ and $k_2 = k + \Delta k$

$$\Psi(x,t) = 2A\underbrace{\cos(\varpi t - kx)}_{\text{High Frequency "Carrier"}}\underbrace{\cos((\Delta\varpi)t - (\Delta k)x)}_{\text{Low Frequency "Signal"}}$$

High Frequency "Carrier" — Low Frequency "Signal"



Addition of two waves of slightly different frequency

If ω_1 and ω_2 are similar, $\Delta\omega \ll \omega$ and thus the slowly varying group (relative to the quickly varying "carrier") has a "group velocity" of:

$$v_{Group} = \frac{\Delta\varpi}{\Delta k} \Rightarrow \text{and in the infinitesmal limit} \Rightarrow \boxed{v_{Group} = \frac{d\varpi}{dk}}$$